Computer Language Processing (CS-320)

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Computer Language Processing = ?

A **language** can be:

- natural language (English, French, . . .)
- **computer language** (Scala, Java, C, SQL, . . .)
- language used to write mathematical statements: $\forall \varepsilon . \exists \delta . \forall x . \, (|x| < \delta \Rightarrow |f(x)| < \varepsilon|)$

We can define languages mathematically as **sets of strings**

We can **process** languages: define algorithms working on strings

In this course we study algorithms to process computer languages

Interpreters and Compilers

We are particularly interested in processing general-purpose programming languages.

Two main approaches:

- interpreter: execute instructions while traversing the program (Python)
- compiler: traverse program, generate executable code to run later (Rust, C)

Portable compiler (Java, Scala, C#):

- compile (`javac`) to platform-independent **bytecode** (`.class`)
- use a combination of interpretation and compilation to run bytecode (`java`)
  - compile or interpret fast, determine important code fragments (inner loops)
  - **optimize** important code and swap it in for subsequent iterations

Compilers for Programming Languages

A typical compiler processes a Turing-complete programming language and translates it into the form where it can be efficiently executed (e.g. machine code).

Source code in a programming language

↓ compiler

machine code

- gcc, clang: map C into machine instructions
- Java compiler: map Java source into bytecodes (.class files)
- Just-in-time (JIT) compiler inside the Java Virtual Machine (JVM): translate .class files into machine instructions (while running the program)

Java compiler (javac) and JIT compiler (java)

```
class Counter {
 public static void main( ... ) {
   int i = 0; int j = 0;
   while (i < 10) {
     System.out.println(j);
     i = i + 2;
     j = j + 2*i + 1; }}}
```

↓ javac -g

```
Counter.class bytecode

cafe babe 0000 0034
0018 0a00 0500 0b09
000c 000d 0a00 0e00
0f07 0010 0700 1101
```

java ⟶

```
0
5
14
27
44
```

Inside a Java class file

```
class Counter {
 public static void main( ... ) {
  int i = 0; int j = 0;
  while (i < 10) {
   System.out.println(j);
   i = i + 2;
   j = j + 2*i + 1; }}}
```

↓ javac

```
Counter.class bytecode

cafe babe 0000 0034
0018 0a00 0500 0b09
000c 000d 0a00 0e00
0f07 0010 0700 1101
```

javap -c ⟶

```
 0: iconst_0
 1: istore_1
 2: iconst_0
 3: istore_2
 4: iload_1
 5: bipush 10
 7: if_icmpge 32
    ...
21: iload_2
22: iconst_2
23: iload_1
24: imul
25: iadd
26: iconst_1
27: iadd
28: istore_2
29: goto 4
32: return
```

Compilers are Important

Source code (e.g. Scala, Java, C, C++, Python)

- designed to be easy **for programmers** to use
- should correspond to way programmers think and help them be productive: avoid errors, write at a **higher level**, use abstractions, interfaces

Target code (e.g. x86, arm, JVM, .NET)

- designed **to efficiently run on hardware**
- low level
- fast to execute, low power use

Compilers **bridge these two worlds**

- essential for building complex, performant software

Some Skills and Knowledge Learned in the Course

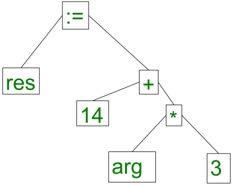
- Develop a compiler for a functional language
  - Write a compiler from start to end
  - Generates Web Assembly
  - generated code runs in browser or in nodejs
- libraries (e.g. parsing combinators) to build compilers: using and making them
- Analyze complex text
- Automatically detecting errors in code:
  - type checking
  - abstract interpretation
- (byte)code generation
- Foundations: automata, regular expressions, grammars, parsing

Examples of the Use of This Knowledge

- understand how compilers work, use them and choose them better
- gain experience with building complex software
- build compiler for your next great language
- extend language with a new construct you need
- adapt existing compiler to new target platform
  (e.g. embedded CPU or graphics processor)
- regular expression handling in editors and search tools
- analyze HTML pages
- process complex input boxes in your applications
  (make own spreadsheet software, expression evaluators)
- process LaTeX, build computer algebra system or a proof assistant
- parse simple natural language fragments

