Computer Language Processing (CS-320)

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https://lara.epfl.ch/w/cc

Computer Language Processing = ?

A **language** can be:

- natural language (English, French, ...)
- **computer language** (Scala, Java, C, SQL, ...)
- language used to write mathematical statements: $\forall \varepsilon . \exists \delta . \forall x . \ (|x| < \delta \Rightarrow |f(x)| < \varepsilon|)$

We can define languages mathematically as **sets of strings**

We can **process** languages: define algorithms working on strings

In this course we study algorithms to process computer languages

Interpreters and Compilers

We are particularly interested in processing general-purpose programming languages.

Two main approaches:

- interpreter: execute instructions while traversing the program (Python)
- compiler: traverse program, generate executable code to run later (Rust, C)

Portable compiler (Java, Scala, C#):

- compile (`javac`) to platform-independent **bytecode** (`.class`)
- use a combination of interpretation and compilation to run bytecode (`java`)
  - compile or interpret fast, determine important code fragments (inner loops)
  - **optimize** important code and swap it in for subsequent iterations

Compilers for Programming Languages

A typical compiler processes a Turing-complete programming language and translates it into the form where it can be efficiently executed (e.g. machine code).

Source code in a programming language

↓ compiler

machine code

- gcc, clang: map C into machine instructions
- Java compiler: map Java source into bytecodes (.class files)
- Just-in-time (JIT) compiler inside the Java Virtual Machine (JVM): translate .class files into machine instructions (while running the program)

Java compiler (javac) and JIT compiler (java)

```
class Counter {
 public static void main( ... ) {
   int i = 0; int j = 0;
   while (i < 10) {
     System.out.println(j);
     i = i + 2;
     j = j + 2*i + 1; }}}
```

↓ javac -g

```
Counter.class bytecode

cafe babe 0000 0034
0018 0a00 0500 0b09
000c 000d 0a00 0e00
0f07 0010 0700 1101
```

java ⟶

```
0
5
14
27
44
```

Inside a Java class file

```
class Counter {
 public static void main( ... ) {
  int i = 0; int j = 0;
  while (i < 10) {
   System.out.println(j);
   i = i + 2;
   j = j + 2*i + 1; }}}
```

↓ javac

```
Counter.class bytecode

cafe babe 0000 0034
0018 0a00 0500 0b09
000c 000d 0a00 0e00
0f07 0010 0700 1101
```

javap -c ⟶

```
 0: iconst_0
 1: istore_1
 2: iconst_0
 3: istore_2
 4: iload_1
 5: bipush 10
 7: if_icmpge 32
    ...
21: iload_2
22: iconst_2
23: iload_1
24: imul
25: iadd
26: iconst_1
27: iadd
28: istore_2
29: goto 4
32: return
```

Compilers are Important

Source code (e.g. Scala, Java, C, C++, Python)

- designed to be easy **for programmers** to use
- should correspond to way programmers think and help them be productive: avoid errors, write at a **higher level**, use abstractions, interfaces

Target code (e.g. x86, arm, JVM, .NET)

- designed **to efficiently run on hardware**
- low level
- fast to execute, low power use

Compilers **bridge these two worlds**

- essential for building complex, performant software

Some Skills and Knowledge Learned in the Course

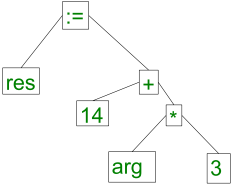
- Develop a compiler for a functional language
  - Write a compiler from start to end
  - Generates Web Assembly
  - generated code runs in browser or in nodejs
- libraries (e.g. parsing combinators) to build compilers: using and making them
- Analyze complex text
- Automatically detecting errors in code:
  - type checking
  - abstract interpretation
- (byte)code generation
- Foundations: automata, regular expressions, grammars, parsing

Examples of the Use of This Knowledge

- understand how compilers work, use them and choose them better
- gain experience with building complex software
- build compiler for your next great language
- extend language with a new construct you need
- adapt existing compiler to new target platform
  (e.g. embedded CPU or graphics processor)
- regular expression handling in editors and search tools
- analyze HTML pages
- process complex input boxes in your applications
  (make own spreadsheet software, expression evaluators)
- process LaTeX, build computer algebra system or a proof assistant
- parse simple natural language fragments

