Homework related to Parikh's theorem

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We use the same notation as in the lecture's slides.

1 Problem 1

Parikh's theorem can be stated in the following two ways:

1. Any context-free language is letter-equivalent to a regular language.
2. The Parikh image of any context-free language is a semilinear set.

Prove that 1 is equivalent to 2.

2 Problem 2

The set of states of a k-Parikh automaton is defined as follows:

$$Q = \{(x_1, \ldots, x_n) \in \mathbb{N}^n | \sum_{i=1}^{n} x_i \leq k\}$$

Prove that $|Q| = \binom{n+k}{k}$ and that $|Q| = O(4^n)$.

3 Problem 3

Let $n \in \mathbb{N}$ and define grammar G by productions $\{A_k \to A_{k-1}A_{k-1} | 2 \leq k \leq n\} \cup \{A_1 \to a\}$ and axiom $S = A_n$. Consider the smallest non-deterministic finite automaton M such that $L(M) =_{\Pi} L(G)$. How many states does M have? What can we conclude about the size of the $(n+1)$-Parikh automaton ?

4 Problem 4

Given grammar G in Chomsky Normal Form, find a counter-example or prove correct the following statement: $L(G) = L_{n+1}(G)$.

$L_{n+1}(G)$ is the language obtained by considering only derivations of G of index up to $n+1$.