Faster Algorithms for Algebraic Path Properties in Recursive State Machines with Constant Treewidth

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Interprocedural Analysis

Classic algorithmic problem on the control-flow graphs of programs

- reachability
- constant propagation
- live variable analysis
- ...

Algorithmic Improvements

1. Multiple queries
   - Preprocess phase
   - Query phase
2. Control-flow graphs have special structure

Recursive State Machines

A **Recursive State Machine** is a collection of modules $\{A_1, A_2, \ldots, A_k\}$. Each A_i consists of

1. A set B_i of boxes
2. A map $Y_i : B_i \to \{1, \ldots k\}$ from each box to a module
3. A set V_i of nodes:
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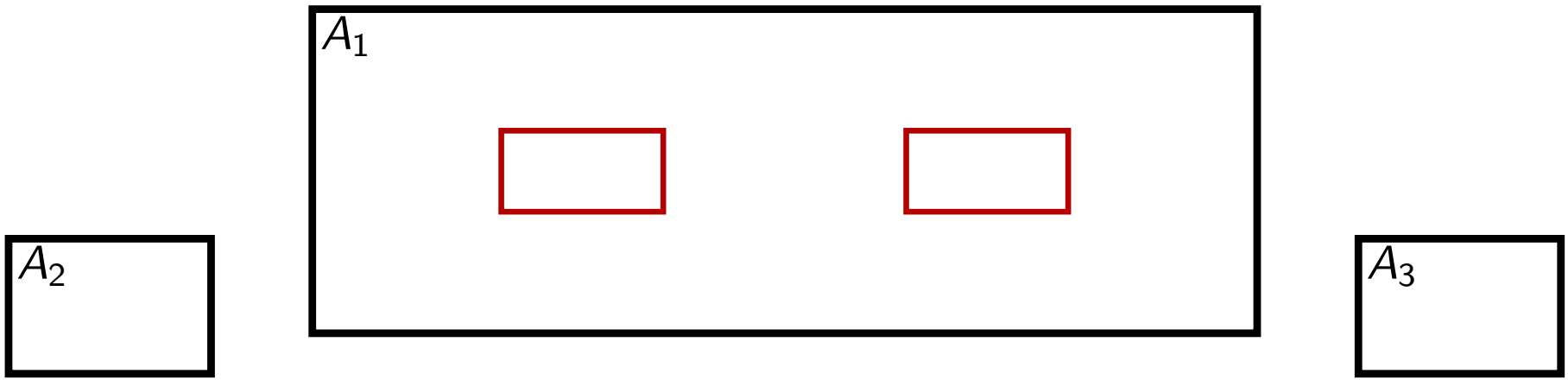

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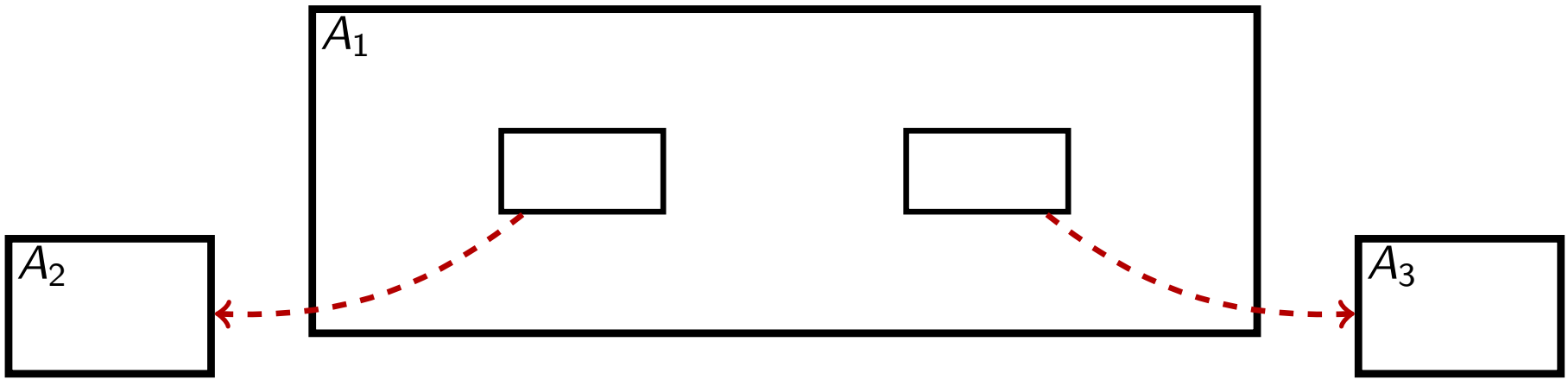

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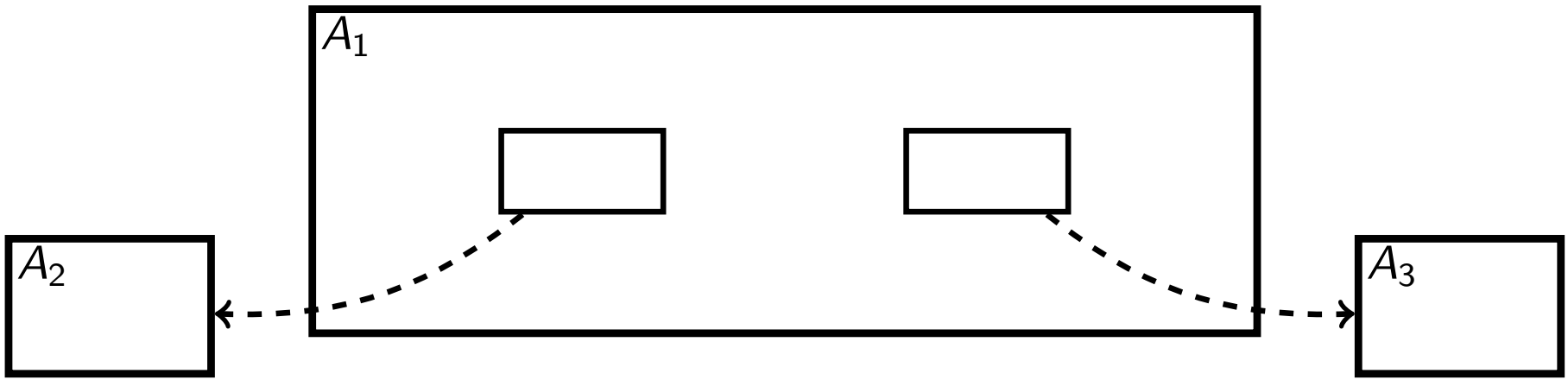

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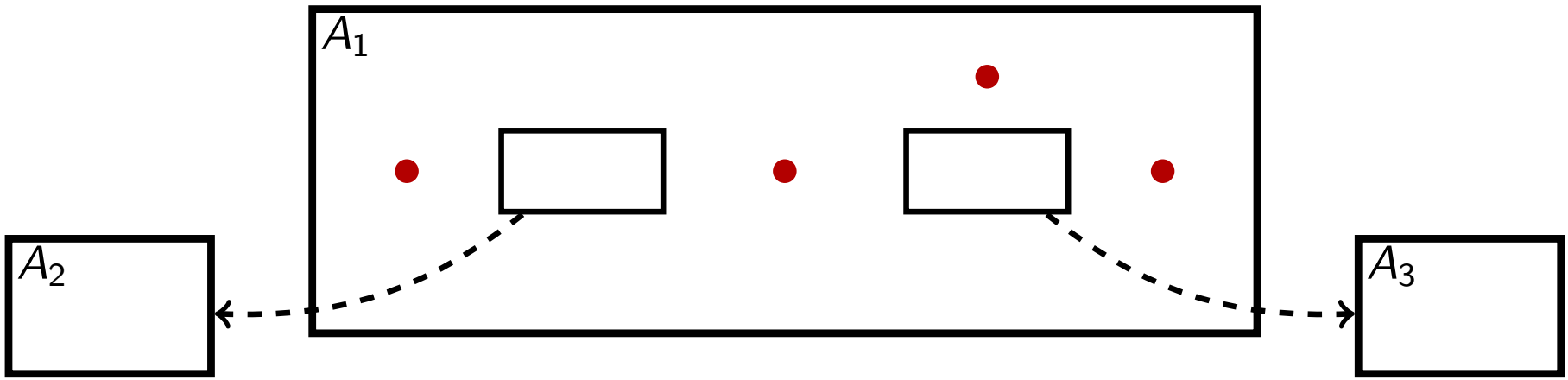

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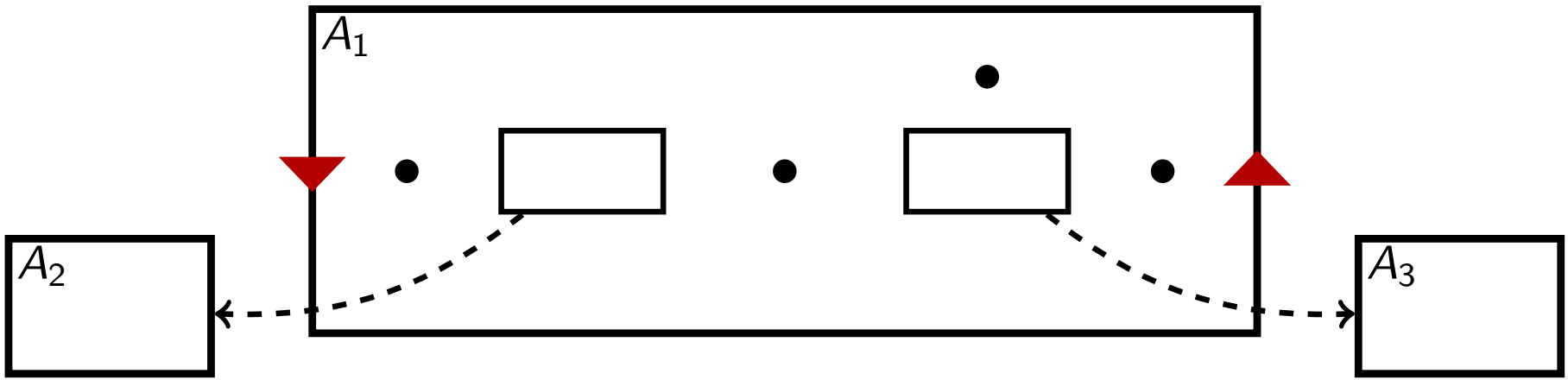