Compilers Bridge the Source-Target Gap in Phases

characters	`res = 14 + arg * 3`
↓ lexical analyzer	
words	`res` | `=` | `14` | `+` | `arg` | `*` | `3`
↓ parser	
trees	Assign(res, Plus(C(14), Times(V(arg),C(3))))
↓ name analyzer	
graphs	(variables mapped to declarations)
↓ type checker	
graphs	Assign(res:Int, Plus(C(14), Times(V(arg):Int,C(3)))):Unit
↓ intermediate code generator	
intermediate code	e.g. LLVM bitcode, JVM bytecode, Web Assembly
↓ JIT compiler or platform-specific back end	
machine code	e.g. x86, ARM, RISC-V



Front End and Back End

front end

characters
↓ lexical analyzer
words
↓ parser
trees
↓ name analyzer
graphs
↓ type checker
graphs
↓ intermediate code generator

back end

intermediate code
↓ JIT compiler or platform-specific back end
machine code e.g. x86, ARM, RISC-V

Benefits of modularity:

- do one thing in one phase
- swap different front-end: add languages
  (C or Rust, Java or Scala)
- swap different back-end: add various architectures
  (Linux on x86 and ARM)

Interpreters

characters
↓ lexical analyzer
words
↓ parser
trees ⟵——————— program input
↓
program result

Comparison to a compiler:

- same front end: front end techniques apply to interpreters
- no back end: compute result using trees and graphs

Program Trees are Crucial for Interpreters and Compilers

We call a program tree **Abstract Syntax Tree** (AST)

- a language implementation today that does *not* use AST-s is a joke

Structure of trees:

- Nodes represent arithmetic operations, statements, blocks
- Leaves represent constants, variables, methods

Representation of trees:

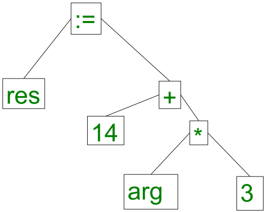
- classes in object-oriented languages
- algebraic data types in functional languages like Haskell, ML

A Simple AST Definition in Scala

```
abstract class Expression
case class C(n: Int) extends Expression // constant
case class V(s: String) extends Expression // variable
case class Plus(e1: Expression, e2: Expression) extends Expression
case class Times(e1: Expression, e2: Expression) extends Expression

abstract class Statement
case class Assign(id:String, e:Expression) extends Statement
case class Block(s: List[Statement]) extends Statement

val program = Assign("res", Plus(C(14), Times(V("arg"),C(3))))
```



Transforming Text Into a Tree

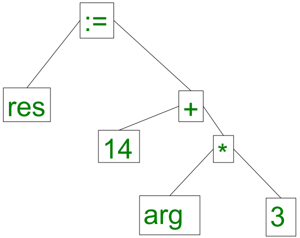
characters `res = 14 + arg * 3`

↓ lexical analyzer

words `res` `=` `14` `+` `arg` `*` `3`

↓ parser

trees Assign(res, Plus(C(14), Times(V(arg),C(3))))



First two phases:

1. lexical analyzer (lexer): sequence of characters → sequence of words
2. syntax analyzer (parser): sequence of words → tree

We will study *linear-time algorithms* for these problems.

We start with the underlying *theory of formal languages*.

Formal Languages: Concepts

- Alphabet (A) - any finite non-empty set of letters (used to write the input)
  e.g. $A = \{0, 1\}$, $E = \{a, b, c, \ldots, z\}$
- Word (w) (akka string) - finite sequence of letters (elements of the alphabet A)
  $w \in A^*$ (here A^* is the set of all finite sequences of elements of A)
  $A^* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, \ldots\}$ (all words)
  We write sequence denoting a word by just writing one letter after another
  ε is the word of length zero (empty string)
  Length of the word $|w|$ is the number of symbols (repetitions count): $|01011| = 5$
- Language (L) - a set of words (possibly empty, possibly infinite)
  $L \subseteq A^*$
  e.g. $L_1 = \{1, 11, 111, \ldots\}$ (words of length one or more, containing only 1-s)
  $L_2 = \{\varepsilon, 00, 01, 10, 11, 0000, 0001, 0010, \ldots\}$ (words of even length)
  $L_3 = \{0, 101, 111, 00000\}$ (finite language with these specific four words)

