Attribute Grammars

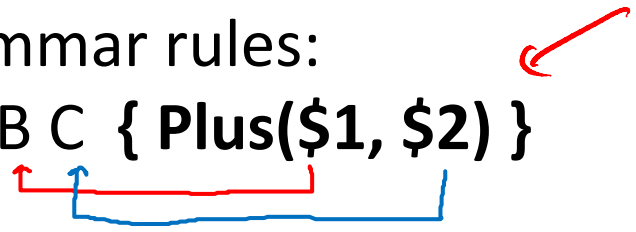
- They extend context-free grammars to give parameters to non-terminals, have rules to combine attributes
- Attributes can have any type, but often they are trees
- Example:
  - context-free grammar rule:

    A ::= B C
  - attribute grammar rules:

    A ::= B C **{ Plus($1, $2) }**

    or, e.g.

    A ::= B**:x** C**:y {: RESULT := new Plus(x.v, y.v) :}**



Semantic actions indicate how to compute attributes

- attributes computed bottom-up, or in more general way

Parser Generators:
Attribute Grammar -> Parser

1) Embedded: parser combinators (Scala, Haskell)

They ***are*** code in some (functional) language

```
def ID : Parser = "x" | "y" | "z"
def expr : Parser = factor ~ (( "+" ~ factor | "-" ~ factor )
                            | epsilon)
def factor : Parser = term ~ (( "*" ~ term | "/" ~ term )
                             | epsilon)
def term : Parser = ( "(" ~ expr ~ ")" | ID | NUM )
```

implicit conversion: string s to skip(s)

concatenation

<- often not really LL(1) but "try one by one", must put first non-empty, then epsilon

implementation in Scala: use **overloading** and **implicits**

2) Standalone tools: JavaCC, Yacc, ANTLR, CUP

- ***generate*** code in a conventional programming languages (e.g. Java)

Example in CUP - LALR(1) (not LL(1))

```
precedence left PLUS, MINUS;
precedence left TIMES, DIVIDE, MOD;  // priorities disambiguate
precedence left UMINUS;


expr ::= expr PLUS expr          // ambiguous grammar works here
        | expr MINUS expr
        | expr TIMES expr
        | expr DIVIDE expr
        | expr MOD expr
        | MINUS expr %prec UMINUS
        | LPAREN expr RPAREN
        | NUMBER ;
```

Adding Java Actions to CUP Rules

```
expr ::= expr:e1 PLUS expr:e2
        {: RESULT = new Integer(e1.intValue() + e2.intValue()); :}
        | expr:e1 MINUS expr:e2
        {: RESULT = new Integer(e1.intValue() - e2.intValue()); :}
        | expr:e1 TIMES expr:e2
        {: RESULT = new Integer(e1.intValue() * e2.intValue()); :}
        | expr:e1 DIVIDE expr:e2
        {: RESULT = new Integer(e1.intValue() / e2.intValue()); :}
        | expr:e1 MOD expr:e2
        {: RESULT = new Integer(e1.intValue() % e2.intValue()); :}
        | NUMBER:n   {: RESULT = n; :}
        | MINUS expr:e
      {: RESULT = new Integer(0 - e.intValue()); :} %prec UMINUS
        | LPAREN expr:e RPAREN {: RESULT = e; :} ;
```

A CYK Algorithm Producing Results

input word: $w = w_{(0)}w_{(1)} \ldots w_{(N-1)}$, $w_{p..q} = w_{(p)}w_{(p+1)} \ldots w_{(q-1)}$
Non-terminals $A_1,\ldots,A_K$, tokens $t_1,\ldots.t_L \in T$

Rule **$(A::=B_1 \ldots B_m , f) \in G$** with semantic action f. **R** - result (e.g. tree)

$f : ((A \times N \times N) \times (R \cup T))^m \rightarrow R$

Useful parser: returning a set of result (e.g. syntax trees)

((A, p, q),**r**): $A \Rightarrow^* w_{p..q}$ and the result can be interpreted as **r**