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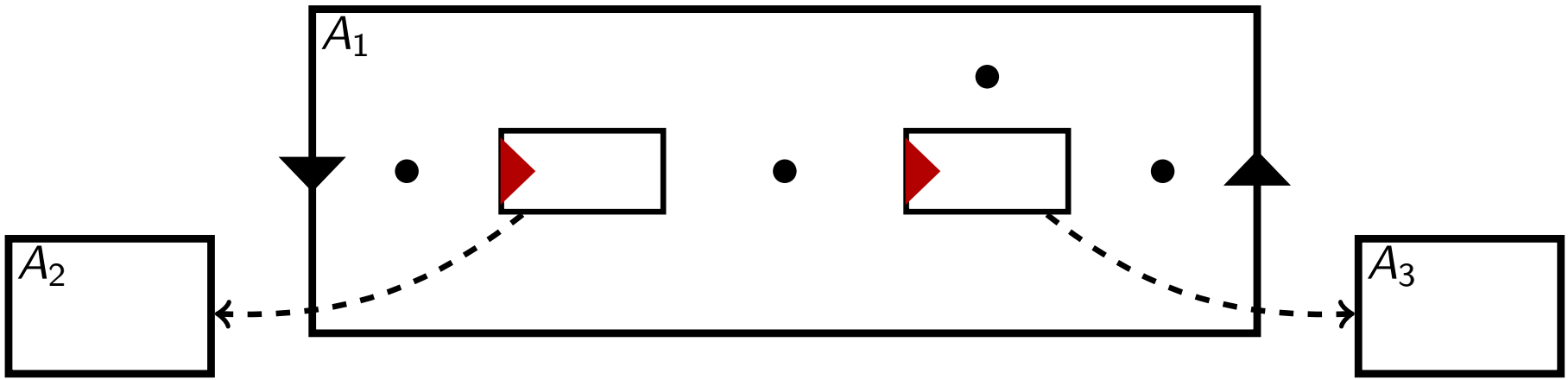

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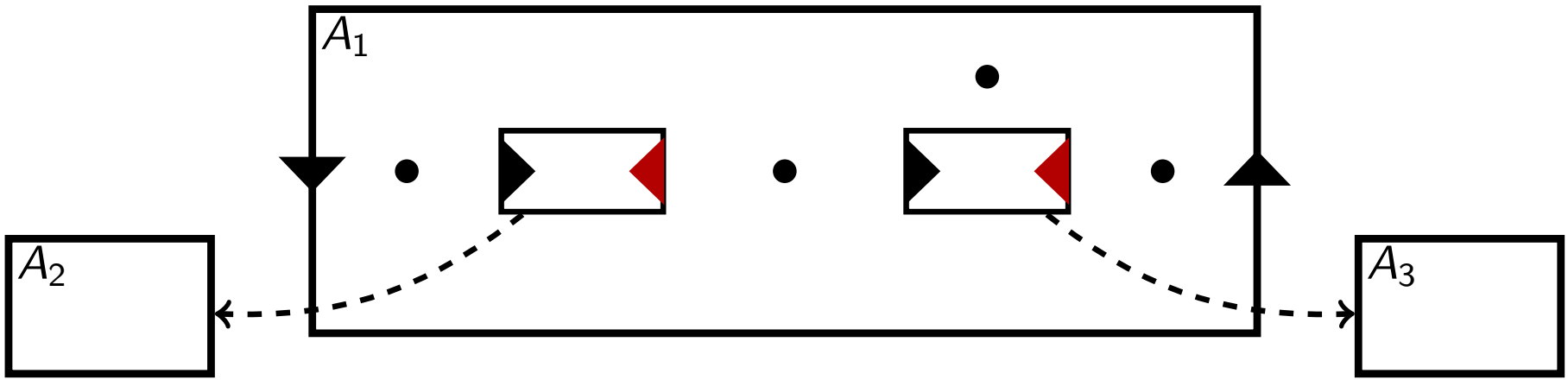

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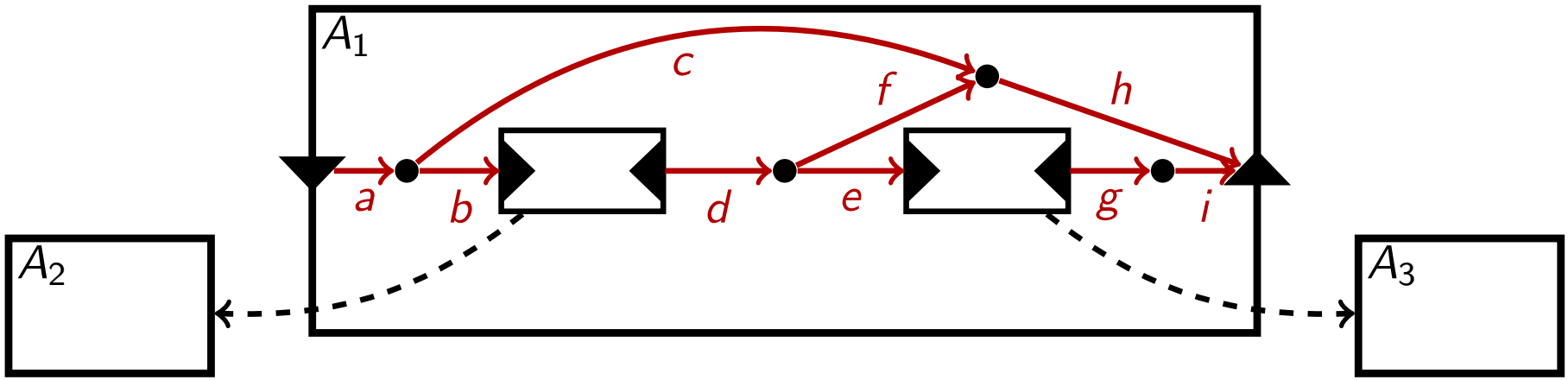
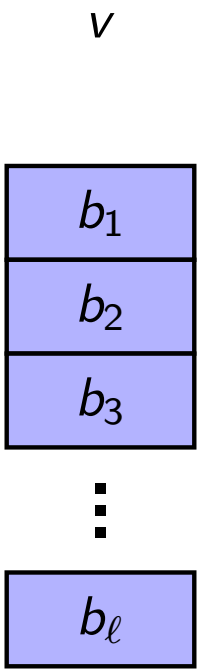


Configurations

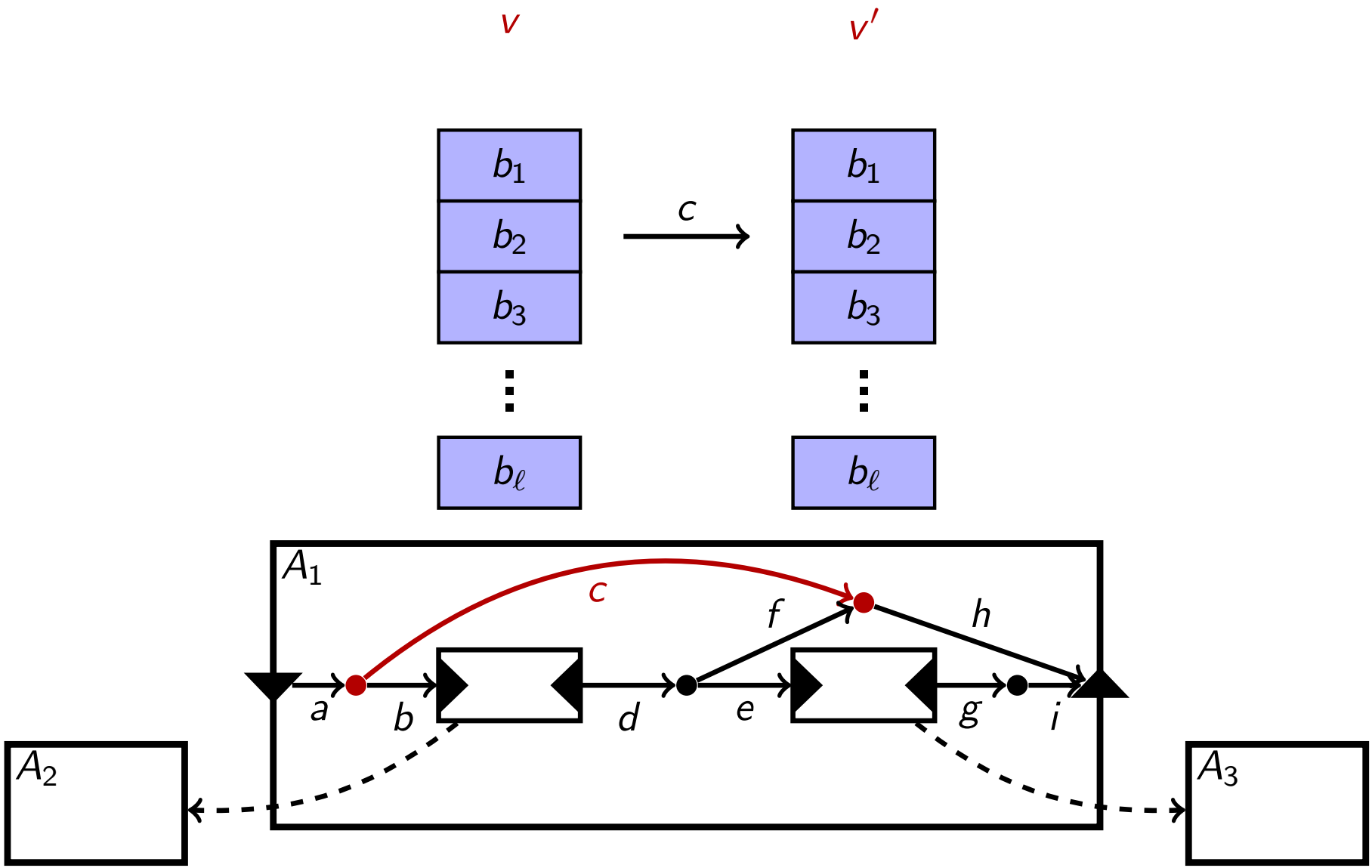
A **configuration** is a pair $c = (v, L)$

