Seminar on Automated Reasoning

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Problem 1 Herbrand interpretations

Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/_0, c/_0\}$ and $\Pi = \{p/_1, q/_2\}$.

(1)

How many different Herbrand interpretations over Σ do exist? Explain briefly.

(2)

How many different Herbrand models over Σ does the universally quantified clause $\forall x.(\neg p(c) \vee q(x, b))$ have? Explain briefly.

Problem 2 Orderings

Let $N = \{M_1, M_2, M_3, M_4, M_5\}$ be a set of multisets of multisets:

$$\begin{aligned}
M_1 &= \{\{a_4\}, \{a_4\}, \{a_1\}, \{a_1\}\} \\
M_2 &= \{\{a_2\}, \{a_1\}, \{a_1\}\} \\
M_3 &= \{\{a_3, a_1\}\} \\
M_4 &= \{\{a_4, a_3\}, \{a_3, a_2\}, \{a_2, a_1, a_1\}\} \\
M_5 &= \{\{a_2\}, \{a_1, a_1\}, \emptyset\}
\end{aligned}$$

(1)

Let the ordering $\succ$ be defined by $a_4 \succ a_3 \succ a_2 \succ a_1$, let $\succ_m$ be the multiset extension of $\succ$, and let $\succ_{mm}$ be the multiset extension of $\succ_m$. Sort the elements of N with respect to $\succ_{mm}$.

(2)

Find another total ordering $\succ'$ on $\{a_1, a_2, a_3, a_4\}$ such that M_3 is maximal and M_1 is minimal in N with respect to $\succ'_{mm}$, where $\succ'_{mm}$ is the twofold multiset extension of $\succ'$.

Problem 3 Model Construction

Let D', D and C be ground clauses such that $D, D' \in N$ and let $D' \succ D \succ C$. Using the notation from the lectures, prove the following.

$$I_D \models C \Rightarrow I_{D'} \models C \text{ and } I_N \models C$$

If, in addition, $C \in N$ or $\max(D) \succ \max(C)$, then:

$$I_D \not\models C \Rightarrow I_{D'} \not\models C \text{ and } I_N \not\models C$$

Problem 4 Theorem Provers in Practice

Install a first-order theorem prover of your choice. If you do not feel like installing, you can also try WebSPASS (`http://www.spass-prover.org/webspass/`). Try to prove, automatically, that a person can be its own grandpa. Or is it not possible? For this purpose use the song "I'm My Own Grandpa". Formulate and state all commonly knows facts and notions of family relations. For example, to express that someones wife's daughter is also a daughter of this person, you need the following axiom:

```
formula(forall([X],forall([Y],forall([Z],
  implies(and(isHusbandTo(X,Y),isMotherTo(Y,Z)), isFatherTo(X,Z))
)))).
```

This axiom uses the following predicates:

```
list_of_symbols.
  predicates[(isHusbandTo,2),(isMotherTo,2),(isFatherTo,2)].
end_of_list.
```

Submit your input file and the theorem prover output. Make sure that the input file parses and that we can verify your output. If you use some other theorem prover, you will need to demonstrate that it works on your laptop.

Viel SPASS! (in German: Enjoy!)