Let **f** be **partial** function, we apply it only if the result is defined

$P = \{((w_{(i)},i,i+1),w_{(i)}) \mid 0 \le i < N-1\}$ // set of ((A,p,q), r)

repeat {

choose rule $(A::=B_1 \ldots B_m , f) \in G$

choose $((B_1,p_0,p_1),r_1), \ldots, ((B_m,p_{m-1},p_m),r_2) \in P$

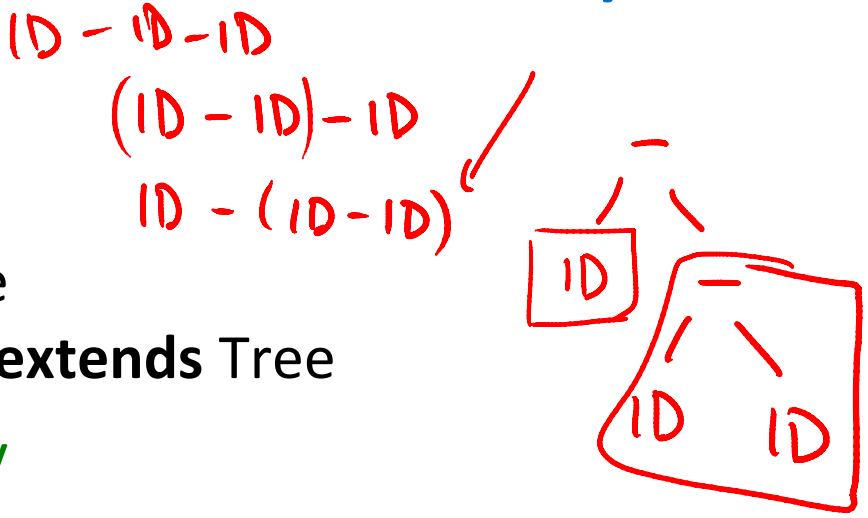
$P := P \cup \{((A,p_0,p_m), f(((B_1,p_0,p_1),r_1), \ldots,((B_m,p_{m-1},p_m),r_2)))\}$

// do nothing if **f** is not defined on those args

} until no more insertions into P possible

Simple Application: Associativity

e ::= e - e | ID



abstract class Tree
case class ID(s:String) **extends** Tree
case class Minus(e1:Tree,e2:Tree) **extends** Tree

Define rules that will return only

Suppose minus is **left associative**

Result attribute type: Tree. Defining semantic action:

General rule: if can parse e from i to j, then minus, then another e from j+1 to k, then can parse e from i to k

f(((e,i,j),t_1), ((-, j, j+1), _), ((e,j+1,k), t_2) = Minus(t_1,t_2)

Restriction: only if t_2 is not Minus(...,...) otherwise undefined

Discuss: right associativity, priority of * over - , parentheses.

Priorities

- In addition to the tree, return the priority of the tree
  - usually the priority is the top-level operator
  - parenthesized expressions have high priority, as do other 'atomic' expressions (identifiers, literals)
- Disallow combining trees if the priority of current right-hand-side is higher than priority of results being combining
- Given: x - y * z with priority of * higher than of -
  - disallow combining x-y and z using *
  - allow combining x and y*z using -

Probabilities: Natural Language Processing

Represent the set of tuples ((A, p, q),$\mathbf{r_1}$),..., ((A, p, q),$\mathbf{r_n}$)

as a map from (A,p,q) to ranked priority queue $\mathbf{r_1}$ **,..., $\mathbf{r_1}$**

Example application: probabilistic context-free grammars (can be learned from corpus).