1. v is a node of a CSM
2. L is a sequence of boxes called the stack



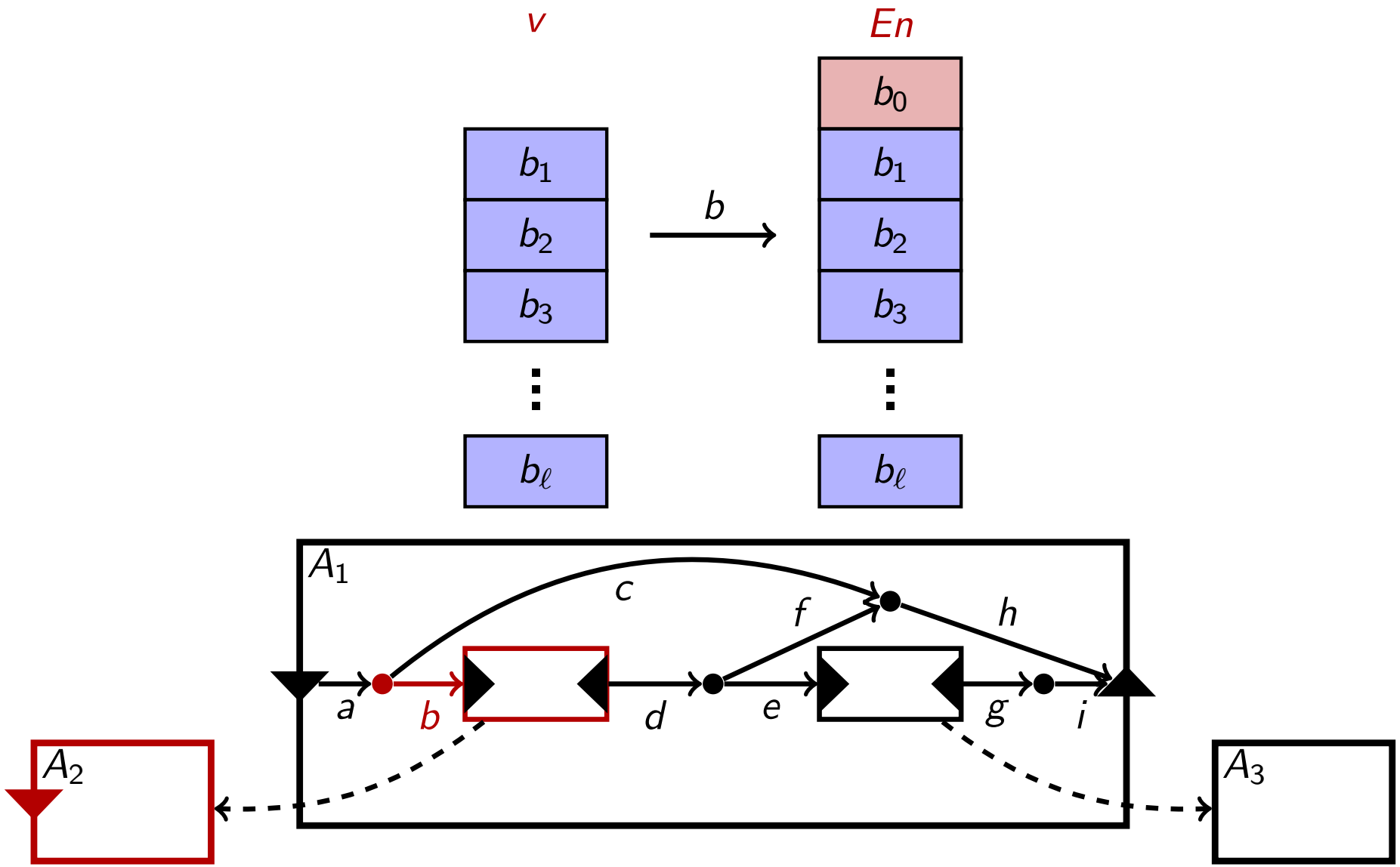
Transitions

Internal transition



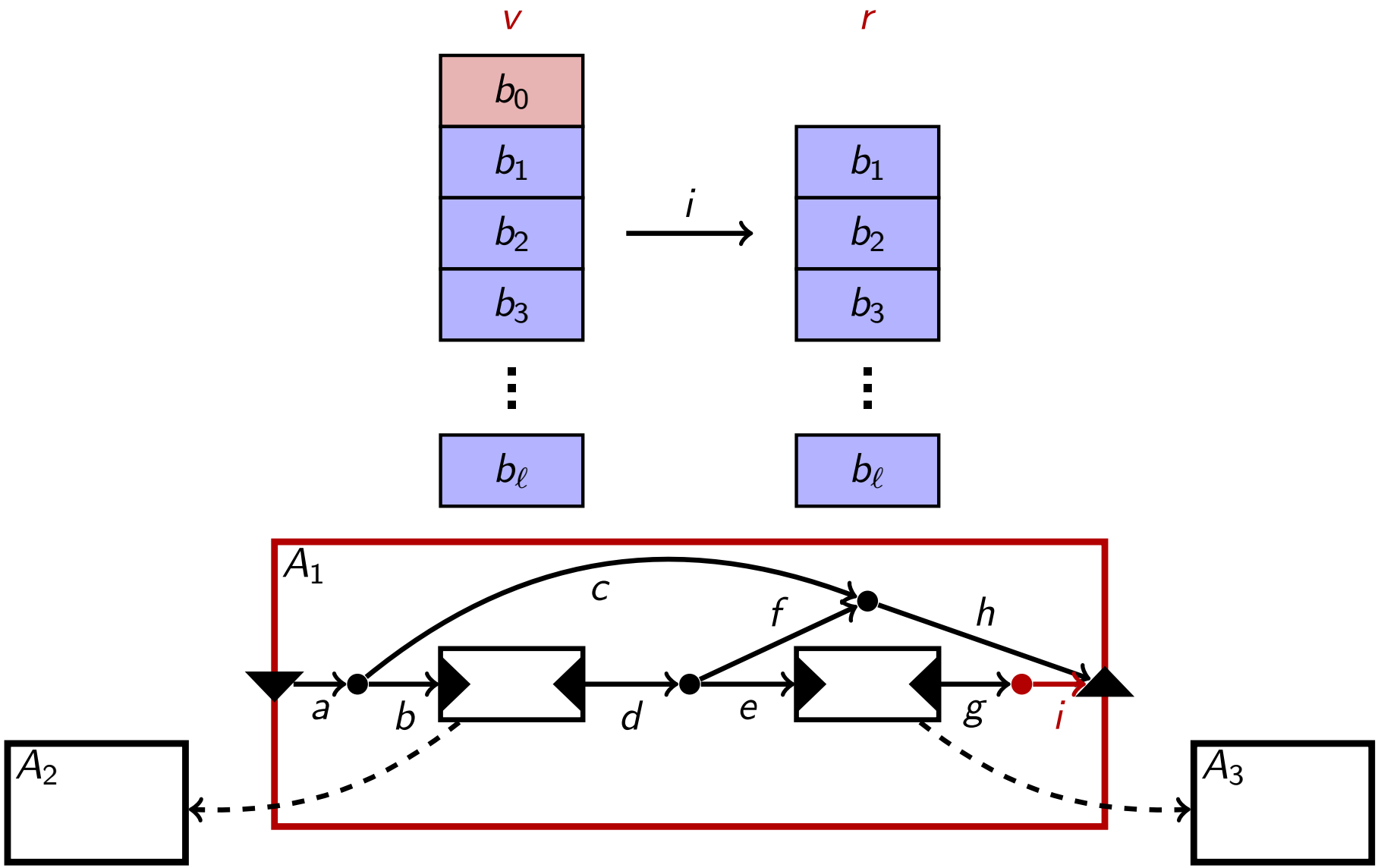
Transitions

Call transition



Transitions

Return transition



Paths & Distances

- A **path** $P = (c_1, \ldots c_\ell)$ is a sequence of labeled transitions
  - Only consider *realizable* paths
- Σ together with $\bigotimes$ and $\bigoplus$ form a semiring
- The weight $\otimes(P)$ of P is $(\sigma_1 \otimes \sigma_2 \cdots \otimes \sigma_\ell)$
- The **distance** from node u to v is

$$d(u, v) = \bigoplus_{P: u \to v} \otimes(P)$$

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Modeled Problems

- Reachability Boolean semiring. $\Sigma = \{\text{True}, \text{False}\}$, $\bigotimes = \wedge$, $\bigoplus = \vee$
- Shortest path Tropical semiring. $\Sigma = \mathbb{R}_{\geq 0} \cup \{\infty\}$, $\bigotimes = +$, $\bigoplus = \min$
- IFDS (Interprocedural, Finite, Distributed, Subset Problems)
  - Finite set D of data facts
  - Universe F of distributive flow functions $f : 2^D \to 2^D$
  - Functions are composed along a path $f_1 \circ f_2, \ldots$
  - Meet $\sqcap$ over all paths

