Seminar on Automated Reasoning

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Problem 1 Terms, Atoms, Literals, and Formulae

Assume that $a/_0$, $b/_0$, $f/_1$ and $g/_2$ are function symbols, $p/_2$ is a predicate symbol and x and y are variables. For each of the following strings, determine whether it is a term, an atom, a literal, or a formula. Note that it may be more than one or it can be syntactically incorrect.

1. a
2. $f(a)$
3. $g(f(a), f)$
4. $p(f(a), x)$
5. $g(x, f(x))$
6. $p(x, y)$
7. $\neg p(a, b)$
8. $\exists a.p(a, b)$
9. $\exists x.p(x, f(a))$
10. $p(x, p(x, x))$
11. $p(a, b) \vee p(b, a)$
12. $p \wedge \exists x.p(x, x)$
13. $\neg \exists x.p(a, b)$
14. $\neg(\exists x. \vee \forall x.p(x, x))$

Problem 2 Peano Arithmetic

Formalize the following statements in the signature Σ_{PA} of Peano arithmetic:

1. 3 is not divisible by 2.

2. All numbers between 1 and 3 are even.

3. There exists exactly one number between 1 and 3.

4. There does not exists a largest square number.

Problem 3 FOL validity

Which of the following formulae is valid. If it is valid, give a proof using the definition of the semantics of first-order logic; otherwise, give a falsifying interpretation.

1. $\exists x.equals(x, y)$

2. $\forall z.equals(z, z) \to \exists x.equals(x, y)$

3. $\forall x, y.(p(x, y) \vee p(y, x)) \to \forall z.p(z, z)$

4. $\exists x.p(x) \to \forall y.p(y)$

5. $\exists x.(p(x) \to \forall y.p(y))$

Problem 4 Clausal Normal Form

1. Transform the following formula into prenex normal form:

$$\Big(\forall z.\big((\forall x.q(x, z)) \to p(x, g(y), z)\big)\Big) \wedge \neg\big(\forall z.\neg(\forall x.q(f(x, y), z))\big)$$

2. Let F be a formula derived in the first step. Universally quantify all the free variables of F and denote this new formula with G

3. Compute the clausal (normal) form of G