Compilers Bridge the Source-Target Gap in Phases

characters	`res = 14 + arg * 3`
↓ lexical analyzer	
words	`res` `=` `14` `+` `arg` `*` `3`
↓ parser	
trees	Assign(res, Plus(C(14), Times(V(arg),C(3))))
↓ name analyzer	
graphs	(variables mapped to declarations)
↓ type checker	
graphs	Assign(res:Int, Plus(C(14), Times(V(arg):Int,C(3)))):Unit
↓ intermediate code generator	
intermediate code	e.g. LLVM bitcode, JVM bytecode, Web Assembly
↓ JIT compiler or platform-specific back end	
machine code	e.g. x86, ARM, RISC-V



Front End and Back End

front end / back end

characters
↓ lexical analyzer
words
↓ parser
trees
↓ name analyzer
graphs
↓ type checker
graphs
↓ intermediate code generator
intermediate code
↓ JIT compiler or platform-specific back end
machine code   e.g. x86, ARM, RISC-V

Benefits of modularity:

- do one thing in one phase
- swap different front-end: add languages
  (C or Rust, Java or Scala)
- swap different back-end: add various architectures
  (Linux on x86 and ARM)

Interpreters

characters
↓ lexical analyzer
words
↓ parser
trees ⟵———————— program input
↓
program result

Comparison to a compiler:

- same front end: front end techniques apply to interpreters
- no back end: compute result using trees and graphs

Program Trees are Crucial for Interpreters and Compilers

We call a program tree **Abstract Syntax Tree** (AST)

- a language implementation today that does *not* use AST-s is a joke

Structure of trees:

- Nodes represent arithmetic operations, statements, blocks
- Leaves represent constants, variables, methods

Representation of trees:

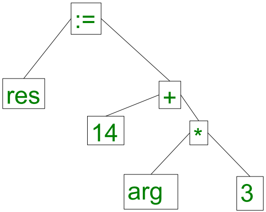
- classes in object-oriented languages
- algebraic data types in functional languages like Haskell, ML

A Simple AST Definition in Scala

```
abstract class Expression
case class C(n: Int) extends Expression // constant
case class V(s: String) extends Expression // variable
case class Plus(e1: Expression, e2: Expression) extends Expression
case class Times(e1: Expression, e2: Expression) extends Expression

abstract class Statement
case class Assign(id:String, e:Expression) extends Statement
case class Block(s: List[Statement]) extends Statement

val program = Assign("res", Plus(C(14), Times(V("arg"),C(3))))
```



Transforming Text Into a Tree

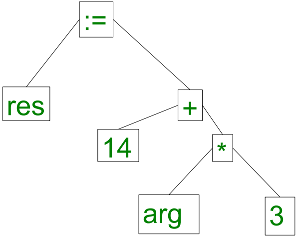
characters `res = 14 + arg * 3`

↓ lexical analyzer

words `res` `=` `14` `+` `arg` `*` `3`

↓ parser

trees Assign(res, Plus(C(14), Times(V(arg),C(3))))



First two phases:

1. lexical analyzer (lexer): sequence of characters → sequence of words
2. syntax analyzer (parser): sequence of words → tree

We will study *linear-time algorithms* for these problems.

We start with the underlying *theory of formal languages*.

Formal Languages: Concepts

- Alphabet (A) - any finite non-empty set of letters (used to write the input)
  e.g. $A = \{0,1\}$, $E = \{a,b,c,\ldots,z\}$
- Word (w) (akka string) - finite sequence of letters (elements of the alphabet A)
  $w \in A^*$ (here A^* is the set of all finite sequences of elements of A)
  $A^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}$ (all words)
  We write sequence denoting a word by just writing one letter after another
  ε is the word of length zero (empty string)
  Length of the word $|w|$ is the number of symbols (repetitions count): $|01011| = 5$
- Language (L) - a set of words (possibly empty, possibly infinite)
  $L \subseteq A^*$
  e.g. $L_1 = \{1, 11, 111, \ldots\}$ (words of length one or more, containing only 1-s)
  $L_2 = \{\varepsilon, 00, 01, 10, 11, 0000, 0001, 0010, \ldots\}$ (words of even length)
  $L_3 = \{0, 101, 111, 00000\}$ (finite language with these specific four words)