  Meet-composition semiring $\Sigma = F$, $\bigotimes = \circ$, $\bigoplus = \sqcap$

Reachability and Shortest Path

Existing algorithms for single instances

Reachability	
Time	Space
$O(n)$	$O(n)$

Shortest Path	
Time	Space
$O(n^4)$	$O(n)$

Consideration 1: Preprocess vs Query

Preprocess vs Query

- **Preprocess phase**: Spend some resources to preprocess the RSM
- **Query phase**:
  - Pair query: Given nodes u, v in A_i, return $d(u, v)$
  - Single-source query: Given a node u in A_i, return $d(u, v)$ for every other node v in A_i

Complete vs No Preprocessing

Preprocessing spectrum

No Preprocessing

Complete Preprocessing

Our Work

Complete vs No Preprocessing

Preprocessing spectrum

No Preprocessing

Our Work

Complete Preprocessing

Consideration 2: Treewidth of control-flow graphs

Our Results (Reachability)

Reachability				
	Preprocess time	Space	Single-source query	Pair query
Complete preprocessing	$O(n^2)$	$O(n^2)$	$O(n)$	$O(1)$
No preprocessing	-	$O(n)$	$O(n)$	$O(n)$
Our Results	$\mathbf{O(n \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(n)}$	$\mathbf{O(\log n)}$
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Our Results (Shortest Path)

Shortest Path				
	Preprocess time	Space	Single-source query	Pair query
Complete preprocessing	$O(n^5)$	$O(n^2)$	$O(n)$	$O(1)$
No preprocessing	-	$O(n)$	$O(n^4)$	$O(n^4)$
Our Results	$\mathbf{O(n^2 \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(n)}$	$\mathbf{O(\log n)}$
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Our Results (IFDS)

IFDS				
	Preprocessing time	Space	S.s. query	Pair query
1	$O(n^2 \cdot \|D\|^3)$	$O(n^2 \cdot \|D\|)$	$O(n \cdot \|D\|)$	$O(\|D\|)$
2	-	$O(n \cdot \|D\|)$	$O(n \cdot \|D\|^3)$	$O(n \cdot \|D\|^3)$
3a	$O(n \cdot \log n \cdot \|D\|^3)$	$O(n \cdot \log n \cdot \|D\|^2)$	$O(n \cdot \|D\|^2)$	$O(\|D\|^2)$
3b	$O(n \cdot \log n \cdot \|D\|^3)$	$O(n \cdot \|D\|^2)$	$O(n \cdot \|D\|^2)$	$O(\log n \cdot \|D\|^2)$

- **1.** IFDS complete preprocessing
- **2.** IFDS no preprocessing
- **3a** Our results (time optimized)
- **3b** Our results (space optimized)

Tree-Decompositions and Treewidth

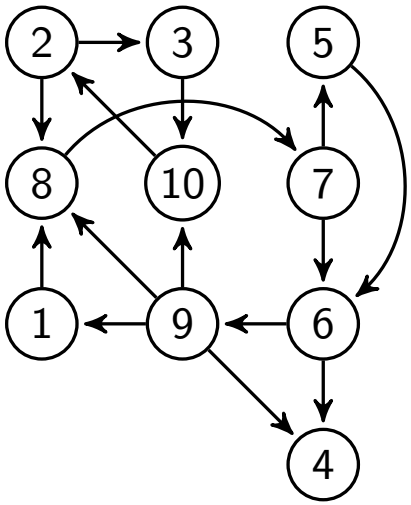
Tree Decomposition

Definition (Tree decomposition)

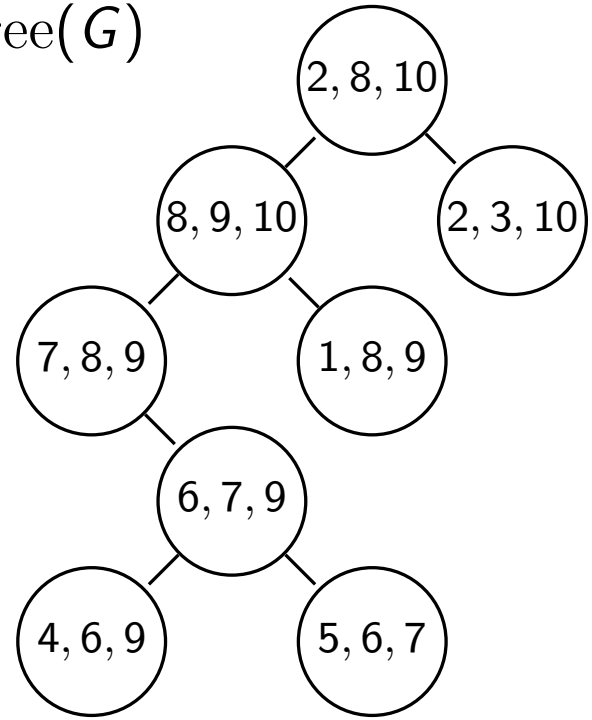
Given a graph $G = (V, E)$, a **tree-decomposition** $\mathrm{Tree}(G) = (V_T, E_T)$ is a tree of bags $B_i \subseteq V$ such that:

1. Every node of G is contained in a bag
2. Every edge of G is contained in a bag
3. Every node of G appears in a contiguous subtree of $\mathrm{Tree}(G)$.

G



$\mathrm{Tree}(G)$



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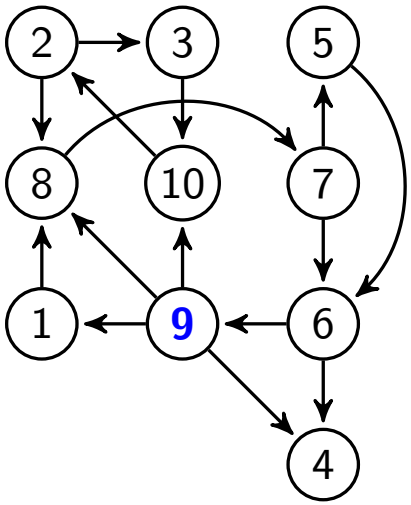

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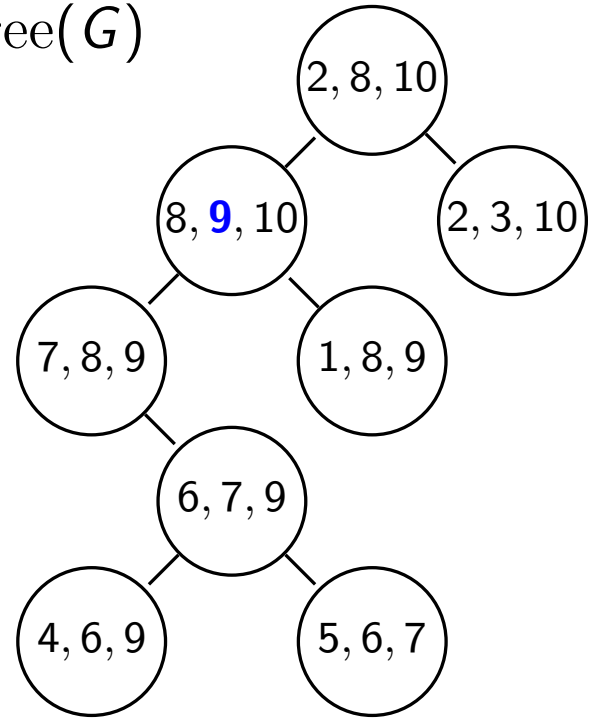

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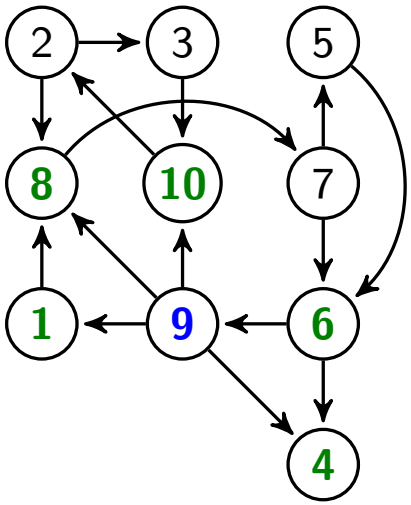

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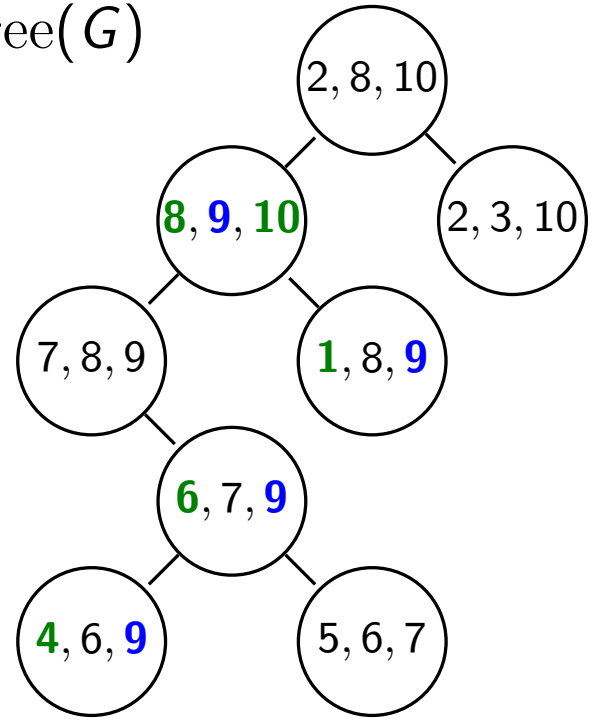

