Chomsky's Classification of Grammars

On Certain Formal Properties of Grammars

(N. Chomsky, INFORMATION AND CONTROL 9., 137-167 (1959)

type 0: arbitrary string-rewrite rules

equivalent to Turing machines!

e X b => e X   e X => Y

type 1: context sensitive, RHS always larger

O(n)-space Turing machines

a X b => a c X b

type 2: context free - one LHS nonterminal

type 3: regular grammars (regular languages)

Parsing Context-Free Grammars

Decidable even for type 1 grammars,
(by eliminating epsilons - Chomsky 1959)

We choose $O(n^3)$ CYK algorithm - simple

Better complexity possible:

General Context-Free Recognition in Less than Cubic Time, JOURNAL OF COMPUTER AND SYSTEM SCIENCES 10, 308--315 (1975)

- problem reduced to matrix multiplication - n^k for k between 2 and 3

More practical algorithms known:

J. Earley **An efficient context-free parsing algorithm,** Ph.D. Thesis, Carnegie Mellon University, Pittsburgh, PA (1968)

can be adapted so that it automatically works in quadratic or linear time for better-behaved grammars

CYK Parsing Algorithm

C:

John **Cocke** and Jacob T. Schwartz (1970). Programming languages and their compilers: Preliminary notes. Technical report, Courant Institute of Mathematical Sciences, New York University.

Y:

Daniel H. **Younger** (1967). Recognition and parsing of context-free languages in time n^3. *Information and Control* 10(2): 189–208.

K:

T. **Kasami** (1965). An efficient recognition and syntax-analysis algorithm for context-free languages. Scientific report AFCRL-65-758, Air Force Cambridge Research Lab, Bedford, MA.

CYK Algorithm Can Handle Ambiguity

Why Parse General Grammars

- General grammars can be ambiguous: for some strings, there are multiple parser trees
- Can be impossible to make grammar unambiguous
- Some languages are more complex than simple programming languages
  - mathematical formulas:

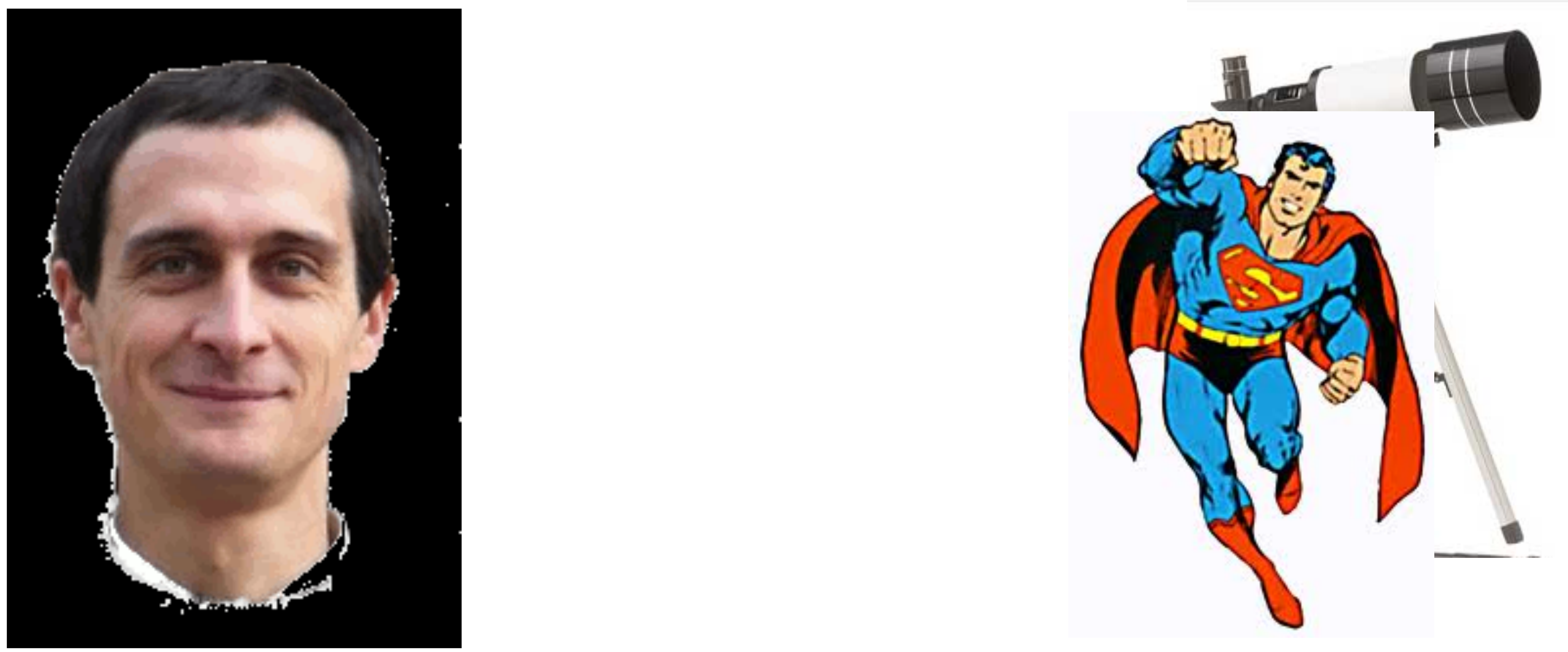
    $x = y \wedge z$? $(x=y) \wedge z$ $x = (y \wedge z)$
  - natural language:

