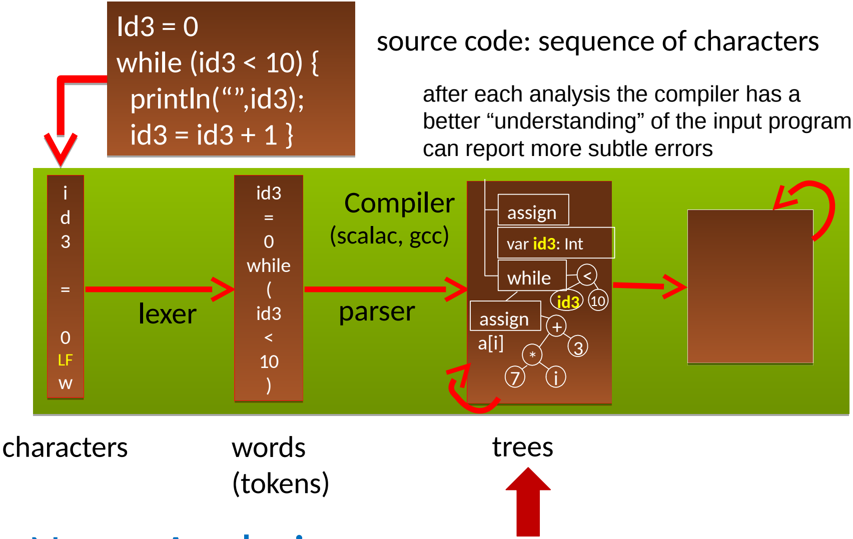


```
Id3 = 0
while (id3 < 10) {
  println("",id3);
  id3 = id3 + 1 }
```

source code: sequence of characters

after each analysis the compiler has a better "understanding" of the input program; can report more subtle errors

Name Analysis:

making sense of trees;
converting them into **graphs**:
connect identifier **uses** and **declarations**

Reporting Errors

Errors Detected So Far

- File input: file does not exist
- Lexer: unknown token, string not closed before end of file, …
- Parser: syntax error - unexpected token, cannot parse given non-terminal
- Name analyzer: unknown identifier
- Type analyzer:
  applying function to arguments of wrong type
- Data-flow analyzer:
  variable read before written, division by zero

Name Analysis Problems Reported: 1

- a class is defined more than once:

```
class A { ...} class B { ... } class A { ... }
```

- a variable is defined more than once:

```
int x; int y; int x;
```

- a class member is overriden without **override** keyword:

```
class A { int x; ... } class B extends A { int x; ... }
```

- a method is **overloaded** (forbidden in Tool):

```
class A { def f(B x) {} def f(C x) {} ... }
```

- a method argument is shadowed by a local variable declaration (forbidden in Java, Tool):

```
def (x:Int) { var x : Int; ...}
```

- two method arguments have the same name:

```
def (x:Int,y:Int,x:Int) { ... }
```

Name Analysis Problems Reported: 2

- a class name is used as a symbol (as parent class or type, for instance) but is not declared:

  class A extends Objekt {}

- an identifier is used as a variable but is not declared:

  def(amount:Int) { total = total + ammount }

- the inheritance graph has a cycle:

  class A extends B {}
  class B extends C {}
  class C extends A

To make it efficient and clean to check for such errors, we associate mapping from each identifier to the symbol that the identifier represents.

- We use Map data structures to maintain this mapping
- The rules that specify how declarations are used to construct such maps are given by ***scoping*** **rules of the programming language.**

Storing and Using Tree Positions

Showing Good Errors with Syntax Trees

Suppose we have undeclared variable ‘i’ in a program of 100K lines

Which error message would you prefer to see from the compiler?

  - An ocurrence of variable ‘i’ not declared (which variable? where?)
  - An ocurrence of variable ‘i’ in procedure P not declared
  - Variable ‘i’ undeclared at line 514, position 12 (and IDE points you there)⚡

How to emit this error message if we only have a syntax trees?

- Abstract syntax tree nodes store positions within file
- For identifier nodes: allows reporting variable uses
  - Variable 'i' in line 11, column 5 undeclared
- For other nodes, supports useful for type errors, e.g. could report for (x + y) * (!ok)
  - Type error in line 13,
  - expression in line 13, column 11-15, has type **Bool**, expected **Int** instead

Showing Good Errors with Syntax Trees

Constructing trees with positions:

- Lexer records positions for tokens
- Each subtree in AST corresponds to some parse tree, so it has first and last token
- Get positions from those tokens
- Save these positions in the constructed tree

