Verifying pattern matching with guards in Scala

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Outline

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.

[1] The Scala Experiment – Can We Provide Better Language Support for Component Systems? http://lamp.epfl.ch/~odersky/talks/google06.pdf

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.
- It removes some of the more arcane constructs of these environments and adds instead:
  1. a uniform object model
  2. pattern matching and higher-order functions
  3. novel ways to abstract and compose programs

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Algebraic Data Types in Scala

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- In Scala:

abstract class Tree

case class Node (left: Tree, value: Int, right: Tree) **extends** Tree

case object EmptyTree **extends** Tree

Pattern matching in Scala

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```
def search(tree: Tree, value: Int): Boolean = tree match {
    case EmptyTree ⇒ false
    case Node(_,v,_) if(v == value) ⇒ true
    case Node(l,v,_) if(v < value) ⇒ search(l,v)
    case Node(_,v,r) if(v > value) ⇒ search(r,v)
    case _ ⇒ throw new Exception("...")
}
```

Pattern matching in Scala - cont'd

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- use wildcards

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Enforcement of these properties varies among languages.

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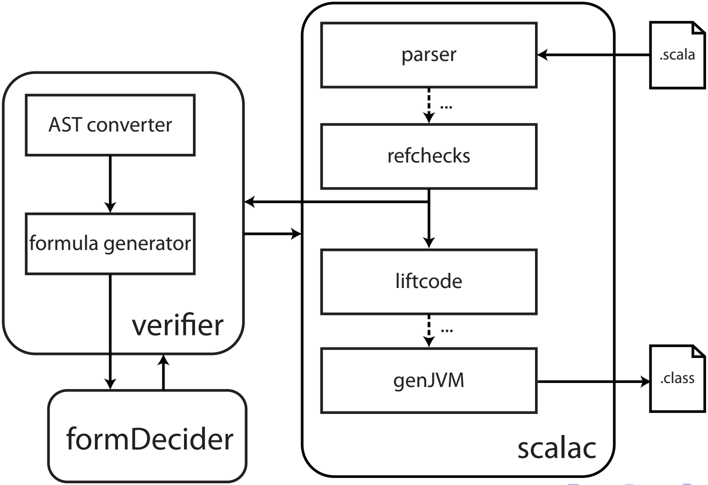
There is room for improvements using formal verification techniques.

Extending the Scala compiler

1. Analysis is implemented as an additional phase in the compiler.
2. Pattern matching subtrees and the related hierarchy are retrieved from the compiler environment and AST.
3. This information is used to generate an intermediate representation.
4. From there, formulas are constructed and fed to formDecider.
5. Based on the results, warning/error messages are sent back to the compiler.

The big picture



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  - define completeness and disjointness
    - what axioms do we need?
    - how do formulas relate to each other?

Formalizing completeness and disjointness

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t **match** $\{$

  case $p_1 \Rightarrow \ldots$

  $\ldots$

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- E is complete $\iff \bigvee_i \xi(t, p_i)$
- E is disjoint $\iff \forall i, j,\ i \neq j \implies \neg(\xi(t, p_i) \wedge \xi(t, p_j))$

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Properties of ADT are used to generate axioms

- $\forall t \in Tree, t \in Node(...) \oplus t \in EmptyTree$

Formalizing patterns – cont'd

Objects are represented as singletons

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  $\forall n \in Node\ (\Psi_{\text{Node,left}}(n) \in Tree \wedge \Psi_{\text{Node,right}} \in Tree)$

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The above transformations, along with the information about the selector's type, define *axioms* about E.

Example – Axioms

```
abstract class Tree
case class Node(left:Tree,right:Tree) extends Tree
case object Leaf extends Tree
```

