

Exercise 1

Complete solution for the first

Long solution for $\text{def } S(x, y, z) = (x(z))(y(z))$

We begin by assigning type variable to nodes:

```
x: t1
y: t2
z: t3
(x(z))(y(z)): t4
x(z): t5
y(z): t6
```

Then, we generate constraints:

```
From the node (x(z))(y(z)) we get:  t5 = (t6 => t4)
From the node x(z) we get:          t1 = (t3 => t5)
From the node y(z) we get:          t2 = (t3 => t6)
```

Then, we solve constraints:

```
t5 = (t6 => t4)
t1 = (t3 => t5)
t2 = (t3 => t6)
```

We first eliminate t_5 (eliminated constraints will appear in *italics*).

```
t5 = (t6 => t4)
t1 = (t3 => (t6 => t4))
t2 = (t3 => t6)
```

Now, all rules leave the equation set untouched. We are done with constraint solving.

We therefore obtain:

```
def S(x: t3 => (t6 => t4), y: t3 => t6, z: t3): t4
```

Which, after generalisation, leads to:

```
def S[A, B, C](x: A => (B => C), y: A => B, z: A): C
```

Short solutions:

```
def cm[A, B, C](f: A => B, g: C => A): C => B
```

```
def cr[A, B, C](f: (A, B) => C): A => B => C
```

```
def uncr[A, B, C](f: A => B => C): ((A, B)) => C
```

```
def pr[A, B, C](x: A, y: B): (A => B => C) => C
```

```
def c1[A, B, C](p: (A => B => A) => C): C
```

```
def c2[A, B, C](p: (A => B => B) => C): C
```

```
def e[A, B, C](x: A, y: B): A  
  // = c1[A, B, A](pr[A, B, A](x,y))
```

Longer solution for the last (occur check):

Long solution for def $S(x, y, z) = (x(z))(z(x))$

We begin by assigning type variable to nodes:

$x: t_1$
 $y: t_2$
 $z: t_3$
 $(x(z))(z(x)): t_4$
 $x(z): t_5$
 $z(x): t_6$

Then, we generate constraints:

From the node $(x(z))(z(x))$ we get: $t_5 = (t_6 \Rightarrow t_4)$
 From the node $x(z)$ we get: $t_1 = (t_3 \Rightarrow t_5)$
 From the node $z(x)$ we get: $t_3 = (t_1 \Rightarrow t_6)$

Then, we (try to) solve constraints:

$t_5 = (t_6 \Rightarrow t_4)$
 $t_1 = (t_3 \Rightarrow t_5)$
 $t_3 = (t_1 \Rightarrow t_6)$

We first eliminate t_5 :

$t_5 = (t_6 \Rightarrow t_4)$
 $t_1 = (t_3 \Rightarrow (t_6 \Rightarrow t_4))$
 $t_3 = (t_1 \Rightarrow t_6)$

Then, we eliminate t_1 :

$t_5 = (t_6 \Rightarrow t_4)$
 $t_1 = (t_3 \Rightarrow (t_6 \Rightarrow t_4))$
 $t_3 = ((t_3 \Rightarrow (t_6 \Rightarrow t_4)) \Rightarrow t_6)$

Then, we stop due to the “occurs check” rule.

The constraints could not be solved and therefore type inference fails. The function definition can not be typed in our type system.