Verifying pattern matching with guards in Scala

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Outline

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.

[1] The Scala Experiment – Can We Provide Better Language Support for Component Systems? http://lamp.epfl.ch/~odersky/talks/google06.pdf

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.
- It removes some of the more arcane constructs of these environments and adds instead:
  1. a uniform object model
  2. pattern matching and higher-order functions
  3. novel ways to abstract and compose programs

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Algebraic Data Types in Scala

- Consider the following ADT definition:

$$\begin{aligned}\textbf{type}\ \mathsf{Tree} = {} & \mathsf{Node\ of\ Tree} * \mathsf{int} * \mathsf{Tree} \\ \mid {} & \mathsf{EmptyTree}\end{aligned}$$

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- In Scala:

abstract class Tree

case class Node (left: Tree, value: Int, right: Tree) **extends** Tree

case object EmptyTree **extends** Tree

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def search(tree: Tree, value: Int): Boolean = tree **match** {
  case EmptyTree ⇒ **false**
  case Node(_,v,_) **if**(v == value) ⇒ **true**
  case Node(l,v,_) **if**(v < value) ⇒ search(l,v)
  case Node(_,v,r) **if**(v > value) ⇒ search(r,v)
  case _ ⇒ **throw new** Exception("...")
}

back

Pattern matching in Scala - cont'd

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- use wildcards

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Enforcement of these properties varies among languages.

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- unreachable patterns are forbidden

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There is room for improvements using formal verification techniques.

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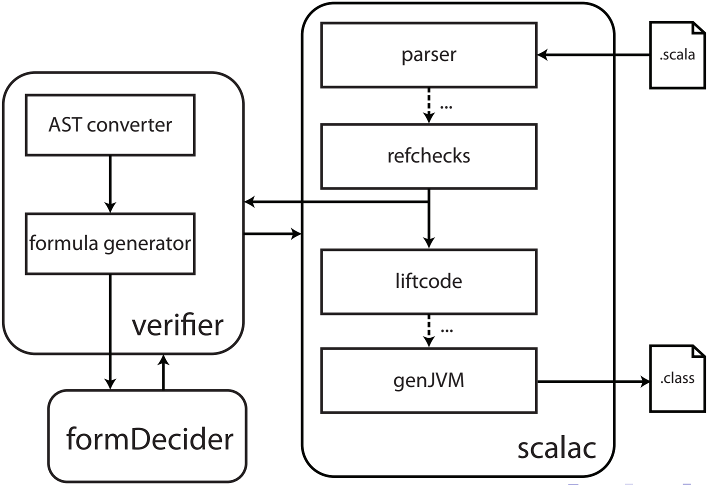
4. From there, formulas are constructed and fed to formDecider.
5. Based on the results, warning/error messages are sent back to the compiler.

The big picture



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    - how about primitive types? and strings?
  - define completeness and disjointness
    - what axioms do we need?
    - how do formulas relate to each other?

Formalizing completeness and disjointness

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t **match** $\{$

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- E is complete $\iff \bigvee_i \xi(t, p_i)$
- E is disjoint $\iff \forall i, j,\ i \neq j \implies \neg(\xi(t, p_i) \wedge \xi(t, p_j))$

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Properties of ADT are used to generate axioms

- $\forall t \in Tree, t \in Node(\ldots) \oplus t \in EmptyTree$

Formalizing patterns – cont'd

Objects are represented as singletons

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The above transformations, along with the information about the selector's type, define *axioms* about E.

Example – Axioms

abstract class Tree
case class Node(left:Tree,right:Tree) **extends** Tree
case object Leaf **extends** Tree

t: Tree **match** { . . . }

$$
\begin{aligned}
& t \in Tree \\
& \quad \wedge\ Node \subseteq Tree \wedge Leaf \subseteq Tree \wedge Leaf = \{leaf_0\} \\
& \quad \wedge\ \forall t_0 \in Tree,\ t_0 \in Node(...) \oplus t_0 \in Leaf \\
& \quad \wedge\ \forall n \in Node\ (\Psi_{\text{Node,left}}(n) \in Tree \wedge \Psi_{\text{Node,right}} \in Tree)
\end{aligned}
$$

Axioms – cont'd

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- Each of these formulas is in the form $A(t) \implies \Pi(p_i)$, where $A(t)$ are the axioms previously mentioned, and $\Pi(p_i)$ a formula depending on p_i.