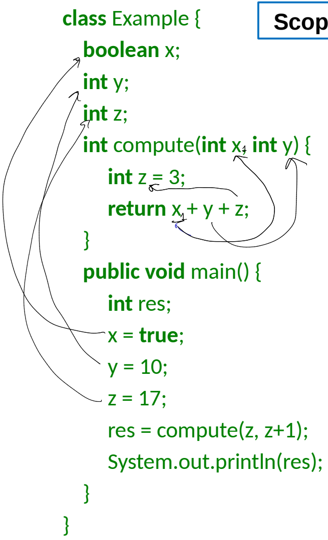
What is important is to save information for leaves

- information for other nodes can often be approximated using information in the leaves

Continuing Name Analysis:

Scope of Identifiers

Example: find program result, symbols, scopes



```
class Example {
   boolean x;
   int y;
   int z;
   int compute(int x, int y) {
      int z = 3;
      return x + y + z;
   }
   public void main() {
      int res;
      x = true;
      y = 10;
      z = 17;
      res = compute(z, z+1);
      System.out.println(res);
   }
}
```

Scope of a variable = part of the program where it is visible

Draw an arrow from occurrence of each identifier to the point of its declaration.

For each declaration of identifier, identify where the identifier can be referred to (its scope).

Name analysis:

- computes those arrows
  - = maps, partial functions (math)
  - = environments (PL theory)
  - = symbol table (implementation)
- report some simple semantic errors

We usually introduce **symbols** for things denoted by identifiers.
Symbol tables map identifiers to symbols.

Usual **static** scoping: What is the result?

```
class World {
 int sum;
 int value;
 void add() {
   sum = sum + value;
   value = 0;
 }
 void main() {
    sum = 0;
    value = 10;
    add();
    if (sum % 3 == 1) {
      int value;
      value = 1;
      add();
      print("inner value = ", value);  1
      print("sum = ", sum);   10
    }
    print("outer value = ", value);   0
  }
}
```

Identifier refers to the symbol that was declared “closest” to the place **in program structure** (thus "static").

We will assume static scoping unless otherwise specified.

Renaming Statically Scoped Program

```
class World {
 int sum;
 int value;
 void add(int foo) {
    sum = sum + value;
    value = 0;
 }
 void main() {
     sum = 0;
     value = 10;
     add();
     if (sum % 3 == 1) {
       int value1;
       value1 = 1;
       add(); // cannot change value1
       print("inner value = ", value1); 1
       print("sum = ", sum);   10
     }
     print("outer value = ", value);   0
  }
}
```

Identifier refers to the symbol that was declared “closest” to the place **in program structure** (thus "static").

We will assume static scoping unless otherwise specified.

Property of static scoping:
Given the entire program, we can **rename variables** to avoid any shadowing (**make** all vars **unique**!)

Dynamic scoping: What is the result?

```
class World {
 int sum;
 int value;
 void add() {
    sum = sum + value;
    value = 0;
 }
 void main() {
     sum = 0;
     value = 10;
     add();
     if (sum % 3 == 1) {
        int value;
        value = 1;
        add();
        print("inner value = ", value);  0
        print("sum = ", sum);   11
     }
     print("outer value = ", value);   0
  }
}
```

Symbol refers to the variable that was most **recently declared within program execution.**

Views variable declarations as executable statements that establish which symbol is considered to be the ‘current one’. (Used in old LISP interpreters.)

Translation to normal code: access through a dynamic environment.

Dynamic scoping translated using global map, working like stack

```
class World {
 int sum;
 int value;
 void add() {
   sum = sum + value;
   value = 0;
 }
 void main() {
    sum = 0;
    value = 10;
    add();
    if (sum % 3 == 1) {
      int value;
      value = 1;
      add();
      print("inner value = ", value);  0
      print("sum = ", sum);  11
    }
    print("outer value = ", value);  0
 }
}
```

```
class World {
 pushNewDeclaration('sum);
 pushNewDeclaration('value);
 void add(int foo) {
   update('sum, lookup('sum) + lookup('value));
   update('value, 0);
 }
 void main() {
    update('sum, 0);
    update('value,10);
    add();
    if (lookup('sum) % 3 == 1) {
      pushNewDeclaration('value);
      update('value, 1);
      add();
      print("inner value = ", lookup('value));
      print("sum = ", lookup('sum));
      popDeclaration('value)
    }
    print("outer value = ", lookup('value));
  }
}
```

Object-oriented programming has scope for each object, so we have a nice controlled alternative to dynamic scoping (objects give names to scopes).

Good Practice for Scoping

- Static scoping is almost universally accepted in modern programming language design
- It is the approach that is usually easier to reason about and easier to **compile**, since we do not have names at compile time and compile each code piece separately
- Still, various ad-hoc language designs emerge and become successful
  - LISP implementations took dynamic scoping since it was simpler to implement for higher-order functions
  - Javascript

How the **symbol map** changes in case of **static** scoping

Outer declaration
int value is shadowed by
inner declaration **string value**

Map becomes bigger as
we enter more scopes,
later becomes smaller again
Imperatively: need to make
maps bigger, later smaller again.
Functionally: immutable maps,
keep old versions.