Definition of Words in Set Theory

Let A be the alphabet. We define words of length n, denoted A^n

Definition: $A^0 = \{\varepsilon\}$ (only one word of length zero, always denoted ε)
For $n > 0$, $A^n = \{f \mid f : \{0, \ldots, n-1\} \to A\}$
A non-empty word is just a function that tells us what the letters are and in which order.
For $w = \mathbf{1011}$ we thus have:
$w(0) = \mathbf{1} \quad w(1) = \mathbf{0} \quad w(2) = \mathbf{1} \quad w(3) = \mathbf{1}$
(We also write the pretty $w_{(i)}$ instead of $w(i)$)
Set of all words:

$$A^* = \bigcup_{n \geq 0} A^n$$

which means: $w \in A^*$ if and only iff there exists n such that $w \in A^n$.
Note: sometimes people represent e.g. 1011 as $(1, 0, 1, 1)$, but we can think of n-tuple as a function $\{0, \ldots, n-1\} \to A$, so that is equivalent.

Word Equality

Words are equal when they have same length and same letters in the same order:

Let $u, v \in A^*$. Then

$u = v$ if and only if both

1. $|u| = |v|$ and
2. for all i where $0 \leq i < |u|$ we have $u_{(i)} = v_{(i)}$

Words as Scala Lists

```scala
sealed abstract class List[A] { // A is the alphabet
  def ::(t:A): List[A] = Cons(t, this)
  def length: BigInt = this match {
    case Nil() ⇒ BigInt(0)
    case Cons(h, t) ⇒ 1 + t.length }
  def apply(index: BigInt): A = {
    this match {
      case Cons(h,t) ⇒
        if (index == BigInt(0)) h
        else t(index-1) } }
}
case class Nil[A]() extends List[A]
case class Cons[A](h: A, t: List[A]) extends List[A]

val w = 1 :: 0 :: 1 :: 1 :: Nil[Int]() // 1011
val n = w.length // 4
val z = w(1) // 0
```

Words as Inductive Structures

If $a \in A$ and $u \in A^*$, let $a \cdot u$ denote the word that starts with a and then follows with symbols from u (like Cons).

Theorem (Decomposing a word)

Given $w \in A^*$, either $w = \varepsilon$ or $w = a \cdot v$ where $a \in A$ and $v \in A^*$.

Theorem (Equality)

Given $u, v \in A^*$ we have $u = v$ if and only if one of the following conditions hold:

- *$u = \varepsilon$ and $v = \varepsilon$.*
- *there exists $a \in A$ and $u', v' \in A^*$ such that $u = a \cdot u'$, $v = a \cdot v'$ and $u' = v'$.*

Theorem (Structural induction for words)

Given a property of words $P : A^* \to \{true, false\}$, if $P(\varepsilon)$ and, if for every letter $a \in A$ and every u, if $P(u)$ then $P(a \cdot u)$, then $\forall u \in A^*. P(u)$.

Each Word is Finite. The Set of All of Them is Infinite

Each word has a finite length, and each symbol is an element from a finite set. Thus, each word is a finite object that can be written down using finitely many bits.

That set of all words is countably infinite: it is as big as the set of natural numbers. For example, if $A = \{1\}$ then each word is of the form $1 \ldots 1$ and is uniquely given by its length n. Thus, there is a bijection between such words and non-negative integers n, which, by definition, means that these two sets have the same cardinality. Similarly, if $A = \{0, 1\}$, we have a bijection between positive integers and words over A: given a word of length n of the form $k_1 \ldots k_n$ we can assign it to a strictly positive integer whose binary number representation is

$$\overline{1 k_1 \ldots k_n}$$

Such mapping establishes a bijection between A^* and postitive integers. More generally, we can show that, for any alphabet A the set of all words A^* is a countably infinite set. Intuitively, we can take any total ordering on A and use it to sort all words as in a dictionary. This defines a bijection with non-negative integers.

Concatenation

Concatenation is a fundamental operation on words, and denotes putting the words of one word after another. For example, concatenating words 01 and 10, denoted $01 \cdot 10$, results in the word 0110.

