With additional typing rules, the following program type-checks according to the rules given in the course:

```
int x;
int y;
if (1 > 42)
  {return x}
else
  {return y}
x := y;
```

The new typing rules are:

$$\frac{\Gamma \vdash x : Int \qquad \Gamma \vdash y : Int}{\Gamma \vdash (x > y) : Bool}$$

Here is the derivation tree:

$$\dfrac{\dfrac{\dfrac{}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash if\,(0 > 0)\,\{\,if\,(3 > 4)\,\{x = y;\}\,else\,\{\,\{y = x;\}\,\}\,else\,if\,(4 > 3)\,x = x; else\,y = y;\}\ :\ void}\ Tree_0 \quad Tree_1 \quad Tree_3}{\{(x, Int)\} \vdash int\,y;\ if\,(0 > 0)\,\{\,if\,(3 > 4)\,\{x = y;\}\,else\,\{y = x;\}\,\}\,else\,\{\,if\,(4 > 3)\,x = x; else\,y = y;\}\ :\ void}}{int\,x;\ int\,y;\ if\,(0 > 0)\,\{\,if\,(3 > 4)\,\{x = y;\}\,else\,\{y = x;\}\,\}\,else\,\{\,if\,(4 > 3)\,x = x; else\,y = y;\}\ :\ void}$$

$Tree_0$:

$$\dfrac{\dfrac{\dfrac{}{\vdash 0 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash 0 : Bool} \quad \dfrac{\dfrac{}{\vdash 0 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash 0 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash (0 > 0)\ :\ Bool}$$

$Tree_1$ **(... is the same as its left neighbor with x and y swaped)**

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash 3 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash 3 : Bool} \quad \dfrac{\dfrac{}{\vdash 4 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash 4 : Bool}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash (3 > 4) : Bool} \quad \dfrac{\dfrac{\dfrac{}{(x, Int) \in \{(x, Int)\} \oplus \{(y, Int)\}}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash x : Int} \quad \dfrac{\dfrac{}{(y, Int) \in \{(x, Int)\} \oplus \{(y, Int)\}}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash y : Int}}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash x = y; : Void} \quad ...}{\{(x, Int)\} \oplus \{(y, Int)\} \vdash if\,(3 > 4)\,\{x = y;\}\,else\,\{y = x;\}\ :\ Void}$$

$Tree_2$ **is similar to** $Tree_1$