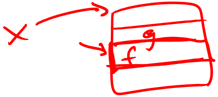
```
class World {
  int sum;  int value;
  // value → int, sum → int
  void add(int foo) {
    // foo → int, value → int, sum → int
    string z;
    // z → string, foo → int, value → int, sum → int
    sum = sum + value; value = 0;
  }
  // value → int, sum → int
  void main(string bar) {
    // bar → string, value → int, sum → int
    int y;
    // y → int, bar → string, value → int, sum → int
    sum = 0;
    value = 10;
    add();
    // y → int, bar → string, value → int, sum → int
    if (sum % 3 == 1) {
      string value;
      // value → string, y → int, bar → string, sum → int
      value = 1;
      add();
      print("inner value = ", value);
      print("sum = ", sum); }
    // y → int, bar → string, value → int, sum → int
    print("outer value = ", value);
} }
```

Representing Data

- In Java, the standard model is a mutable graph of objects
- It seems natural to represent references to symbols using mutable fields (initially null, **resolved** during name analysis)
- Alternative way is functional
  - store the **backbone** of the graph as a algebraic data type (immutable)
  - pass around a map linking from identifiers to their declarations
- Note that a field **class** A { **var** f:T } is like f: Map[A,T]

Symbol Table (Γ) Contents

- Map identifiers to the symbol with relevant information about the identifier
- All information is derived from syntax tree - symbol table is for efficiency
  - in old one-pass compilers there was only symbol table, no syntax tree
  - in modern compiler: we could always go through entire tree, but symbol table can give faster and easier access to the part of syntax tree, or some additional information
- Goal: efficiently supporting phases of compiler
- In the name analysis phase:
  - finding which identifier refers to which definition
  - we store *definitions*



- What kinds of things can we define? What do we need to know for each ID?

variables (globals, fields, parameters, locals):

- need to know types, positions - for error messages
- later: memory layout. To compile x.f = y into memcopy(addr_y, addr_x+6, 4)
  - e.g. 3rd field in an object should be stored at offset e.g. +6 from the address of the object
  - the size of data stored in x.f is 4 bytes
- sometimes more information explicit: whether variable local or global
  methods, functions, classes: recursively have with their own symbol tables

Functional: Different Points, Different Γ

```
class World {
  int sum;
  void add(int foo) {
     sum = sum + foo;
  }
  void sub(int bar) {
     sum = sum - bar;
  }
  int count;
}
```

$\Gamma_0 = \{(sum, int), (count, int)\}$

$\leftarrow \Gamma_1 = \Gamma_0 [foo := int]$

$\leftarrow \Gamma_0$

$\leftarrow \Gamma_1 = \Gamma_0 [bar := int]$

Imperative Way: Push and Pop

```
class World {
  int sum;
  void add(int foo) {
     sum = sum + foo;
  }
  void sub(int bar) {
     sum = sum - bar;
  }
  int count;
}
```

$\Gamma_0 = \{(sum, int), (count, int)\}$

$\Gamma_1 = \Gamma_0 [foo := int]$

change table, record change

Γ_0

revert changes from table

$\Gamma_1 = \Gamma_0 [bar := int]$

change table, record change

revert changes from table

Imperative Symbol Table

- Hash table, mutable Map[ID,Symbol]
- Example:
  - hash function into array
  - array has linked list storing (ID,Symbol) pairs
- Undo stack: to enable entering and leaving scope
- Entering new scope (function,block):
  - add beginning-of-scope marker to undo stack
- Adding nested declaration (ID,sym)
  - lookup old value symOld, push old value to undo stack
  - insert (ID,sym) into table
- Leaving the scope
  - go through undo stack until the marker, restore old values

Functional: Keep Old Version

```
class World {
  int sum;
  void add(int foo) {
     sum = sum + foo;
  }
  void sub(int bar) {
     sum = sum - bar;
  }
  int count;
}
```

$\Gamma_0 = \{(sum, int), (count, int)\}$

$\Gamma_1 = \Gamma_0\, [foo := int]$

create new Γ_1, keep old Γ_0

Γ_0

$\Gamma_2 = \Gamma_0\, [bar := int]$

create new Γ_2, keep old Γ_0

Functional Symbol Table Implemented

- Typical: Immutable Balanced Search Trees

```
sealed abstract class BST
case class Empty() extends BST
case class Node(left: BST, value: Int, right: BST) extends BST
```

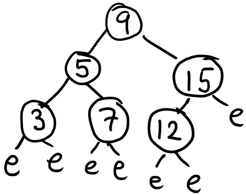
Simplified. In practice, BST[A], store Int key and value A

- Updating returns new map, keeping old one
  - lookup and update both *log(n)*
  - update creates new path (copy *log(n)* nodes, share rest!)
  - memory usage acceptable

Lookup

```
def contains(key: Int, t : BST): Boolean = t match {
  case Empty() => false
  case Node(left,v,right) => {
    if (key == v) true
    else if (key < v) contains(key, left)
    else contains(key, right)
  }
}
```

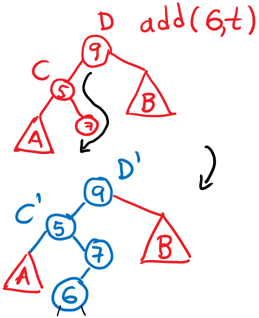
Running time bounded by tree height.



contains(6,t) ?