Definition of Words in Set Theory

Let A be the alphabet. We define words of length n, denoted A^n

Definition: $A^0 = \{\varepsilon\}$ (only one word of length zero, always denoted ε)
For $n > 0$, $A^n = \{f \mid f : \{0, \dots, n-1\} \to A\}$
A non-empty word is just a function that tells us what the letters are and in which order.
For $w = \mathbf{1011}$ we thus have:
$w(0) = \mathbf{1} \quad w(1) = \mathbf{0} \quad w(2) = \mathbf{1} \quad w(3) = \mathbf{1}$
(We also write the pretty $w_{(i)}$ instead of $w(i)$)
Set of all words:

$$A^* = \bigcup_{n \geq 0} A^n$$

which means: $w \in A^*$ if and only iff there exists n such that $w \in A^n$.
Note: sometimes people represent e.g. 1011 as $(1, 0, 1, 1)$, but we can think of n-tuple as a function $\{0, \dots, n-1\} \to A$, so that is equivalent.

Word Equality

Words are equal when they have same length and same letters in the same order:

Let $u, v \in A^*$. Then

$u = v$ if and only if both

1. $|u| = |v|$ and
2. for all i where $0 \leq i < |u|$ we have $u_{(i)} = v_{(i)}$

Words as Scala Lists

```scala
sealed abstract class List[A] { // A is the alphabet
  def ::(t:A): List[A] = Cons(t, this)
  def length: BigInt = this match {
    case Nil() => BigInt(0)
    case Cons(h, t) => 1 + t.length }
  def apply(index: BigInt): A = {
    this match {
      case Cons(h,t) =>
        if (index == BigInt(0)) h
        else t(index-1) } }
}
case class Nil[A]() extends List[A]
case class Cons[A](h: A, t: List[A]) extends List[A]

val w = 1 :: 0 :: 1 :: 1 :: Nil[Int]() // 1011
val n = w.length // 4
val z = w(1) // 0
```

Words as Inductive Structures

If $a \in A$ and $u \in A^*$, let $a \cdot u$ denote the word that starts with a and then follows with symbols from u (like Cons).

Theorem (Decomposing a word)

Given $w \in A^*$, either $w = \varepsilon$ or $w = a \cdot v$ where $a \in A$ and $v \in A^*$.

Theorem (Equality)

Given $u, v \in A^*$ we have $u = v$ if and only if one of the following conditions hold:

- *$u = \varepsilon$ and $v = \varepsilon$.*
- *there exists $a \in A$ and $u', v' \in A^*$ such that $u = a \cdot u'$, $v = a \cdot v'$ and $u' = v'$.*

Theorem (Structural induction for words)

Given a property of words $P : A^* \rightarrow \{true, false\}$, if $P(\varepsilon)$ and, if for every letter $a \in A$ and every u, if $P(u)$ then $P(a \cdot u)$, then $\forall u \in A^*. P(u)$.

Each Word is Finite. The Set of All of Them is Infinite

Each word has a finite length, and each symbol is an element from a finite set. Thus, each word is a finite object that can be written down using finitely many bits.

That set of all words is countably infinite: it is as big as the set of natural numbers. For example, if $A = \{1\}$ then each word is of the form $1 \ldots 1$ and is uniquely given by its length n. Thus, there is a bijection between such words and non-negative integers n, which, by definition, means that these two sets have the same cardinality. Similarly, if $A = \{0, 1\}$, we have a bijection between positive integers and words over A: given a word of length n of the form $k_1 \ldots k_n$ we can assign it to a strictly positive integer whose binary number representation is

$$\overline{1k_1 \ldots k_n}$$

Such mapping establishes a bijection between A^* and postitive integers. More generally, we can show that, for any alphabet A the set of all words A^* is a countably infinite set. Intuitively, we can take any total ordering on A and use it to sort all words as in a dictionary. This defines a bijection with non-negative integers.

Concatenation

Concatenation is a fundamental operation on words, and denotes putting the words of one word after another. For example, concatenating words 01 and 10, denoted $01 \cdot 10$, results in the word 0110.