Each rule has a probability **p**

This assigns probability to the space of all possible parse trees

r stores pointers to sub-trees and probability of the parse tree **q**

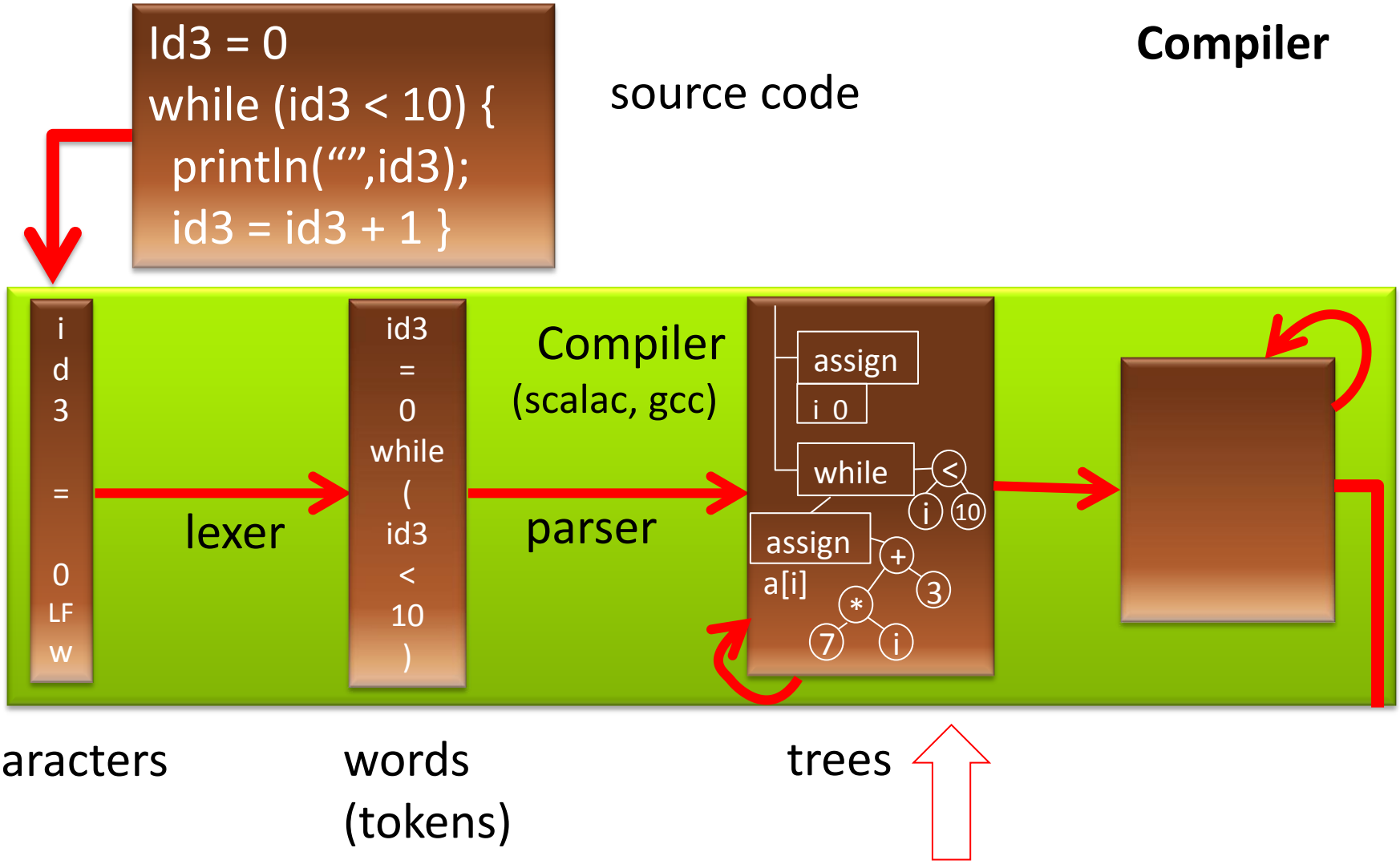
$$f(((B_1,p_0,p_1),(\_,\mathbf{q_1})), ..., ((B_m,p_{m-1},p_m),(\_,\mathbf{q_m})))$$

$$= ((B_1,p_0,p_1)...(B_m,p_{m-1},p_m) , \mathbf{p \cdot q_1 \cdot ... \cdot q_m})$$