    I saw the man with the telescope.
  - future programming languages

Ambiguity 1

1)

2)



I saw the man with the telescope.

Ambiguity 2

Time flies like an arrow.

Indeed, time passes by quickly.

Those special “time flies” have an “arrow” as their favorite food.

You should regularly measure how fast the flies are flying, using a process that is much like an arrow.

…

Two Steps in the Algorithm

1) Transform grammar to normal form

   called Chomsky Normal Form

2) Parse input using transformed grammar

   dynamic programming algorithm

“a method for solving complex problems by breaking them down into simpler steps.
It is applicable to problems exhibiting the properties of overlapping subproblems”

Dynamic Programming to Parse Input

Assume Chomsky Normal Form, 3 types of rules:

$S' \to \varepsilon \mid S$	(only for the start non-terminal)
$N_i \to t$	(names for terminals)
$N_i \to N_j \ N_k$	(just **2** non-terminals on RHS)

Decomposing long input:

N_i

N_j N_k

(	(	(	)	(	)	)	(	)	)	(	(	)	)

find all ways to parse substrings of length 1,2,3,…

Balanced Parentheses Grammar

Original grammar G

B → ε | B B | (B)

Modified grammar in Chomsky Normal Form:

B1 → ε | B B | O M | O C

B → B B | O M | O C

M → B C

O → '('

C → ')'

Terminals: ( )

Nonterminals: B, B1, O, C, M, B

Parsing an Input

B1 → ε | B B | O M | O C

B → B B | O M | O C

M → B C

O → '('

C → ')'

6								
5								
4								
3								
2								
1	O	O	C	O	C	O	C	C
	(	(	)	(	)	(	)	)
	1	2	3	4	5	6	8	9

Algorithm Idea

w_{pq} – substring from p to q

d_{pq} – all non-terminals that could expand to w_{pq}

Initially d_{pp} has $N_{w(p,p)}$

key step of the algorithm:

if $X \rightarrow Y\ Z$ is a rule,

Y is in $d_{p\,r}$, and

Z is in $d_{(r+1)q}$

then put X into d_{pq}

($p <= r < q$),

in increasing value of (q-p)

Algorithm

```
INPUT:  grammar G in Chomsky normal form
             word w to parse using G
OUTPUT: true iff (w in L(G))
N = |w|
var d : Array[N][N]
for p = 1 to N {
  d(p)(p) = {X | G contains X->w(p)}
  for q in {p + 1 .. N} d(p)(q) = {} }
for k = 2 to N // substring length
  for p = 0 to N-k // initial position
    for j = 1 to k-1 // length of first half
      val r = p+j-1; val q = p+k-1;
      for (X::=Y Z) in G
        if Y in d(p)(r) and Z in d(r+1)(q)
          d(p)(q) = d(p)(q) union {X}
return  S in d(0)(N-1)
```

What is the running time as a function of grammar size and the size of input?

O(   )

(	(	)	(	)	(	)	)

Number of Parse Trees

Let w denote word ()()()

- it has two parse trees

Give a lower bound on number of parse trees of the word w^n (n is positive integer)

w^5 is the word

()()() ()()() ()()() ()()() ()()()

CYK represents all parse trees compactly

- can re-run algorithm to extract first parse tree, or enumerate parse trees one by one