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- The formula for completeness $\bigvee_i \xi(t, p_i)$ hence becomes $\bigvee_i (A(t) \implies \Pi(p_i))$

Simplified, this becomes: $A(t) \implies \bigvee_i \Pi(p_i)$

Translation of patterns

The “root” type in the pattern is assigned to the selector

- t **match** { **case** Node(...) $\Rightarrow$ ...} $\longmapsto t \in \mathit{Node}$

[2] the practical implementation slightly differs when proving completeness

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Aliases[2] are bound to fresh names

- **case** Node(left: Node, ...) ⇒ ...
  $\longmapsto \mathit{left}_{\text{fresh}} = \Psi_{\text{Node,left}}(t) \wedge \mathit{left}_{\text{fresh}} \in \mathit{Node}$

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  $\longmapsto left_{\text{fresh}} = \Psi_{\text{Node,left}}(t) \wedge left_{\text{fresh}} \in Node$

Wildcards generate no constraints

- **case** _ ⇒ ... $\longmapsto true$

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Translation of patterns – cont'd

Guards are, to some extent, translated to formulas:

- equality and arithmetic operators are kept "as it"
- `equals` is always considered side-effect free
- dynamic type tests are converted to set membership
  - `o.isInstanceOf[`*Type*`]` $\longmapsto o \in Type$
- other method calls are ignored

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The result of the transformation is a predicate, whose parameters are the selector and the aliases defined in the pattern.

It is added as a conjunction to the main formula.

Matching on lists

Scala, as a language making an extensive use of lists, has a dedicated syntax for them:

z match {
    case Nil ⇒ . . .
    case x :: xs ⇒ . . .
}

. . . but this is essentially syntactic sugar for the following hierarchy:

sealed abstract class List
case final class ::(List, List) **extends** List
case object Nil **extends** List

Implementation status

- large supported subset of Scala pattern matching expressions
- generation of formulas for completeness and disjointness
- integration with formDecider
- scalac integration under way . . .

Future work

Some issues we want to address in the future:

- . . . complete scalac integration :)
- allow matching on string constants
- improve support for primitive types
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- implement limited support for external variables and functions
- . . . oh, well, you always find something to do

Questions ?

One for the road...

```
sealed abstract class Arith
case class Sum(l: Arith, r: Arith) extends Arith
case class Prod(n: Num, f: Arith) extends Arith
case class Num(n: Int) extends Arith

def eval(a: Arith): Int = (a: @verified) match {
    case Sum(l, r) => eval(l) + eval(r)
    case Prod(Num(n), f) if(n == 0) => 0
    case Prod(Num(n), f) if(n != 0) => n * eval(f)
    case Num(n) => n
}
```

$$a \in Arith \wedge Sum \subseteq Arith \wedge Prod \subseteq Arith \wedge Num \subseteq Arith$$
$$\wedge \forall a_0 \in Arith, ((a_0 \in Sum \oplus a_0 \in Prod) \wedge (a_0 \in Sum \oplus a_0 \in Num)$$
$$\wedge (a_0 \in Prod \oplus a_0 \in Num)) \wedge \forall s_0 \in Sum, (\Psi_{\text{Sum,l}}(s_0) \in Arith$$
$$\wedge \Psi_{\text{Sum,r}}(s_0) \in Arith) \wedge \forall p_0 \in Prod, (\Psi_{\text{Prod,n}}(p_0) \in Num$$
$$\wedge \Psi_{\text{Prod,f}}(s_0) \in Arith) \wedge \forall n_0 \in Num, \Psi_{\text{Num,n}}(n_0) \in \mathbb{N}$$
$$\Longrightarrow$$
$$((l_{\text{fresh}} = \Psi_{\text{Sum,l}}(a) \wedge r_{\text{fresh}} = \Psi_{\text{Sum,r}}(a)) \implies a \in Sum)$$
$$\vee((f_{\text{fresh}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod$$
$$\wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh}} = 0)$$
$$\vee((f_{\text{fresh'}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh'}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod$$
$$\wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh'}} \neq 0)$$
$$\vee(n_{\text{fresh''}} = \Psi_{\text{Num,n}}(a) \implies a \in Num)$$