For binary rules: how we split it, and what probability we got.

more (optional) in book: **Daniel Jurafsky, James H. Martin: Speech and Language Processing, Pearson (2nd edition), (Part III)**

http://www.cs.colorado.edu/~martin/SLP/


Id3 = 0
while (id3 < 10) {
println(“”,id3);
id3 = id3 + 1 }
source code
Compiler
i
d
3
=
0
LF
w
lexer
id3
=
0
while
(
id3
<
10
)
Compiler
(scalac, gcc)
parser
assign
i 0
while
<
i
10
assign
a[i]
+
*
3
7
i
characters
words
(tokens)
trees
Name Analysis
(step towards type checking)

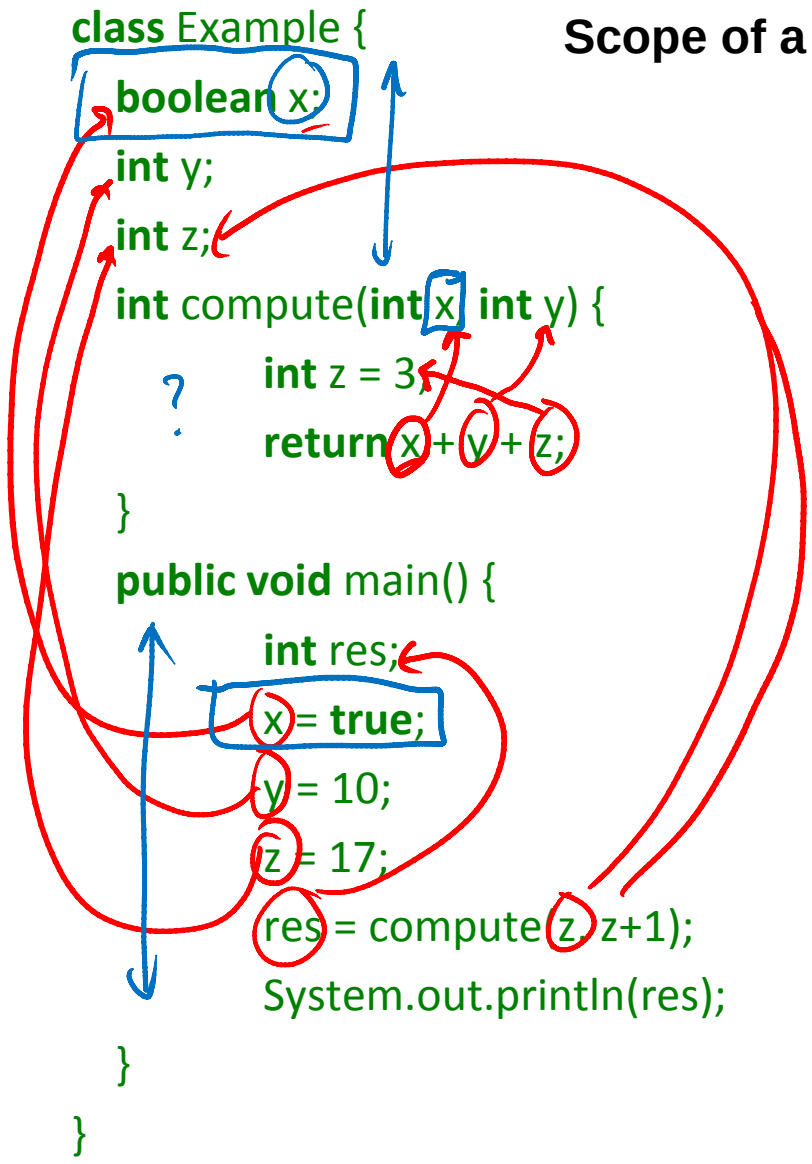
Name Analysis Problems Detected

- a class is defined more than once: **class A { ...} class B { ... } class A { ... }**
- a variable is defined more than once: **int x; int y; int x;**
- a class member is overloaded (forbidden in Tool, requires **override** keyword in Scala):
  class A { int x; ... } class B extends A { int x; ... }
- a method is overloaded (forbidden in Tool, requires **override** keyword in Scala):
  class A { int x; ... } class B extends A { int x; ... }
- a method argument is shadowed by a local variable declaration (forbidden in Java, Tool):
  def (x:Int) { var x : Int; ...}
- two method arguments have the same name: **def (x:Int,y:Int,x:Int) { ... }**
- a class name is used as a symbol (as parent class or type, for instance) but is not declared:
  class A extends Objekt {}
- an identifier is used as a variable but is not declared:
  def(amount:Int) { total = total + ammount }
- the inheritance graph has a cycle: **class A extends B {} class B extends C {} class C extends A**

To make it efficient and clean to check for such errors, we associate mapping from each identifier to the symbol that the identifier represents.

- We use Map data structures to maintain this mapping (Map, what else?)
- The rules that specify how declarations are used to construct such maps are given by ***scoping* rules of the programming language.**

Example: program result, symbols, scopes