Concatenation of $u = u_{(0)} \dots u_{(n-1)}$ and $v = v_{(0)} \dots v_{(m-1)}$, denoted $u \cdot v$, or uv for short, is the word

$$u_{(0)} \dots u_{(n-1)} v_{(0)} \dots v_{(m-1)}$$

Definition

$u \cdot v$ is the unique word w such that $|w| = |u| + |v|$ and for all i where $0 \leq i < |w|$,

$$w_{(i)} = \begin{cases} u_{(i)}, & \text{if } 0 \leq i < |u| \\ v_{(i-|u|)}, & \text{if } |u| \leq i < |u| + |v| \end{cases}$$

Note that it follows: $w \cdot \varepsilon = w$ and $\varepsilon \cdot w = w$

Associativity of Concatenation

Theorem

For all $u, v, w \in A$,

$$u \cdot (v \cdot w) = (u \cdot v) \cdot w$$

First, we show that the two words have the same length. Indeed,
$|u \cdot (v \cdot w)| = |u| + |v \cdot w| = |u| + |v| + |w|$ and likewise
$|(u \cdot v) \cdot w| = |u \cdot v| + |w| = |u| + |v| + |w|$.
Next, we show that the letters are same at all positions i where $0 \leq i < |u| + |v| + |w|$.
Pick any such i. There are three cases, depending on the interval to which i belongs.
Case $i < |u|$. We have $(u \cdot (v \cdot w))_{(i)} = u_{(i)}$ by the definition of concatenation.
Similarly, because $i < |u \cdot v|$, we have that likewise $((u \cdot v) \cdot w)_{(i)} = (u \cdot v)_{(i)} = u_{(i)}$.
Case $|u| \leq i < |u| + |v|$. We have $(u \cdot (v \cdot w))_{(i)} = (v \cdot w)_{i-|u|} = v_{i-|u|}$ and also
$((u \cdot v) \cdot w)_{(i)} = (u \cdot v)_i = v_{i-|u|}$.
Case $|u| + |v| \leq i$. We have $(u \cdot (v \cdot w))_{(i)} = (v \cdot w)_{i-|u|} = w_{i-|u|-|v|}$ and also
$((u \cdot v) \cdot w)_{(i)} = w_{i-|u \cdot v|} = w_{i-|u|-|v|}$.

Free Monoid of Words

The neutral element and associativity law imply that the structure $(A^*, \cdot, \varepsilon)$ is an algebraic structure called *monoid*. The monoid of words is called the *free monoid*. Word monoid satisfies, among others, the following additional properties (which do not hold in all monoids).

Theorem (Left cancellation law)

For every three words $u, v, w \in A^*$, if $wu = wv$, then $u = v$.

Theorem (Right cancellation law)

For every three words $u, v, w \in A^*$, if $uw = vw$, then $u = v$.

Reversal

Reversal of a word is a word of same length with symbols but in the reverse order.
Example: the reversal of the word 011, denoted $(011)^{-1}$, is the word 110.

Definition

Given $w \in A^*$, its reversal w^{-1} is the unique word such that $|w^{-1}| = |w|$ and $w^{-1}_{(i)} = w_{|w|-i}$ for all i where $0 \leq i < |w|$.

From definition it follows that $\varepsilon^{-1} = \varepsilon$ and that $a^{-1} = a$ for all $a \in A$.

Theorem

For all $u, v \in A^*$, $(u^{-1})^{-1} = u$ and $(uv)^{-1} = v^{-1}u^{-1}$.

Every law about words has a dual version.
Here is the dual of induction principle, where we peel of last elements.

Theorem (Structural induction for words (dual))

Given a property of words $P : A^* \to \{true, false\}$, if $P(\varepsilon)$ and, if for every letter $a \in A$ and every u, if $P(u)$ then $P(u \cdot a)$, then $\forall u \in A^*. P(u)$.

Prefix, Postfix, and Slice

Definition
Let $u, v, w \in A^*$ such that $uv = w$. We then say that u *is a prefix of* w and that v *is a suffix of* w.

