Ex 1

1.1

- $\underbrace{c}_{T_4}\ \underbrace{a\ c}_{T_2}\ \underbrace{c}_{T_4}\ \underbrace{a}_{T_1}\ \underbrace{b\ a\ c\ a\ c}_{T_2}\ \dots$

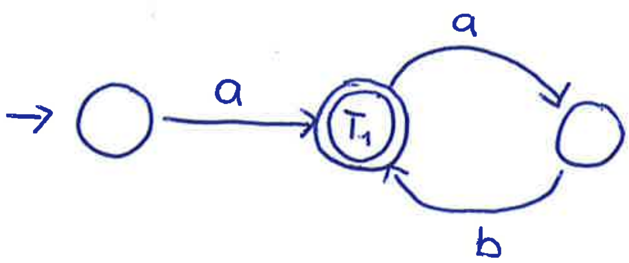
  $\underbrace{c\ b\ a}_{T_3}\ \underbrace{b}_{T_2}\ \underbrace{c}_{T_4}$

- $\underbrace{c\ c\ c}_{T_4}\ \underbrace{a\ a\ b\ a\ b}_{T_1}\ \underbrace{a\ c}_{T_2}\ \dots$

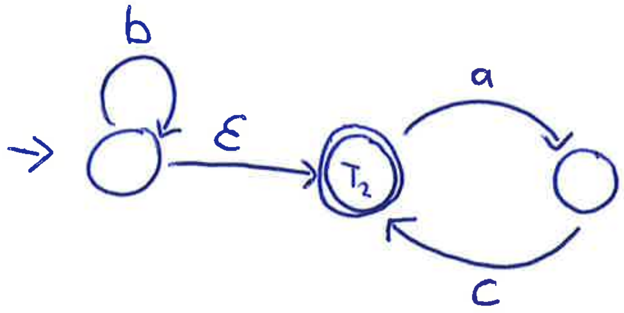
  $\underbrace{c\ b\ a}_{T_3}\ \underbrace{b}_{T_2}\ \underbrace{c\ c}_{T_4}\ \underbrace{b}_{T_2}\ \underbrace{a}_{T_1}\ \underbrace{b\ a\ c}_{T_2}$