Insertion

```
def add(x : Int, t : BST) : Node = t match {
  case Empty() => Node(Empty(),x,Empty())
  case t @ Node(left,v,right) => {
   if (x < v) Node(add(x, left), v, right)
   else if (x==v) t
   else Node(left, v, add(x, right))
  }
}
```

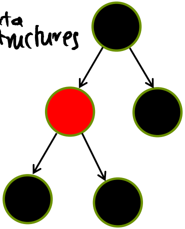


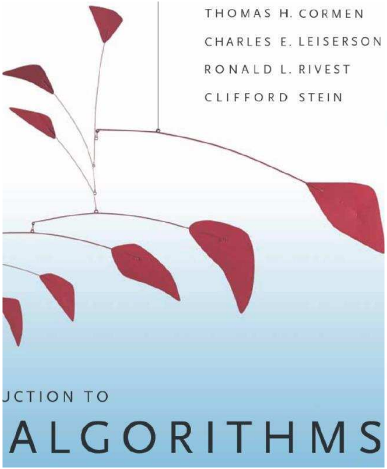
Both add(x,t) and t remain accessible.

Running time and newly allocated nodes bounded by tree height.

Chris Okasaki : Purely Functional Data Structures

Balanced Trees: Red-Black Trees





THOMAS H. CORMEN

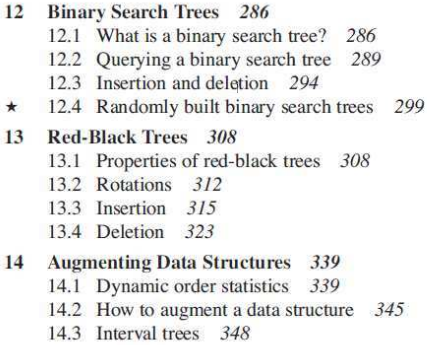
CHARLES E. LEISERSON

RONALD L. RIVEST

CLIFFORD STEIN

UCTION TO

ALGORITHMS



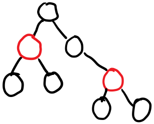
Balanced Tree: Red Black Tree

Goals:

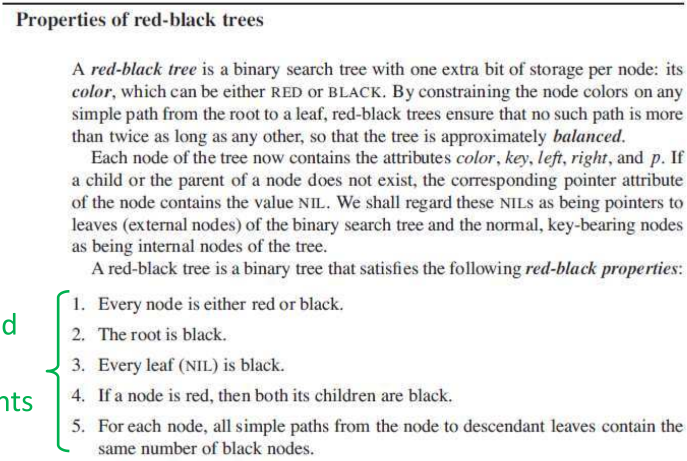
- ensure that tree height remains at most log(size)
  ↳ add(1,add(2,add(3,...add(n,Empty())...))) ~ linked list ✗
- preserve efficiency of individual operations:
  rebalancing arbitrary tree: could cost O(n) work

Solution: maintain mostly balanced trees: height still O(log size)

```
sealed abstract class Color
case class Red() extends Color
case class Black() extends Color
```



```
sealed abstract class Tree
case class Empty() extends Tree
case class Node(c: Color,left: Tree,value: Int, right: Tree)
                        extends Tree
```



Properties of red-black trees

A ***red-black tree*** is a binary search tree with one extra bit of storage per node: its ***color***, which can be either RED or BLACK. By constraining the node colors on any simple path from the root to a leaf, red-black trees ensure that no such path is more than twice as long as any other, so that the tree is approximately ***balanced***.

Each node of the tree now contains the attributes *color*, *key*, *left*, *right*, and *p*. If a child or the parent of a node does not exist, the corresponding pointer attribute of the node contains the value NIL. We shall regard these NILs as being pointers to leaves (external nodes) of the binary search tree and the normal, key-bearing nodes as being internal nodes of the tree.