```
t: Tree match { ... }
```

$$
\begin{aligned}
& t \in Tree \\
& \quad \wedge Node \subseteq Tree \wedge Leaf \subseteq Tree \wedge Leaf = \{leaf_0\} \\
& \quad \wedge \forall t_0 \in Tree, t_0 \in Node(...) \oplus t_0 \in Leaf \\
& \quad \wedge \forall n \in Node\ (\Psi_{\text{Node,left}}(n) \in Tree \wedge \Psi_{\text{Node,right}} \in Tree)
\end{aligned}
$$

Axioms – cont'd

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- Each of these formulas is in the form $A(t) \implies \Pi(p_i)$, where $A(t)$ are the axioms previously mentioned, and $\Pi(p_i)$ a formula depending on p_i.

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Simplified, this becomes: $A(t) \implies \bigvee_i \Pi(p_i)$

Translation of patterns

The "root" type in the pattern is assigned to the selector

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[2] the practical implementation slightly differs when proving completeness

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Aliases[2] are bound to fresh names

- **case** Node(left: Node, ...) ⇒ ...
  $\longmapsto left_{\text{fresh}} = \Psi_{\text{Node,left}}(t) \wedge left_{\text{fresh}} \in Node$

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Wildcards generate no constraints

- **case** _ ⇒ ... $\longmapsto true$

[2] the practical implementation slightly differs when proving completeness

Translation of patterns – cont'd

Guards are, to some extent, translated to formulas:

- equality and arithmetic operators are kept "as it"
- `equals` is always considered side-effect free
- dynamic type tests are converted to set membership
  - `o.isInstanceOf[`*Type*`]` $\longmapsto o \in \mathit{Type}$
- other method calls are ignored

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The result of the transformation is a predicate, whose parameters are the selector and the aliases defined in the pattern.

It is added as a conjunction to the main formula.

Matching on lists

Scala, as a language making an extensive use of lists, has a dedicated syntax for them:

z **match** {
    case Nil ⇒ . . .
    case x :: xs ⇒ . . .
}

. . . but this is essentially syntactic sugar for the following hierarchy:

sealed abstract class List
case final class ::(List, List) **extends** List
case object Nil **extends** List

Implementation status

- large supported subset of Scala pattern matching expressions
- generation of formulas for completeness and disjointness
- integration with formDecider
- scalac integration under way ...

Future work

Some issues we want to address in the future:

- . . . complete scalac integration :)
- allow matching on string constants
- improve support for primitive types
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- implement limited support for external variables and functions
- . . . oh, well, you always find something to do

Questions ?

One for the road...

```
sealed abstract class Arith
case class Sum(l: Arith, r: Arith) extends Arith
case class Prod(n: Num, f: Arith) extends Arith
case class Num(n: Int) extends Arith

def eval(a: Arith): Int = (a: @verified) match {
    case Sum(l, r) => eval(l) + eval(r)
    case Prod(Num(n), f) if(n == 0) => 0
    case Prod(Num(n), f) if(n != 0) => n * eval(f)
    case Num(n) => n
}
```

$$a \in Arith \wedge Sum \subseteq Arith \wedge Prod \subseteq Arith \wedge Num \subseteq Arith$$
$$\wedge \forall a_0 \in Arith, ((a_0 \in Sum \oplus a_0 \in Prod) \wedge (a_0 \in Sum \oplus a_0 \in Num)$$
$$\wedge (a_0 \in Prod \oplus a_0 \in Num)) \wedge \forall s_0 \in Sum, (\Psi_{\text{Sum,l}}(s_0) \in Arith$$
$$\wedge \Psi_{\text{Sum,r}}(s_0) \in Arith) \wedge \forall p_0 \in Prod, (\Psi_{\text{Prod,n}}(p_0) \in Num$$
$$\wedge \Psi_{\text{Prod,f}}(s_0) \in Arith) \wedge \forall n_0 \in Num, \Psi_{\text{Num,n}}(n_0) \in \mathbb{N}$$

$$\Longrightarrow$$

$$((l_{\text{fresh}} = \Psi_{\text{Sum,l}}(a) \wedge r_{\text{fresh}} = \Psi_{\text{Sum,r}}(a)) \implies a \in Sum)$$
$$\vee((f_{\text{fresh}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod$$
$$\wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh}} = 0)$$
$$\vee((f_{\text{fresh'}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh'}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod$$
$$\wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh'}} \neq 0)$$
$$\vee(n_{\text{fresh''}} = \Psi_{\text{Num,n}}(a) \implies a \in Num)$$