1.2

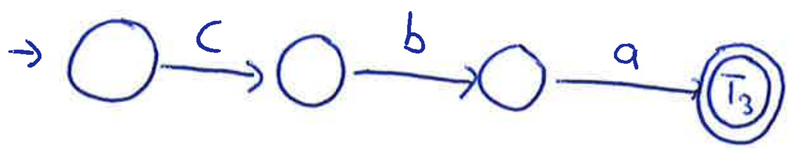
- $a(ab)^*$



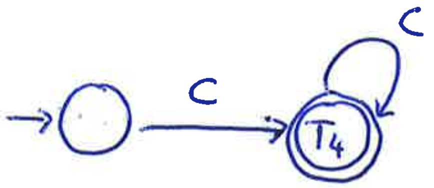
- $b^*(ac)^*$



- cba

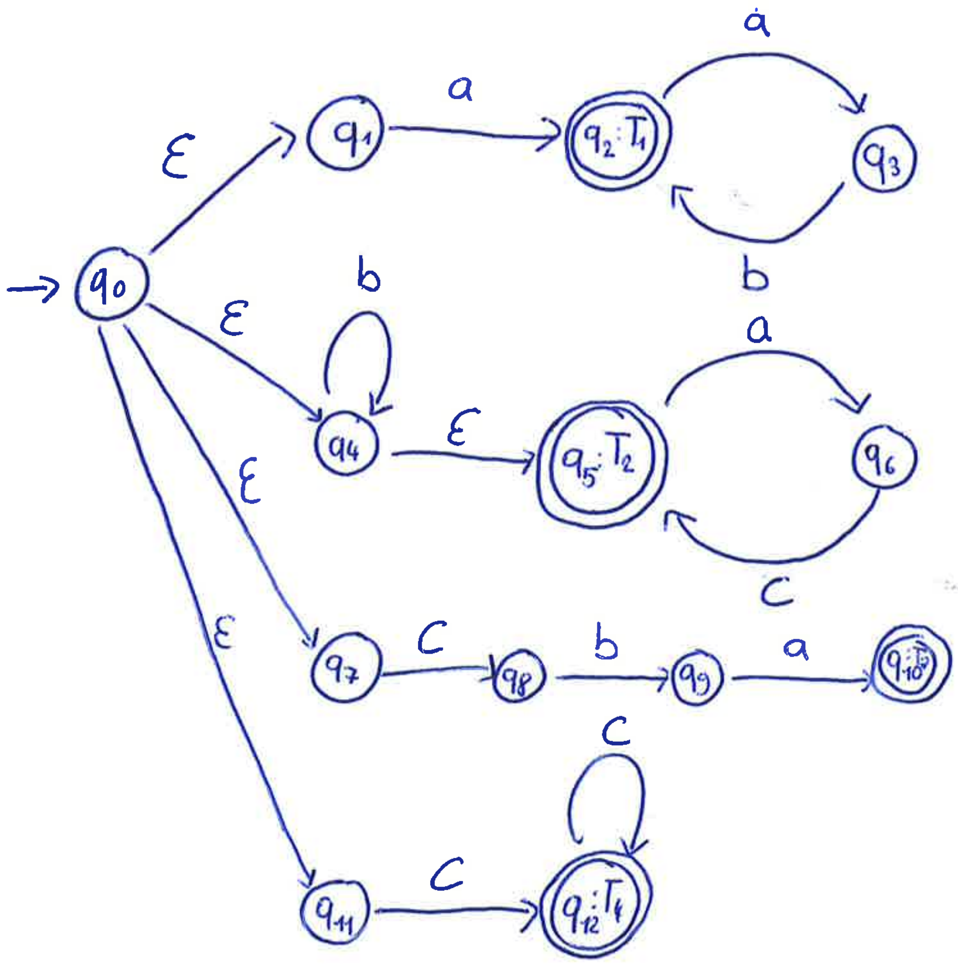


cc^*



1.3

NFA



DFA

Initial state $q_0' = E(q_0) = \{q_0, q_1, q_4, q_5, q_7, q_{11}\}$

From q_0' - on "a" : $\{q_2, q_6\} = q_1'$

- on "b" : $\{q_4, q_5\} = q_2'$

- on "c" : $\{q_8, q_{12}\} = q_3'$

From q'_1 - on "a" : $\{q_3\} = q'_4$
- on "b" : $\{\} = q'_5$
- on "c" : $\{q_5\} = q'_6$

From q'_2 - on "a" : $\{q_6\} = q'_7$
- on "b" : $\{q_4, q_5\} = q'_2$
- on "c" : $\{\} = q'_5$

From q'_3 - on "a" : $\{\} = q'_5$
- on "b" : $\{q_9\} = q'_8$
- on "c" : $\{q_{12}\} = q'_9$

From q'_4 - on "a" : $\{\} = q'_5$
- on "b" : $\{q_2\} = q'_{10}$
- on "c" : $\{\} = q'_5$

From q'_5 - on "a" : $\{\} = q'_5$
- on "b" : $\{\} = q'_5$
- on "c" : $\{\} = q'_5$

From q'_6 - on "a" : $\{q_6\} = q'_7$
- on "b" : $\{\} = q'_5$
- on "c" : $\{\} = q'_5$