Scope of a variable = part of program where it is visible

Draw an arrow from occurrence of each identifier to the point of its declaration.

For each declaration of identifier, identify where the identifier can be referred to (its scope).

Name analysis:

- compute those arrows
  - = maps, partial functions (math)
  - = environments (PL theory)
  - = symbol table (implementation)
- report some simple semantic errors

Usually introduce **symbols** for things denoted by identifiers.
Symbol tables map identifiers to symbols.

Usual **static** scoping: What is the result?

```
class World {
  int sum;
  int value;
  void add() {
     sum = sum + value;
     value = 0;
  }
  void main() {
      sum = 0;
      value = 10;
      add();
      if (sum % 3 == 1) {
        int value;
        value = 1;
        add();
        print("inner value = ", value);  1
        print("sum = ", sum);  10
      }
      print("outer value = ", value);  0
   }
}
```

Identifier refers to the symbol that was declared closest to the place **in program text** (thus "static").

We will assume static scoping unless otherwise specified.

Cool property: we could always rename variables to avoid any shadowing (make all vars unique).

Renaming Statically Scoped Program

```
class World {
  int sum;
  int value;
  void add(int foo) {
     sum = sum + value;
     value = 0;
  }
  void main() {
      sum = 0;
      value = 10;
      add();
      if (sum % 3 == 1) {
        int value1;
        value1 = 1;
        add(); // cannot change value1
        print("inner value = ", value1); 1
        print("sum = ", sum);  10
      }
      print("outer value = ", value);  0
   }
}
```

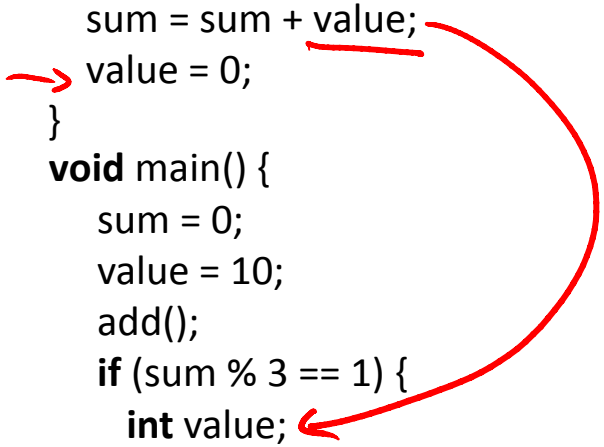
Identifier refers to the symbol that was declared closest to the place **in program text** (thus "static").

We will assume static scoping unless otherwise specified.

Cool property: we could always rename variables to avoid any shadowing (make all vars unique).