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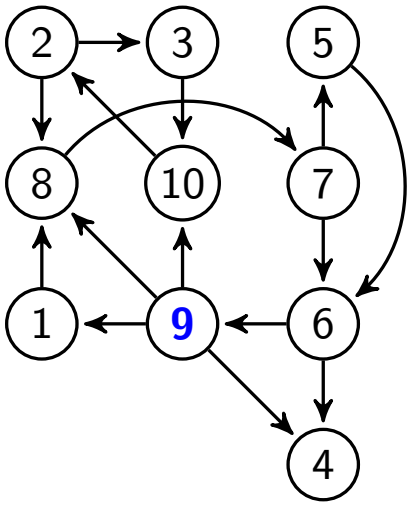

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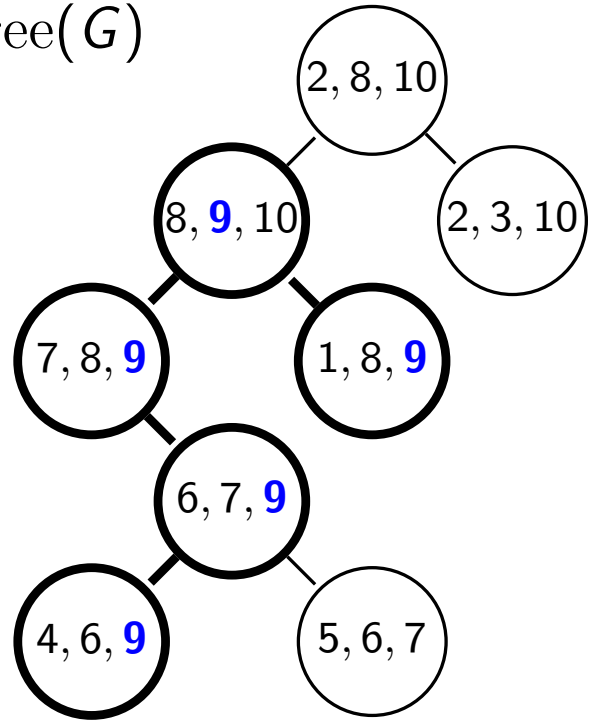
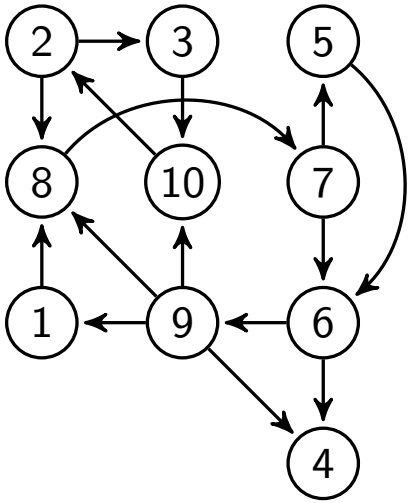


Treewidth

Definition

G has **constant treewidth** if every bag of $\mathrm{Tree}(G)$ has constant size.

G



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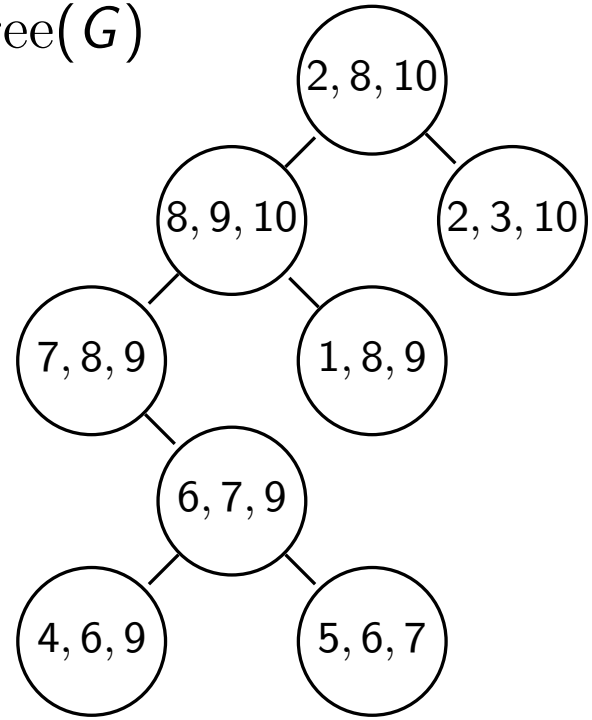


Treewidth of Structured Programs

Theorem (Treewidth of structured programs)

Control-flow graphs of goto-free programs have constant treewidth.

Theorem (Tree decomposition)

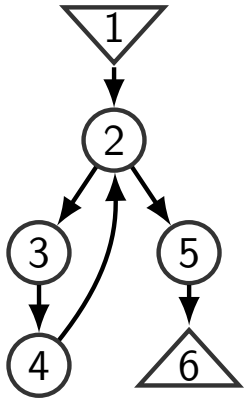
For constant treewidth graphs, $\mathrm{Tree}(G)$ *can be constructed in* $O(n)$ *time.*

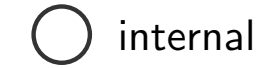
Example: RSM & Tree Decompositions

Method 1: dot_vector

Input: $x, y \in \mathbb{R}^n$
Output: The dot product $x^\top y$

```
1 result ← 0
2 for i ← 1 to n do
3     z ← x[i] · y[i]
4     result ← result + z
5 end
6 return result
```

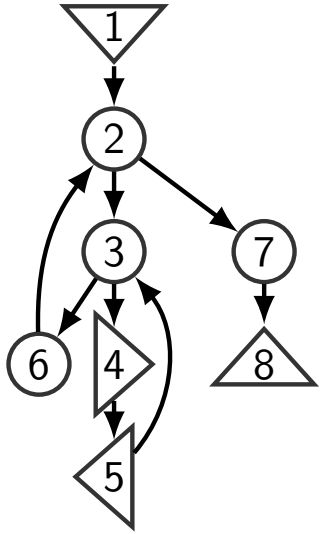




Method 2: dot_matrix

Input: $A \in \mathbb{R}^{n \times k}, B \in \mathbb{R}^{k \times m}$
Output: The dot product $A \times B$

```
1 C ← zero matrix of size n × m
2 for i ← 1 to n do
3     for j ← 1 to m do
4         Call dot_vector(A[i, :], B[:, j])
5         C[i, j] ← the value returned by the
          call of line 4
6     end
7 end
8 return C
```



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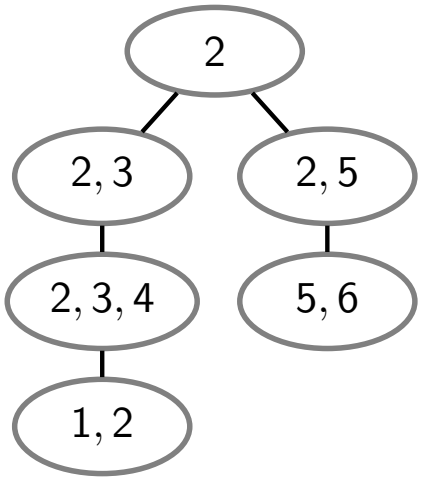
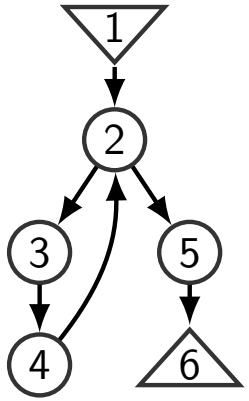
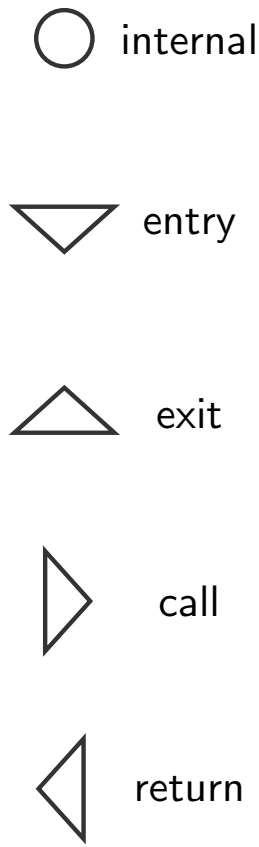
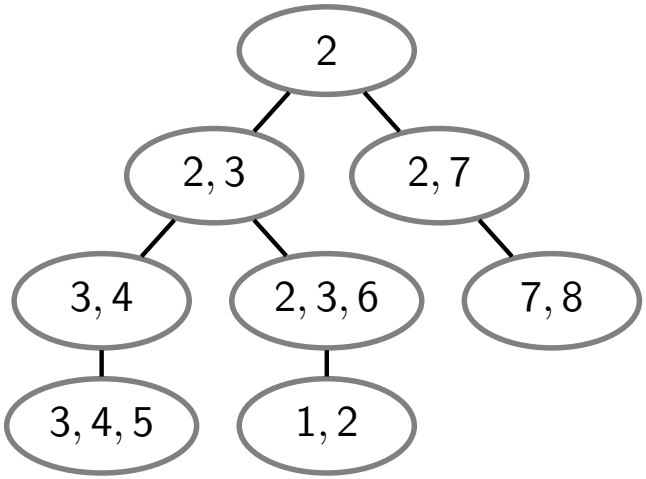
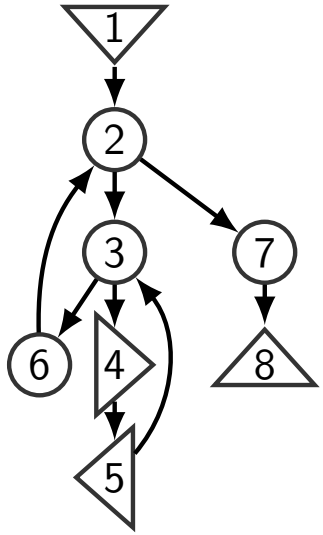










