Formal definition of the syntax of QFPAbit

Let V be a finite set of integer variables. Then the set of terms is defined as follows with $c \in \mathbb{Z}$ and $x \in V$:

$$T := c|x|T + T|cT|\bar{\neg}T|T\bar{\wedge}T|T\bar{\vee}T$$

The set of formulas is defined as follows with $\% \in \{=, \neq, <, \leq, >, \geq\}$:

$$F := T\%T|\neg F|F \wedge F|F \vee F|F \rightarrow F|F \leftrightarrow F;$$

The semantics of everything except the bitvector operators should be clear. The action of the bitvector operators is as follows: Given $x =< x_k, ...x_0 >_{\mathbb{Z}}$ and $y =< y_k, ...y_0 >_{\mathbb{Z}}$ where x_i, y_i are bits, we define

$$\bar{\neg}x :=< \neg x_k, ...\neg x_0 >_{\mathbb{Z}}$$
$$x\bar{\vee}y :=< x_k \vee y_k, ...x_0 \vee y_0 >_{\mathbb{Z}}$$
$$x\bar{\wedge}y :=< x_k \wedge y_k, ...x_0 \wedge y_0 >_{\mathbb{Z}}$$

The Two's Complement Encoding is given by

$$< x_k, ...x_0 >_{\mathbb{Z}}= -2^k x_k + \Sigma_{i=0}^{k-1} 2^i x_i.$$

Recall that we can replicate the most significant bit of a number written in this encoding without changing the value:

$$< x_k, ...x_0 >_{\mathbb{Z}}=< x_k, x_k, ...x_0 >_{\mathbb{Z}}$$

Question 1

Consider the language QFPAbitNoPlus which is the same as QFPAbit but has a slightly restricted signature - we remove the "+" sign. That means we change the definition of terms to

$$T := c|x|cT|\bar{\neg}T|T\bar{\wedge}T|T\bar{\vee}T$$

Find a QFPAbitNoPlus formula F over the set of variables $V = \{x, y, z, w_1, ...w_k\}$ that has the following property: Given a partial assignment $I : \{x, y, z\} \rightarrow \mathbb{Z}$, I can be extended to a satisfying total assignment for F if and only if $I(x) + I(y) = I(z)$.

Think about the reasons why this can not be done for multiplication and what would be the consequences for Peano Arithmetic if it could.

Question 2

In my lecture I was using the fact that $\neg x = -1 - x$. Prove this.