Definition
Given a word $w \in A^*$ and two integers p, q such that $0 \leq p \leq q < |w|$, the $[p,q)$*-slice of* w, denoted $w_{p..q}$, is the word u such that $|u| = q - p$ and $u_{(i)} = w_{(p+i)}$ for all i where $0 \leq i < q - p$.

Theorem
Let $w \in A^*$ and $u = w_{p..q}$ where $0 \leq p \leq q < |w|$. Then the exist words $x, y \in A^*$ such that $|x| = p$, $|y| = |w| - q$, and $w = xuy$.

Theorem
Let $w, u, x, y \in A^*$ and $w = xuy$. Then $x = w_{0..|x|}$, $u = w_{|x|..(|x|+|u|)}$ and $v = w_{(|x|+|u|)..|w|}$.

Language

Definition

A *language* over alphabet A is a set $L \subseteq A^*$.

A language can be empty, it can be finite, it can be infinite while being very difficult to describe, like the set of positions at which a transcendental number has a digit "1"). Last but not the least, the set can also be infinite yet have some structure, where words follow a certain "pattern" that we can describe precisely. It is such last type of languages that will be the most interesting for us.

$L_2 = \{01, 0101, 010101, \ldots\}$ consists of all those non-empty words that are of the form $01 \ldots 01$ where the block 01 is repeated some finite positive number of times. Using notation $(01)^n$ for a word consisting of block 01 repeated n times, we can write $L_2 = \{(01)^n \mid n \geq 1\}$.

Because languages are sets, we can take their union ($\cup$), intersection ($\cap$), and apply other set operations on languages.

Languages $\emptyset$ and $\{\varepsilon\}$ are very different: $\emptyset$ is a set that contains no words, whereas $\{\varepsilon\}$ contains precisely one word, the word of length zero.

Concatenating Languages

In addition to operations such as intersection and union that apply to sets in general, languages support additional operations, which we can define because their elements are words. The first one translates concatenation of words to sets of words, as follows.

Definition (Language concatenation)

Given $L_1 \subseteq A^*$ and $L_2 \subseteq A^*$, define $L_1 \cdot L_2 = \{w_1 w_2 \mid w_1 \in L_1, w_2 \in L_2\}$

The definition above states that $w \in L_1 L_2$ if and only if there is a way to split w into two words w_1 and w_2, so that $w = w_1 w_2$ and such that $w_1 \in L_1$ and $w_2 \in L_2$.

Definition (Language exponentiation)

Given $L \subseteq A^*$, define

$$
\begin{aligned}
L^0 &= \{\varepsilon\} \\
L^{n+1} &= L \cdot L^n
\end{aligned}
$$

Theorem

Given $L \subseteq A^*$, $L^n = \{w_1 \dots w_n \mid w_1, \dots, w_n \in L\}$

Expanding the Definition

If L is an arbitrary language, compute each of the following:

- $L\emptyset$
- $\emptyset L$
- $L\{\varepsilon\}$
- $\{\varepsilon\}L$
- $\emptyset\{\varepsilon\}$
- LL
- $\{\varepsilon\}^n$

Concatenation of Languages

Let A be alphabet. Consider the set of all languages $L \subseteq A^*$

Is this a monoid?

Concatenation of Languages

Let A be alphabet. Consider the set of all languages $L \subseteq A^*$

Is this a monoid?

Does the cancelation law hold?

Repetition of a Language: Kleene Star

Definition (Kleene star)

Given $L \subseteq A^*$, define

$$L^* = \bigcup_{n \geq 0} L^n$$

Theorem

For $L \subseteq A^*$, for every $w \in A^*$ we have $w \in L^*$ if and only if

$$\exists n \geq 0. \exists w_1, \ldots, w_n \in L.\ w = w_1 \ldots w_n$$

$\{a\}^* = \{\varepsilon, a, aa, aaa, \ldots\}$
$\{a, bb\}^* = \{\varepsilon, a, bb, abb, bba, aa, bbbb, aabb, \ldots\}$ (describe this language)
Can L^* be finite for some L?

Starring: $\{a, ab\}$

Let $A = \{a, b\}$ and $L = \{a, ab\}$.
Come up with a property "…" that describes the language L^*:

$$L^* = \{w \in A^* \mid \ldots\}$$

Prove that the property and L^* denote the same language.