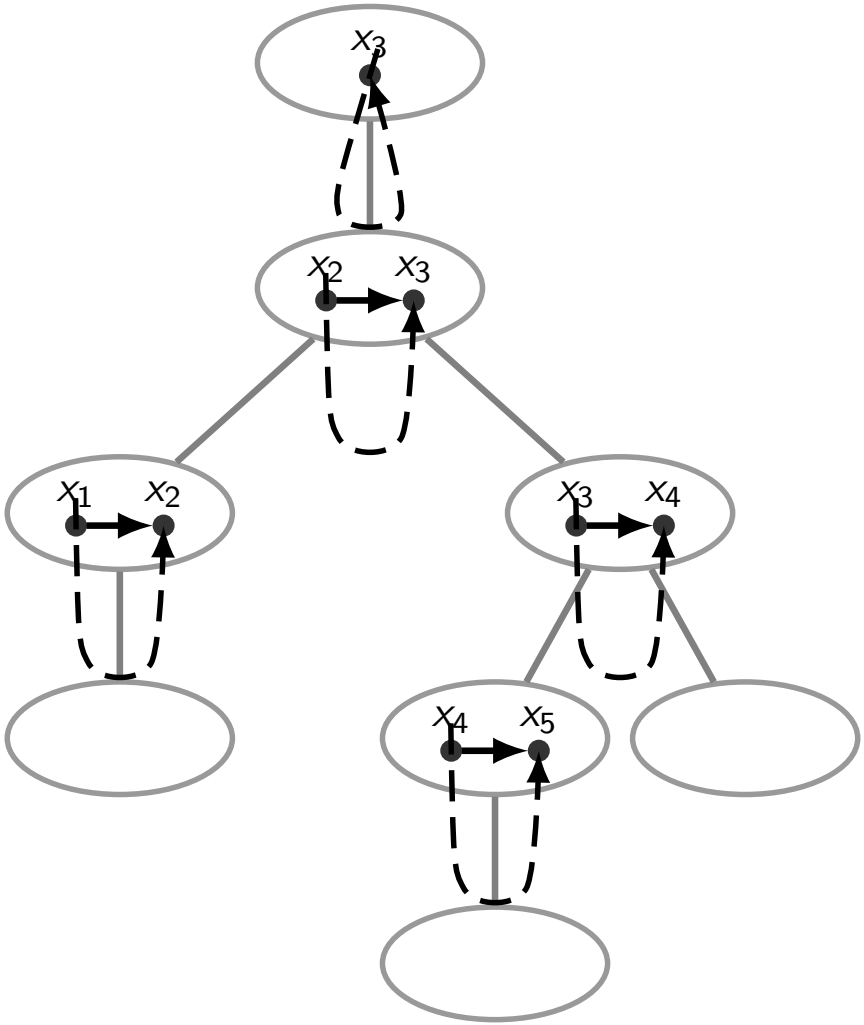
Algorithms for Preprocess, Update & Query on $\mathrm{Tree}(G)$

Focus on graph $G = (V, E)$ and a tree-decomposition $\mathrm{Tree}(G)$

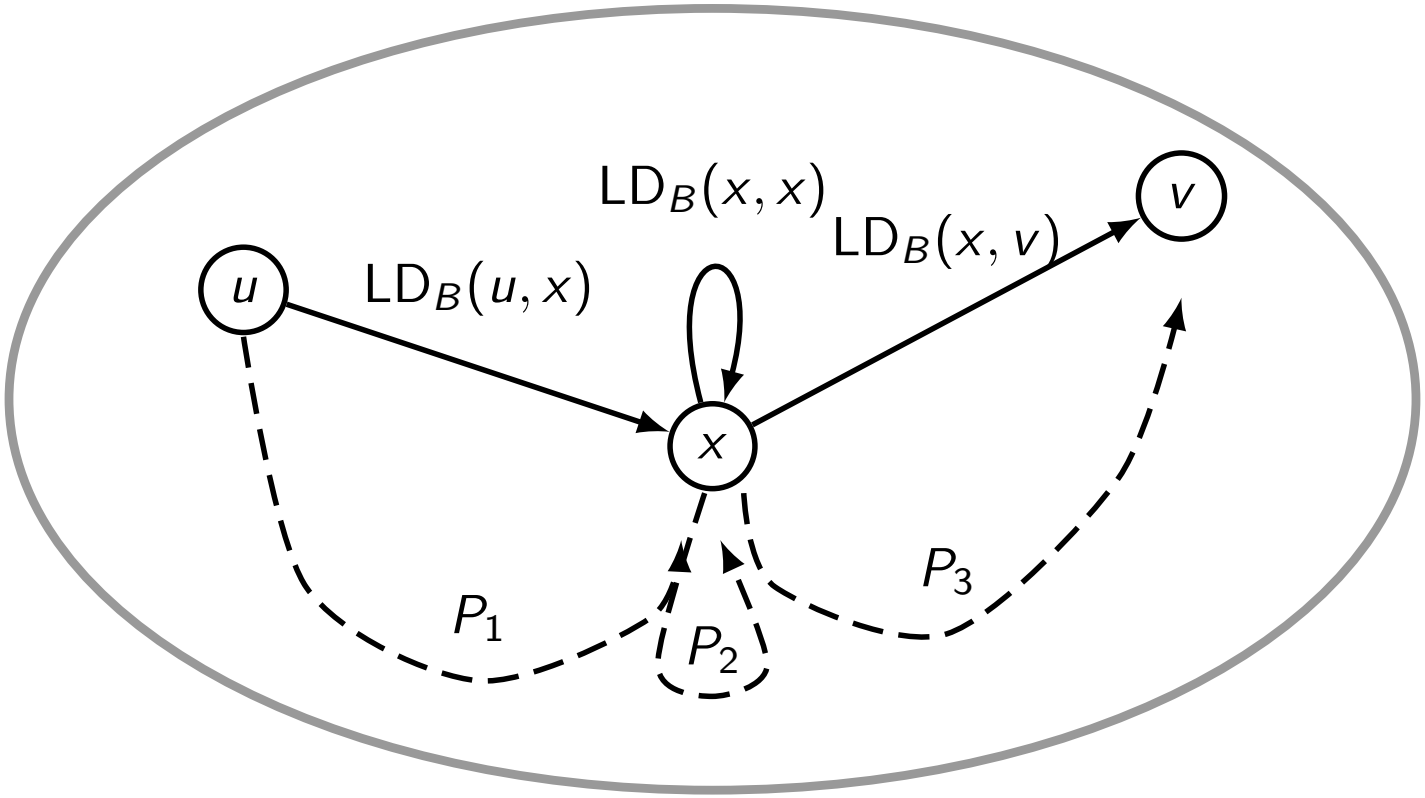
- Preprocess $\mathrm{Tree}(G)$ in $O(n)$ time
- Query for the distance $d(u, v)$ in $O(\log n)$ time
- Update the weight of an edge of G in $O(\log n)$ time

Local Distances

- **Local distances** in a bag B: $\mathrm{LD}_B(x_i, x_j) = \bigoplus_{P: x_i \to x_j} \otimes(P)$
  - where P is a U-shaped path



Path Shortening



$$LD_B(u,v) = LD_B(u,v) \bigoplus LD_B(u,x) \bigotimes (LD_B(x,x))^* \bigotimes LD_B(x,v)\}$$