A red-black tree is a binary tree that satisfies the following ***red-black properties***:

1. Every node is either red or black.
2. The root is black.
3. Every leaf (NIL) is black.
4. If a node is red, then both its children are black.
5. For each node, all simple paths from the node to descendant leaves contain the same number of black nodes.

balanced tree constraints

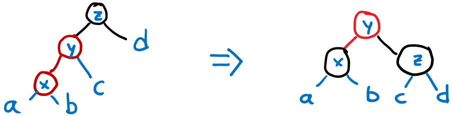
From 4. and 5.: tree height is O(log size).

Analysis is similar for mutable and immutable trees.

for immutable trees: see book by Chris Okasaki

Balancing

```
def balance(c: Color, a: Tree, x: Int, b: Tree): Tree = (c,a,x,b) match {
  case (Black(),Node(Red(),Node(Red(),a,xV,b),yV,c),zV,d) =>
  Node(Red(),Node(Black(),a,xV,b),yV,Node(Black(),c,zV,d))
```



```
  case (Black(),Node(Red(),a,xV,Node(Red(),b,yV,c)),zV,d) =>
  Node(Red(),Node(Black(),a,xV,b),yV,Node(Black(),c,zV,d))
  case (Black(),a,xV,Node(Red(),Node(Red(),b,yV,c),zV,d)) =>
  Node(Red(),Node(Black(),a,xV,b),yV,Node(Black(),c,zV,d))
  case (Black(),a,xV,Node(Red(),b,yV,Node(Red(),c,zV,d))) =>
  Node(Red(),Node(Black(),a,xV,b),yV,Node(Black(),c,zV,d))
  case (c,a,xV,b) => Node(c,a,xV,b)
}
```

Insertion

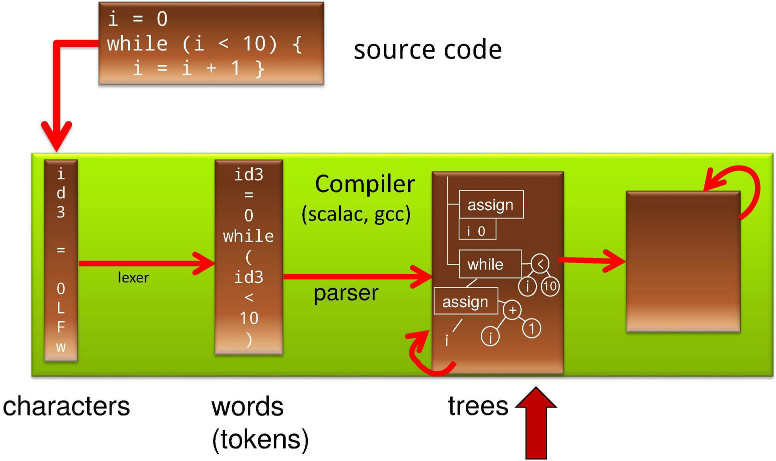
```
def add(x: Int, t: Tree): Tree = {
 def ins(t: Tree): Tree = t match {
  case Empty() => Node(Red(),Empty(),x,Empty())
  case Node(c,a,y,b) =>
   if (x < y) balance(c, ins(a), y, b)
   else if (x == y) Node(c,a,y,b)
   else balance(c,a,y,ins(b))
 }
 makeBlack(ins(t))
}
def makeBlack(n: Tree): Tree = n match {
  case Node(Red(),l,v,r) => Node(Black(),l,v,r)
  case _ => n
}
```

Modern object-oriented languages (e.g. Scala) support abstraction and functional data structures. Just use Map from Scala.

Exercise

Determine the output of the following program assuming static and dynamic scoping. Explain the difference, if there is any.

```
object MyClass {
 val x = 5
 def foo(z: Int): Int = { x + z }
 def bar(y: Int): Int = {
   val x = 1; val z = 2
   foo(y)
 }
 def main() {
  val x = 7
  println(foo(bar(3)))
 }
}
```



```
i = 0
while (i < 10) {
  i = i + 1 }
```

source code

characters

words (tokens)

trees

Type Checking

Evaluating an Expression

scala prompt:

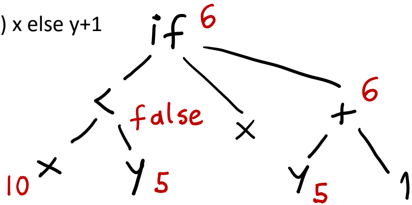
```
>def min1(x : Int, y : Int) : Int = { if (x < y) x else y+1 }
min1: (x: Int,y: Int)Int
>min1(10,5)
res1: Int = 6
```

How can we think about this evaluation?

x → 10

y → 5

if (x < y) x else y+1



Computing types using the evaluation tree

scala prompt:

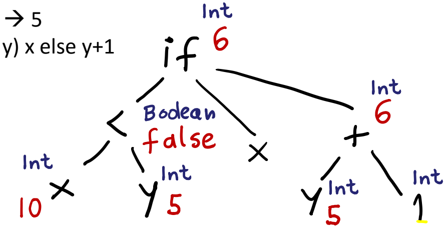
```
>def min1(x : Int, y : Int) : Int = { if (x < y) x else y+1 }
min1: (x: Int,y: Int)Int
>min1(10,5)
res1: Int = 6
```

How can we think about this evaluation?

x : Int → 10

y : Int → 5

if (x < y) x else y+1



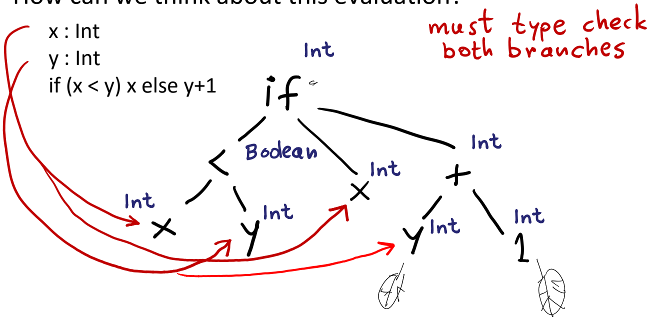
We can compute types without values

scala prompt:

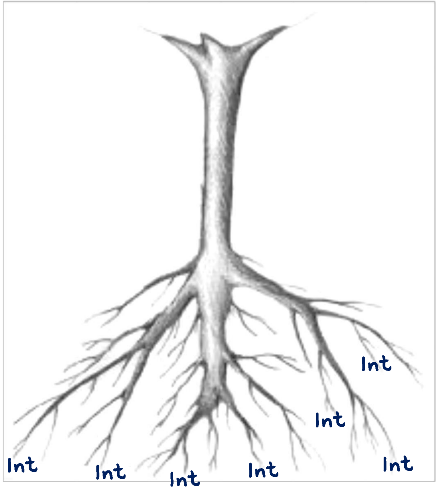
```
>def min1(x : Int, y : Int) : Int = { if (x < y) x else y+1 }
min1: (x: Int,y: Int)Int
>min1(10,5)
res1: Int = 6
```

How can we think about this evaluation?

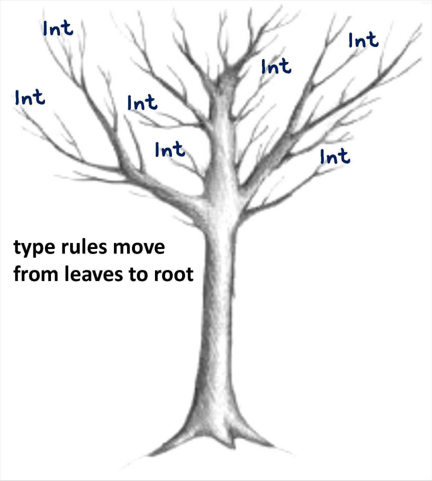
x : Int
y : Int
if (x < y) x else y+1



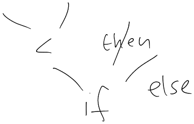
We do not like trees upside-down



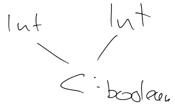
Leaves are Up



type rules move
from leaves to root



Type Judgements and Type Rules



- e type checks to T under Γ (type environment)

$$\Gamma \vdash e : T$$

  - Types of constants are predefined
  - Types of variables are specified in the source code

- If e is composed of sub-expressions

$$\frac{\Gamma \vdash e_1 : T_1 \cdots \Gamma \vdash e_n : T_n}{\Gamma \vdash e : T}$$

↓ type check from leaves

Type Judgements and Type Rules

$$\Gamma \vdash e : T$$

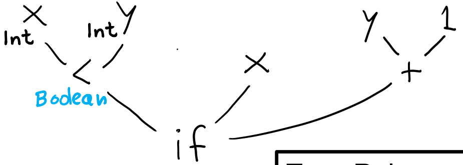
if the (free) variables of e have types given by the type environment gamma, then e (correctly) type checks and has type T

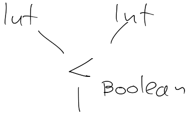
type rule
$$\frac{\Gamma \vdash e_1 : T_1 \quad \cdots \quad \Gamma \vdash e_n : T_n}{\Gamma \vdash e : T}$$

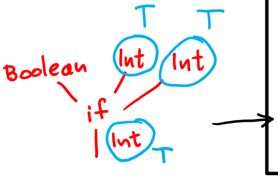
If e_1 type checks in gamma and has type T_1 and ...
and e_n type checks in gamma and has type T_n
then e type checks in gamma and has type T