Dynamic scoping: What is the result?

```
class World {
  int sum;
  int value;
  void add() {
     sum = sum + value;
    value = 0;
  }
  void main() {
      sum = 0;
      value = 10;
      add();
      if (sum % 3 == 1) {
        int value;
        value = 1;
        add();
        print("inner value = ", value);  0
        print("sum = ", sum);  11
      }
      print("outer value = ", value);  0
   }
}
```



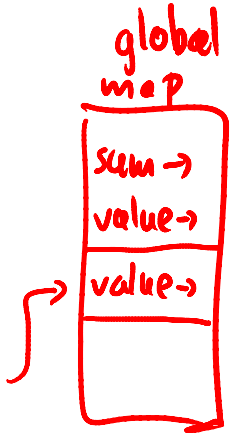
Symbol refers to the variable that was most **recently declared within program execution.**

Views variable declarations as executable statements that establish which symbol is considered to be the ‘current one’. (Used in old LISP interpreters.)

Translation to normal code: access through a dynamic environment.

Dynamic scoping translated using global map, working like stack

```
class World {
  int sum;
  int value;
  void add() {
    sum = sum + value;
    value = 0;
  }
  void main() {
      sum = 0;
      value = 10;
      add();
      if (sum % 3 == 1) {
        int value;
        value = 1;
        add();
        print("inner value = ", value);  0
        print("sum = ", sum);  11
      }
      print("outer value = ", value);  0
    }
}
```




```
class World {
  pushNewDeclaration('sum);
  pushNewDeclaration('value);
  void add(int foo) {
    update('sum, lookup('sum) + lookup('value));
    update('value, 0);
  }
  void main() {
      update('sum, 0);
      update('value,10);
      add();
      if (lookup('sum) % 3 == 1) {
        pushNewDeclaration('value);
        update('value, 1);
        add();
        print("inner value = ", lookup('value));
        print("sum = ", lookup('sum));
        popDeclaration('value)
      }
      print("outer value = ", lookup('value));
    }
}
```

How map changes with **static** scoping

```
class World {
 int sum;  int value;
 // value → int, sum → int
 void add(int foo) {
    // foo → int, value → int, sum → int
    string z;
    // z → string, foo → int, value → int, sum → int
    sum = sum + value; value = 0;
 }
 // value → int, sum → int
 void main(string bar) {
    // bar → string, value → int, sum → int
    int y;
    // y → int, bar → string, value → int, sum → int
    sum = 0;
    value = 10;
    add();
    // y → int, bar → string, value → int, sum → int
    if (sum % 3 == 1) {
      string value;
      // value → string, y → int, bar → string, sum → int
      value = 1;
      add();
      print("inner value = ", value);
      print("sum = ", sum); }
    // y → int, bar → string, value → int, sum → int
    print("outer value = ", value);
} }
```

Outer declaration
int value is shadowed by
inner declaration **string value**

Map becomes bigger as
we enter more scopes,
later becomes smaller again
Imperatively: need to make
maps bigger, later smaller again.
Functionally: immutable maps,
keep old versions.

Abstract Trees of Simple Language with Arbitrarily Nested Blocks

program ::= **class World {** varDecl* method* **}**
method ::= varDecl **(** varDecl* **) {** thing* **return** expr **}**
varDecl ::= type ID
type ::= **int** | **boolean** | **void**
thing ::= varDecl | stmt
stmt ::= expr | if | while | block
if ::= **if (**expr**)** stmt **else** stmt
while ::= **while** expr stmt
block ::= **{** thing* **}**
expr ::= **ID** | expr **+** expr | expr **<=** expr| assign | call | condExpr
assign ::= **ID =** expr
condExpr ::= expr **?** expr **:** expr
call ::= **ID (** expr* **)**

NOTATION FOR MAPS

Mathematical notion of map $f : A \rightharpoonup B$ is a partial function, that is, a function from a subset of A to B.

- $f \subseteq A \times B$
- $\forall x. \forall y_1. \forall y_2.\ (x, y_1) \in f \wedge (x, y_2) \in f \rightarrow y_1 = y_2$

We define $dom(f) = \{x \mid \exists y. (x, y) \in f\}$

$f[k := v] = f \oplus \{k \mapsto v\}$

Key operation is function update

$$f[k := v] = \{(x, y) \mid (x = k \wedge y = v) \vee (x \neq k \wedge (x, y) \in f)\}$$

If the value was defined before, now we redefine it.