Preprocessing of an RSM

Preprocessing of an RSM

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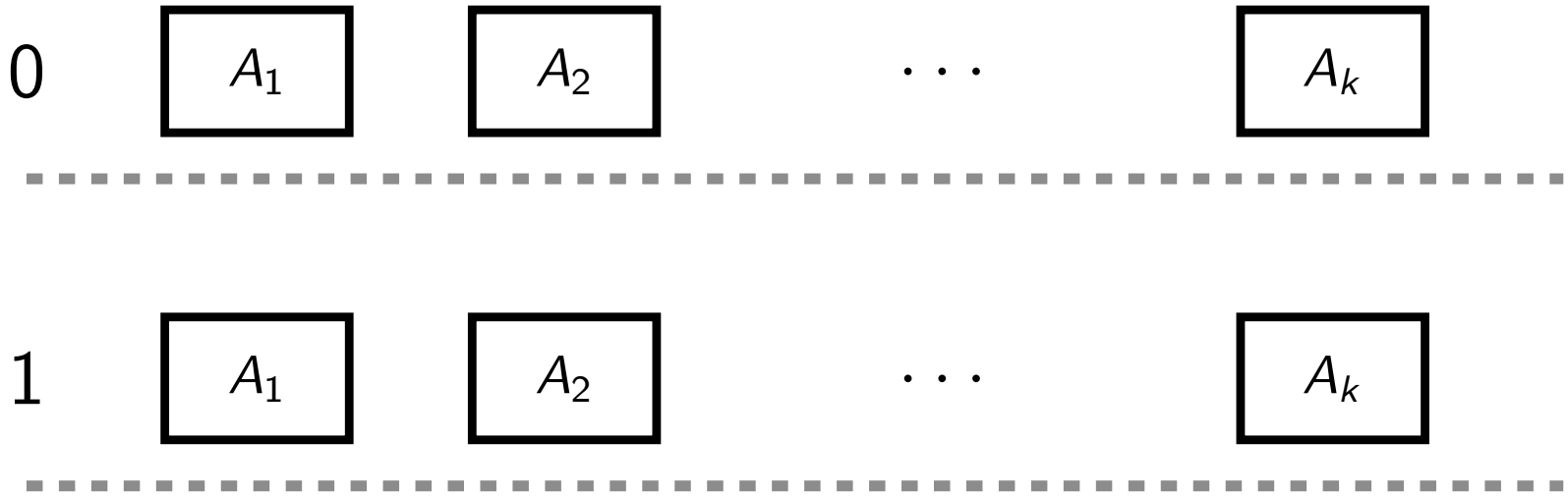
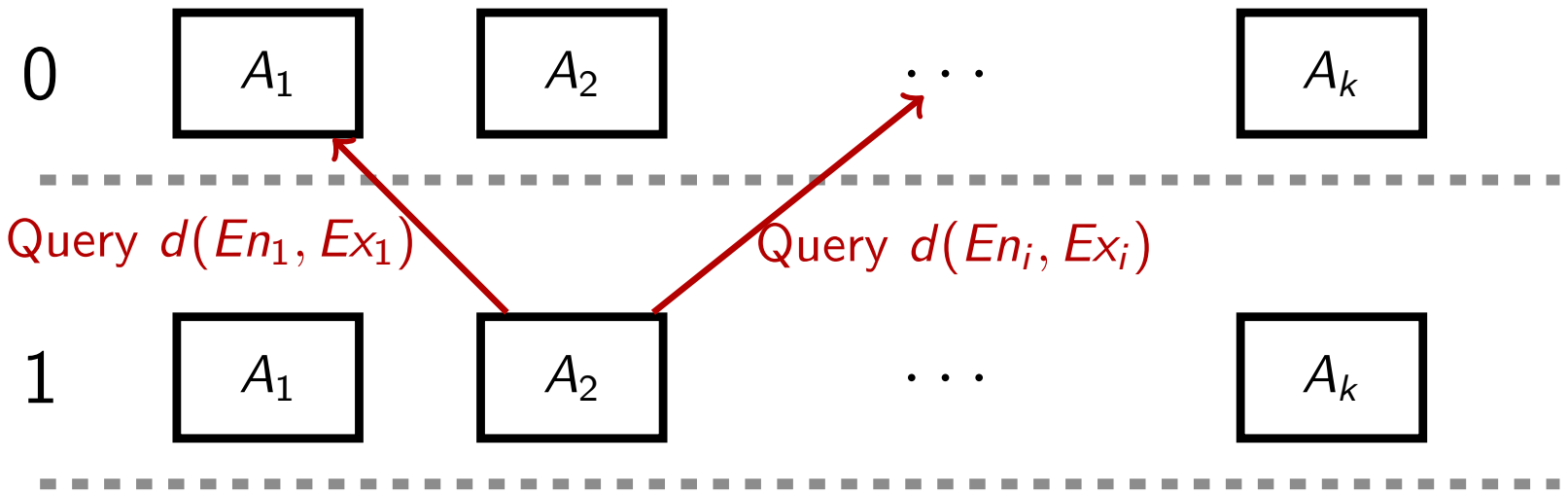


Preprocessing of an RSM



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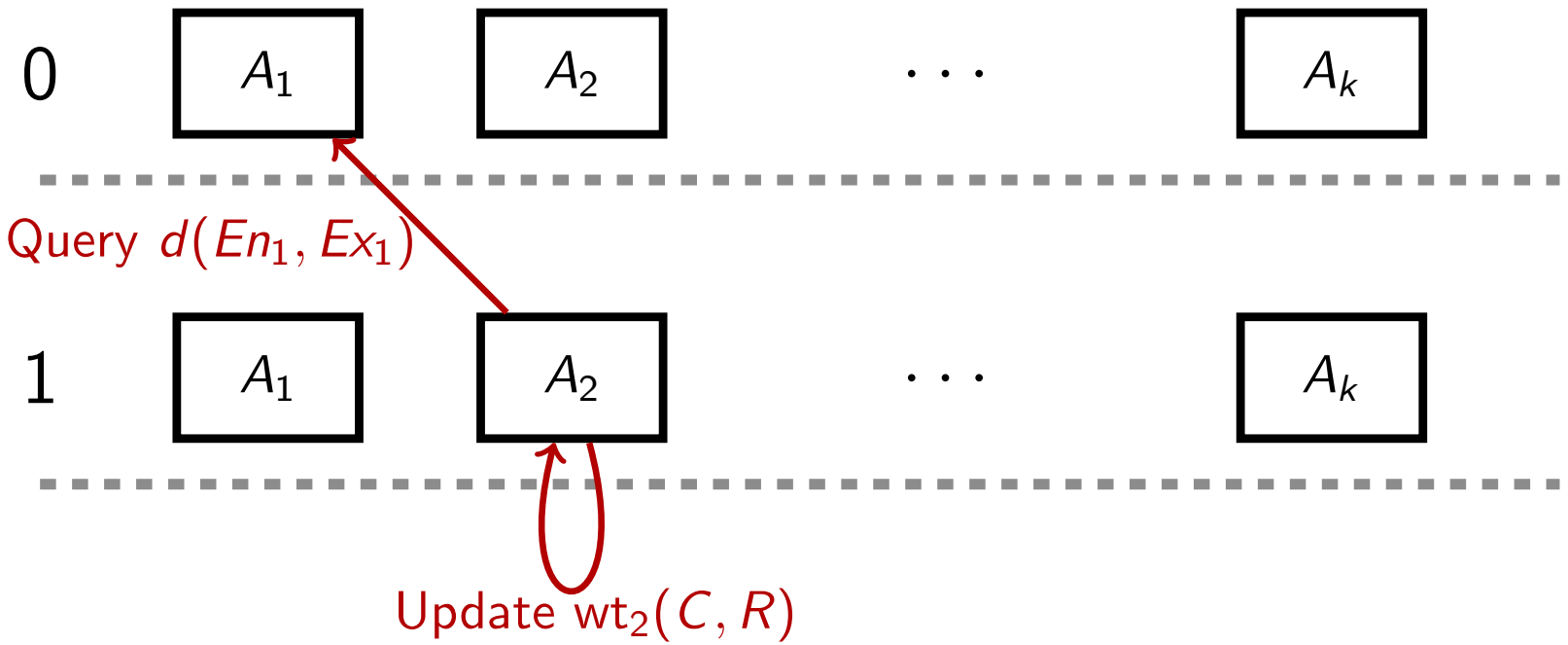

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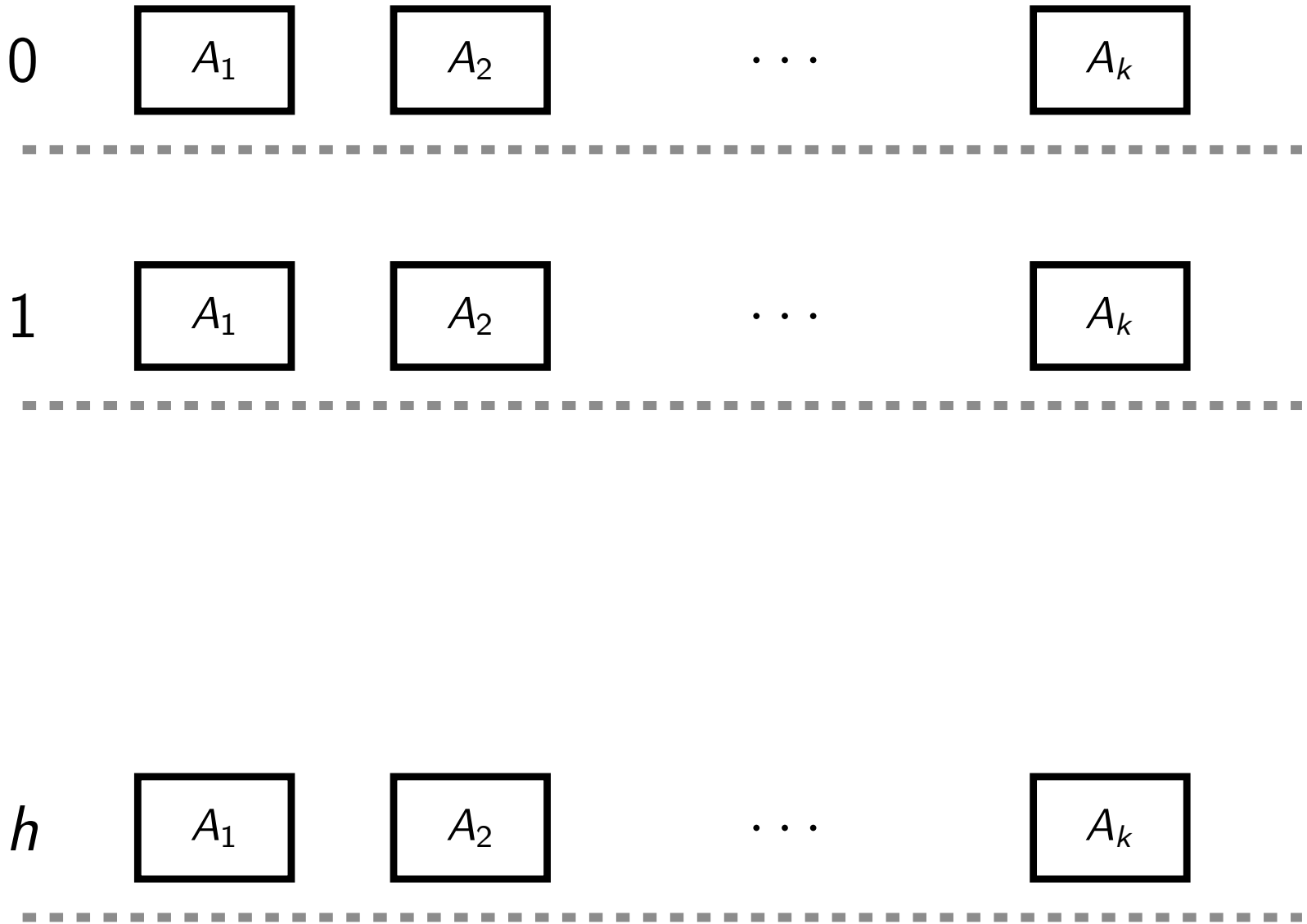