Type Rules as Local Tree Constraints

x : Int
y : Int







Type Rules

$$\frac{e_1 : \text{Int} \qquad e_2 : \text{Int}}{e_1 < e_2 : \text{Boolean}}$$

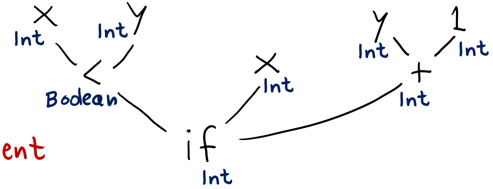
for every type T, if
b has type Boolean, and ...
then

$$\frac{b : \text{Boolean} \qquad e_1 : T \qquad e_2 : T}{\text{if}(b)\ e_1\ \text{else}\ e_2 : T}$$

Type Rules with Environment

x : Int
y : Int

type environment
Γ



Type Rules

$$\frac{(x:T) \in \Gamma}{\Gamma \vdash x : T}$$

$$\frac{}{\text{Int Const}(k) : \text{Int}}$$

$$\frac{\Gamma \vdash e_1 : \text{Int} \qquad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash (e_1 < e_2) : \text{Boolean}}$$

...(then) in the (same) environment Γ
the expression $e_1 < e_2$ has type Bool.

$$\frac{\Gamma \vdash e_1 : \text{Int} \qquad \Gamma \vdash e_2 : \text{Int}}{\Gamma \vdash (e_1 + e_2) : \text{Int}}$$

$$\frac{\Gamma \vdash b : \text{Boolean} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash (\text{if } (b)\ e_1 \text{ else } e_2) : T}$$

Type Checker Implementation Sketch

```
def typeCheck(Γ : Map[ID, ~~Type~~ TypeTree],  e : ExprTree) : TypeTree = {
 e match {
   case Var(id) => {  ??   }
   case If(c,e1,e2) => { ?? }
   ...
}}
```

```
   case Var(id) => { Γ(id) match
     case Some(t) => t
     case None => error(UnknownIdentifier(id,id.pos))
   }
```

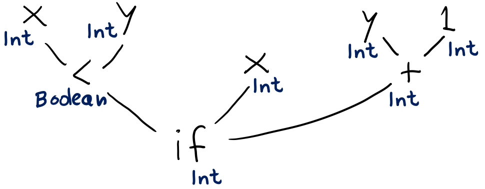
Type Checker Implementation Sketch

- **case** If(c,e1,e2) => {

```
  val tc = typeCheck(Γ,c)
  if (tc != BooleanType) error(IfExpectsBooleanCondition(e.pos))
  val t1 = typeCheck(Γ, e1); val t2 = typeCheck(Γ, e2)
  if (t1 != t2) error(IfBranchesShouldHaveSameType(e.pos))
  t1
  }
```

Derivation Using Type Rules

x : Int
y : Int



Let $\Gamma = \{(x,\text{Int}),(y,\text{Int})\}$

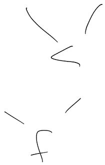
$$\dfrac{\dfrac{\dfrac{(x,\text{Int})\in\Gamma}{\Gamma\vdash x:\text{Int}}\qquad\dfrac{(y,\text{Int})\in\Gamma}{\Gamma\vdash y:\text{Int}}}{\Gamma\vdash (x<y):\text{Boolean}}\qquad \boxed{\dfrac{(x,\text{Int})\in\Gamma}{\Gamma\vdash x:\text{Int}}}\qquad \dfrac{\dfrac{(y,\text{Int})\in\Gamma}{\Gamma\vdash y:\text{Int}}\qquad\dfrac{}{\Gamma\vdash 1:\text{Int}}}{\Gamma\vdash(y+1):\text{Int}}}{\Gamma\vdash \underbrace{(\text{if}(x<y)\ x\ \text{else}\ y+1)}:\text{Int}}$$

Type Rule for Function Application

$$\frac{\Gamma \vdash e_1 \colon\ T_1\ \ \cdots\ \ \Gamma \vdash e_n : T_n \quad \Gamma \vdash f\colon (T_1 \times \cdots \times T_n) \rightarrow T}{\Gamma \vdash f(e_1, \cdots, e_n)\colon T}$$

Type Rule for Function Application [Cont.]