Concatenation of $u = u_{(0)} \dots u_{(n-1)}$ and $v = v_{(0)} \dots v_{(m-1)}$, denoted $u \cdot v$, or uv for short, is the word

$$u_{(0)} \dots u_{(n-1)} v_{(0)} \dots v_{(m-1)}$$

Definition

$u \cdot v$ is the unique word w such that $|w| = |u| + |v|$ and for all i where $0 \leq i < |w|$,

$$w_{(i)} = \begin{cases} u_{(i)}, & \text{if } 0 \leq i < |u| \\ v_{(i-|u|)}, & \text{if } |u| \leq i < |u| + |v| \end{cases}$$

Note that it follows: $w \cdot \varepsilon = w$ and $\varepsilon \cdot w = w$

Associativity of Concatenation

Theorem

For all $u, v, w \in A$,

$$u \cdot (v \cdot w) = (u \cdot v) \cdot w$$

First, we show that the two words have the same length. Indeed, $|u \cdot (v \cdot w)| = |u| + |v \cdot w| = |u| + |v| + |w|$ and likewise $|(u \cdot v) \cdot w| = |u \cdot v| + |w| = |u| + |v| + |w|$.
Next, we show that the letters are same at all positions i where $0 \leq i < |u| + |v| + |w|$.
Pick any such i. There are three cases, depending on the interval to which i belongs.
Case $i < |u|$. We have $(u \cdot (v \cdot w))_{(i)} = u_{(i)}$ by the definition of concatenation.
Similarly, because $i < |u \cdot v|$, we have that likewise $((u \cdot v) \cdot w)_{(i)} = (u \cdot v)_{(i)} = u_{(i)}$.
Case $|u| \leq i < |u| + |v|$. We have $(u \cdot (v \cdot w))_{(i)} = (v \cdot w)_{i-|u|} = v_{i-|u|}$ and also $((u \cdot v) \cdot w)_{(i)} = (u \cdot v)_i = v_{i-|u|}$.
Case $|u| + |v| \leq i$. We have $(u \cdot (v \cdot w))_{(i)} = (v \cdot w)_{i-|u|} = w_{i-|u|-|v|}$ and also $((u \cdot v) \cdot w)_{(i)} = w_{i-|u \cdot v|} = w_{i-|u|-|v|}$.

Free Monoid of Words

The neutral element and associativity law imply that the structure $(A^*, \cdot, \varepsilon)$ is an algebraic structure called *monoid*. The monoid of words is called the *free monoid*. Word monoid satisfies, among others, the following additional properties (which do not hold in all monoids).

Theorem (Left cancellation law)

For every three words $u, v, w \in A^*$, if $wu = wv$, then $u = v$.

Theorem (Right cancellation law)

For every three words $u, v, w \in A^*$, if $uw = vw$, then $u = v$.

Reversal

Reversal of a word is a word of same length with symbols but in the reverse order.
Example: the reversal of the word 011, denoted $(011)^{-1}$, is the word 110.

Definition

Given $w \in A^*$, its reversal w^{-1} is the unique word such that $|w^{-1}| = |w|$ and $w^{-1}_{(i)} = w_{(|w|-1-i)}$ for all i where $0 \leq i < |w|$.

From definition it follows that $\varepsilon^{-1} = \varepsilon$ and that $a^{-1} = a$ for all $a \in A$.

Theorem

For all $u, v \in A^*$, $(u^{-1})^{-1} = u$ *and* $(uv)^{-1} = v^{-1}u^{-1}$.

Every law about words has a dual version.
Here is the dual of induction principle, where we peel of last elements.

Theorem (Structural induction for words (dual))

Given a property of words $P : A^* \to \{true, false\}$, *if* $P(\varepsilon)$ *and, if for every letter* $a \in A$ *and every* u, *if* $P(u)$ *then* $P(u \cdot a)$, *then* $\forall u \in A^*. P(u)$.

Prefix, Postfix, and Slice

Definition

Let $u, v, w \in A^*$ such that $uv = w$. We then say that *u is a prefix of w* and that *v is a suffix of w*.

Definition

Given a word $w \in A^*$ and two integers p, q such that $0 \leq p \leq q < |w|$, the *$[p,q)$-slice of w*, denoted $w_{p..q}$, is the word u such that $|u| = q - p$ and $u_{(i)} = w_{(p+i)}$ for all i where $0 \leq i < q - p$.

Theorem

Let $w \in A^*$ and $u = w_{p..q}$ where $0 \leq p \leq q < |w|$. Then the exist words $x, y \in A^*$ such that $|x| = p$, $|y| = |w| - q$, and $w = xuy$.

Theorem

Let $w, u, x, y \in A^*$ and $w = xuy$. Then $x = w_{0..|x|}$, $u = w_{|x|..(|x|+|u|)}$ and $v = w_{(|x|+|u|)..|w|}$.