From q'_7 - on "a" : $\{\} = q'_5$

- on "b" : $\{\} = q'_5$
- on "c" : $\{q_5\} = q'_6$

From q'_8 - on "a" : $\{q_{10}\} = q'_{11}$

- on "b" : $\{\} = q'_5$
- on "c" : $\{\} = q'_5$

From q'_9 - on "a" : $\{\} = q'_5$

- on "b" : $\{\} = q'_5$
- on "c" : $\{q_{12}\} = q'_9$

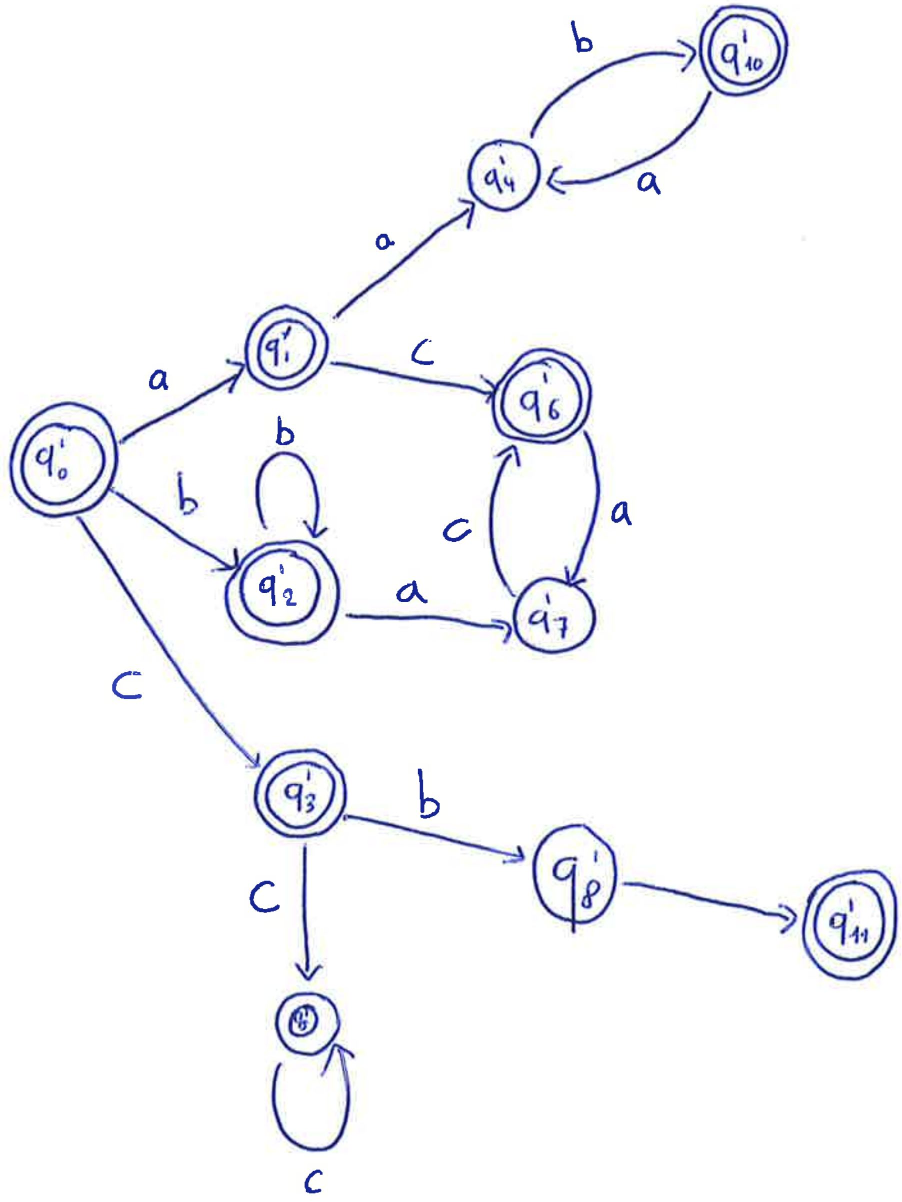
From q'_{10} - on "a" : $\{q_3\} = q'_4$

- on "b" : $\{\} = q'_5$
- on "c" : $\{\} = q'_5$

From q'_{11} - on "a" : $\{\} = q'_5$

- on "b" : $\{\} = q'_5$
- on "c" : $\{\} = q'_5$

Fina states : $q'_0, q'_1, q'_2, q'_3, q'_6, q'_9, q'_{10}, q'_{11}$



Not shown: trap state q'_5

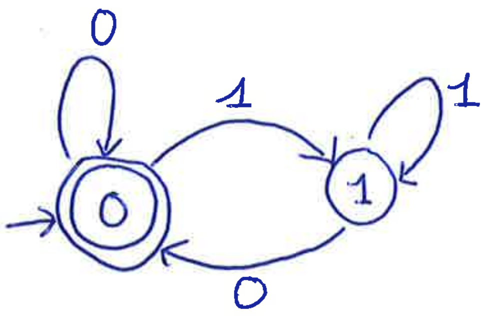
(1.4) DFA already minimal
(Can be shown by building table of state equivalence as discussed).