We can treat operators as variables that have function type



$$+ : \mathrm{Int} \times \mathrm{Int} \rightarrow \mathrm{Int}$$

$$< : \mathrm{Int} \times \mathrm{Int} \rightarrow \mathrm{Boolean}$$

$$\&\& : \mathrm{Boolean} \times \mathrm{Boolean} \rightarrow \mathrm{Boolean}$$

We can replace many previous rules with application rule:

$$\frac{\Gamma \vdash e_1 : T_1 \quad \ldots \quad \Gamma \vdash e_n : T_n \quad \Gamma \vdash f : ((T_1 \times \cdots \times T_n) \rightarrow T)}{\Gamma \vdash f(e_1, \ldots, e_n) : T}$$

$$\frac{\Gamma \vdash e_1 : \mathrm{Bool} \qquad \Gamma \vdash e_2 : \mathrm{Bool} \quad \Gamma \vdash \&\& : (\mathrm{Bool} \times \mathrm{Bool}) \rightarrow \mathrm{Bool}}{\Gamma \vdash e_1 \;\&\&\; e_2 : \mathrm{Bool}}$$

Computing the Environment of a Class

```
object World {
  var data : Int
  var name : String
  def m(x : Int, y : Int) : Boolean { … }
  def n(x : Int) : Int {
    if (x > 0) p(x – 1) else 3
  }
  def p(r : Int) : Int = {
     var k = r + 2
     m(k, n(k))
  }
}
```

$\Gamma_0 = \{$

(data, Int),

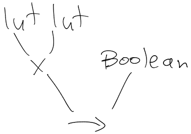
(name, String),

(m, Int x Int → Boolean),

(n, Int → Int),

(p, Int → Int)

$\}$



We can type check each function m,n,p in this global environment

Extending the Environment

$$\Gamma_0 = \{ (data, Int), (name, String), (m, Int \times Int \rightarrow Boolean), (n, Int \rightarrow Int), (p, Int \rightarrow Int) \}$$

```
class World {
  var data : Int
  var name : String
  def m(x : Int, y : Int) : Boolean { ... }
  def n(x : Int) : Int {
    if (x > 0) p(x – 1) else 3
  }
  def p(r : Int) : Int = {
     var k:Int
     k = r + 2
     m(k, n(k))
  }
}
```

$\leftarrow \Gamma_0$

$\leftarrow \Gamma_1 = \Gamma_0 \oplus \{ (r, Int) \}$

$\leftarrow \Gamma_2 = \Gamma_1 \oplus \{ (k, Int) \} = \Gamma_0 \cup \{ (r, Int), (k, Int) \}$

Type Rule for Method Definitions $\text{def } m(x_1 : T_1, \cdots, x_n : T_n) : T = e$

$$\frac{\Gamma \oplus \{(x_1,T_1),\ldots,(x_n,T_n)\} \vdash e : T}{\Gamma \vdash (\text{def } m(x_1:T_1,\ldots,x_n:T_n) : \underset{\uparrow}{T} = e) : \text{OK}}$$

Type Rule for Assignments

$$\frac{(x,T) \in \Gamma \qquad \Gamma \vdash e : T}{\Gamma \vdash (x = e) : \underset{\text{Unit}}{\text{void}}}$$

Type Rules for Block: { var x_1:T_1 ... var x_n:T_n; s_1; ... s_m; e }

$$\frac{\underbrace{\left(\Gamma \oplus \{(x_1,T_1),\ldots,(x_n,T_n)\}\right)}_{\Gamma_1} \qquad \begin{array}{l} \Gamma_1 \vdash s_1 : \text{void} \\ \ldots \\ \Gamma_1 \vdash s_n : \text{void} \\ \Gamma_1 \vdash e : T \end{array}}{\Gamma \vdash \{\underbrace{\text{var } x_1 : T_1; \ldots; \text{var } x_n : T_n}_{};\ \underbrace{s_1; \ldots; s_n}_{};\ e\} : T}$$

Blocks with Declarations in the Middle

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \{e\} : T}$$

just expression

$$\frac{}{\Gamma \vdash \{\} : \mathit{void}}$$

empty

$$\frac{\Gamma \oplus \{(x, T_1)\} \vdash \{t_2; \dots; t_n\} : T}{\Gamma \vdash \{\mathit{var}\ x : T_1; t_2; \dots; t_n\} : T}$$

declaration is first

$$\frac{\Gamma \vdash s_1 : \mathit{void} \qquad \Gamma \vdash \{t_2; \dots; t_n\} : T}{\Gamma \vdash \{s_1; t_2; \dots; t_n\} : T}$$

statement is first

Rule for While Statement

$$\boxed{\dfrac{\Gamma \vdash b : \text{Boolean} \qquad \Gamma \vdash s : \text{void}}{\Gamma \vdash (\text{while}\,(b)\ s) : \text{void}}}$$

Rule for a Method Call

```
class T0 {
  ...
  def m(x1:T1,...,xn:Tn):T = {
    ...
  }
  ...
}
```

$$\frac{\Gamma \vdash x : T_0 \qquad \Gamma_{T_0} \vdash m : T_0 \times T_1 \times \dots \times T_n \to T \qquad \overset{\forall i \in \{1, 2, \dots, n\}}{\Gamma \vdash e_i : T_i}}{\Gamma \vdash x.m(e_1, \dots, e_n) : T}$$

$m(x, e_1, \dots, e_n)$