A generalization of update is overriding one map by another:

$$f \oplus g = \{(x, y) \mid (x, y) \in g \vee (x \notin dom(g) \wedge (x, y) \in f\}$$

Sometimes we denote map $\{(k_1, v_1), \ldots, (k_n, v_n)\}$ by $\{k_1 \mapsto v_1, \ldots, k_n \mapsto v_n\}$

Is $f \oplus g = g \oplus f$?

- $\{x \mapsto b, z \mapsto a\} \oplus \{x \mapsto c\} = \{x \mapsto c, z \mapsto a\}$
- $\{x \mapsto c\} \oplus \{x \mapsto b, z \mapsto a\} = \{x \mapsto b, z \mapsto a\}$

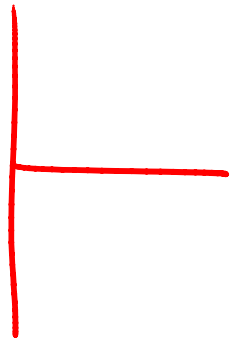
Rules to check that each variable used in a statement is declared

$x_1 \mapsto T_1$

$\Gamma = \{ (x_1, T_1), \ldots (x_n, T_n) \}$ - environment (symbol table)

identifier

symbol (type, ...)



$\Gamma \vdash e$

e uses only variables declared in Γ

$\Gamma = \{ (x, int), (y, string), (z, int) \}$

then:

$\Gamma \vdash x + (z + 1)$

$\Gamma \vdash x = x + 1$

Rules for Checking Variable Use

$$\frac{\Gamma \vdash e_1 \quad \Gamma \vdash e_2}{\Gamma \vdash e_1 + e_2}$$

$$\frac{\Gamma \vdash e_1 \quad \Gamma \vdash e_2}{\Gamma \vdash e_1 * e_2}$$

$$\frac{x \in \mathrm{dom}(\Gamma) \quad (x, \_) \in \Gamma \qquad \Gamma \vdash e}{\Gamma \vdash x = e}$$

$\exists t.\ (x, t) \in \Gamma$

$(x, \_) \in \Gamma$

$$\frac{\Gamma \vdash s \quad \Gamma \vdash \bar{s}}{\Gamma \vdash s; \bar{s}}$$

s - statement

$\bar{s}$ - statement sequence

Local Block Declarations Change Γ

$$\frac{\Gamma[x := \text{int}] \vdash \overline{s}}{\Gamma \vdash (\text{int } x); \overline{s}}$$

$\Gamma \oplus \{(x, \text{int})\}$

$$\frac{\Gamma \vdash e \quad , \quad \Gamma \vdash s_1 \quad , \quad \Gamma \vdash s_2}{\Gamma \vdash \text{if } (e)\ s_1 \text{ else } s_2}$$

Γ

```
if (x>0) {
  int y;   y = x+1
} else { int z;  z = x+1 }
```

$\Gamma = \{(z, \text{int})\}$

$\Gamma[x := \text{int}] = \{(z, \text{int}), (x, \text{int})\}$

$$\frac{\Gamma[x := \text{int}] \vdash x = z + 2}{\Gamma \vdash \text{int } x;\ x = z + 2}$$

Method Parameters are Similar

$$\frac{\Gamma \oplus \{(x_1, T_1), \ldots, (x_n, T_n)\} \vdash \bar{s}}{\Gamma \vdash T\ m\ (T_1\ x_1, \ldots, T_n\ x_n)\ \{\ \bar{s}\ \}}$$

$\Gamma = \{(sum, int), (value, int)\}$

$\Gamma \oplus \{(foo, int)\} \vdash sum = sum + foo;$

$\Gamma \vdash$ void add (int foo){
sum = sum + foo;
}

```
class World {
  int sum;
  int value;
  void add(int foo) {
     sum = sum + foo;
  }
}
```