(1.5) —

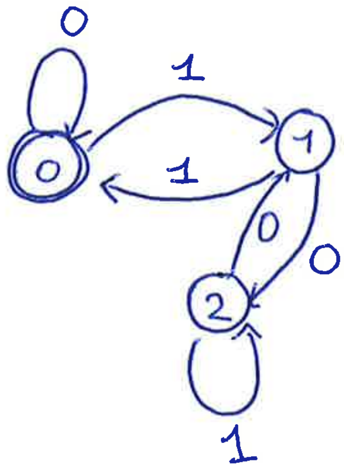
Ex 2

Remark: Adding a 0 after a binary number multiplies it by 2
Adding a 1 multiplies it by 2 and then adds 1.
States in the automata will correspond to
remainders.

2.1

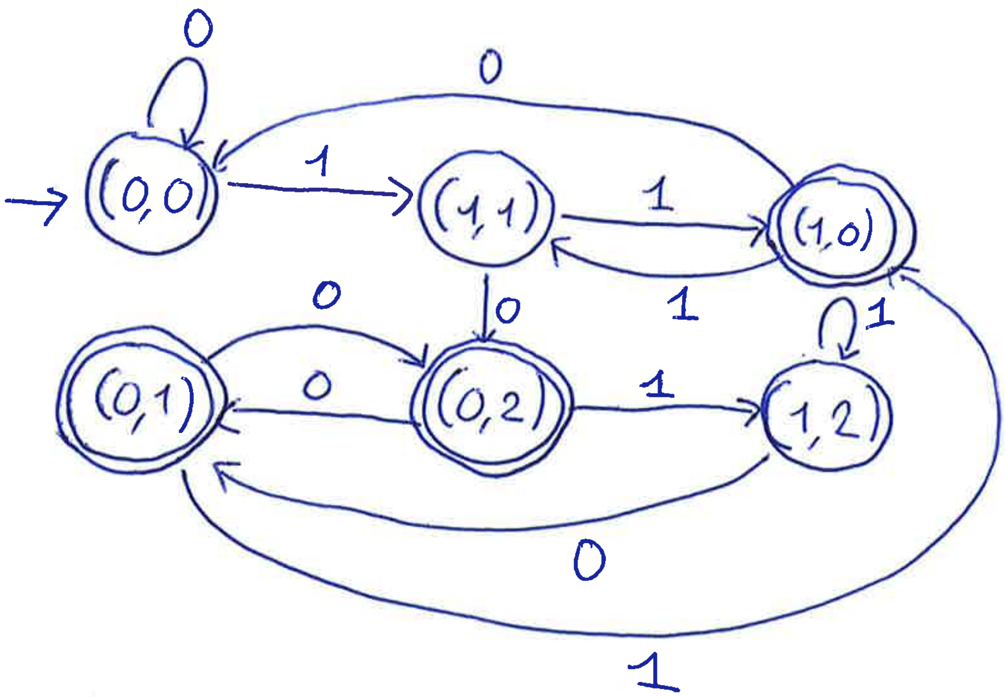


Multiples of 2



Multiples of 3

2.2
&
2.3



Ex 3

Towards a contradiction, assume L is regular. Pumping lemma therefore applies. Let w be a word of L of length at least the pumping constant p.* (Such a word exists due to the infiniteness of the set of primes.)

According to the lemma,

$$w = xyz$$

with $|y| > 0$.

* We also pick w with length strictly greater than 1.

Moreover, for any i, we must have that $xy^iz \in L$, and thus $|xy^iz|$ is prime.

$$|xy^iz| = |x| + i|y| + |z| = |xyz| + (i-1)|y|$$

Consider the case $i = |xyz| + 1$.

We must have

$$|xyz| + |xyz| \cdot |y| =$$

$$\underbrace{(|y|+1)}_{a} \cdot \underbrace{|xyz|}_{b} \quad \text{is prime}$$

Since $|y| > 0$, both a and b are > 1, and thus $a \cdot b$ is not prime. $\lightning$