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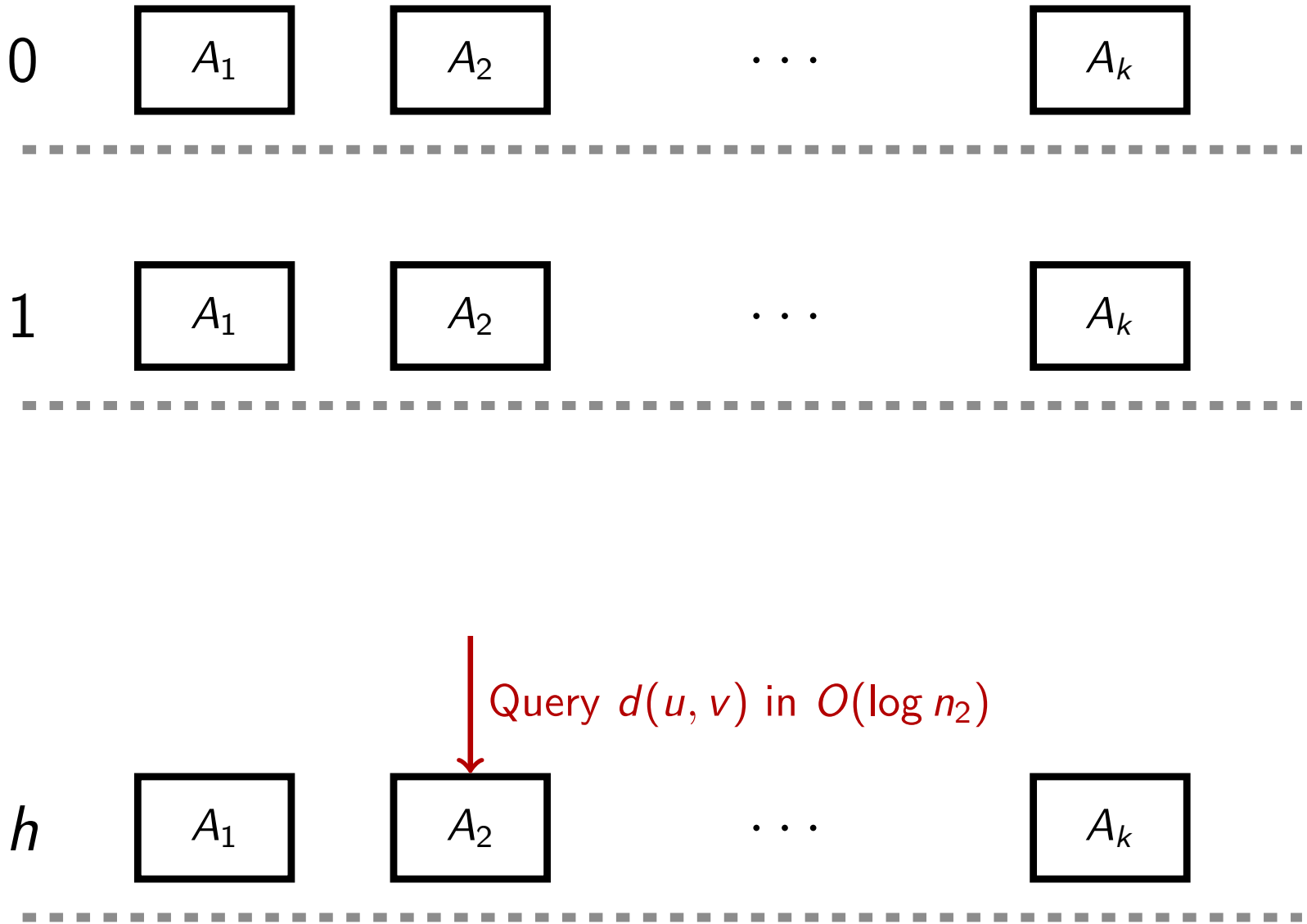

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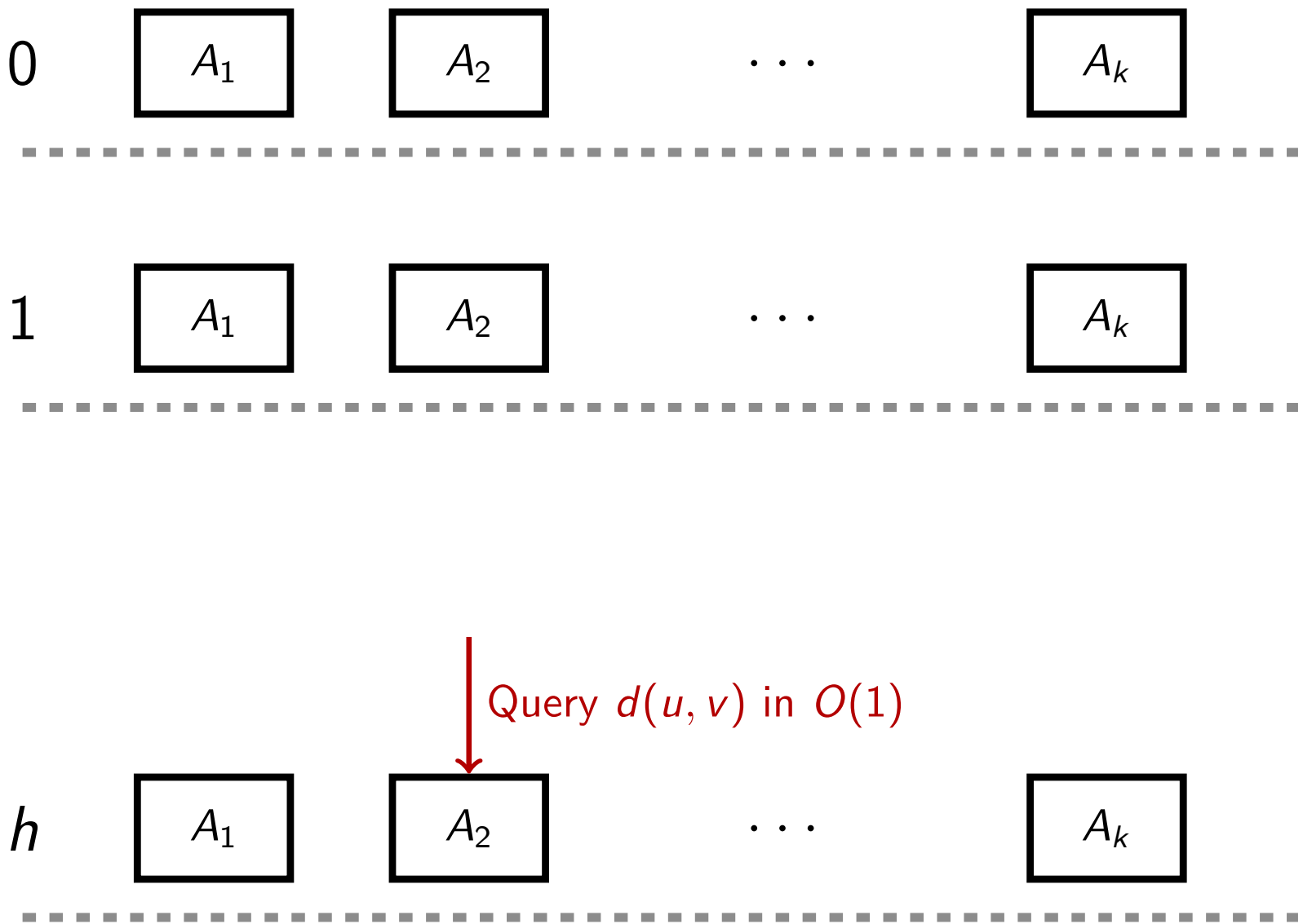


Results

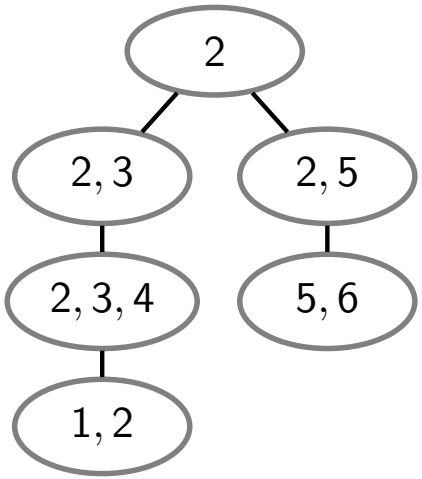
Reachability				
	Preprocess time	Space	Single-source query	Pair query
Complete preprocessing	$O(n^2)$	$O(n^2)$	$O(n)$	$O(1)$
No preprocessing	-	$O(n)$	$O(n)$	$O(n)$
Our Results	$\mathbf{O(n \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(n)}$	$\mathbf{O(\log n)}$
	$\mathbf{O(n \cdot \log n)}$	$\mathbf{O(n \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(1)}$

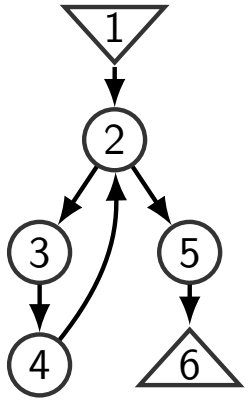
Shortest Path				
	Preprocess time	Space	Single-source query	Pair query
Complete preprocessing	$O(n^5)$	$O(n^2)$	$O(n)$	$O(1)$
No preprocessing	-	$O(n)$	$O(n^4)$	$O(n^4)$
Our Results	$\mathbf{O(n^2 \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(n)}$	$\mathbf{O(\log n)}$
	$\mathbf{O(n^2 \cdot \log n)}$	$\mathbf{O(n \cdot \log n)}$	$\mathbf{O(n)}$	$\mathbf{O(1)}$

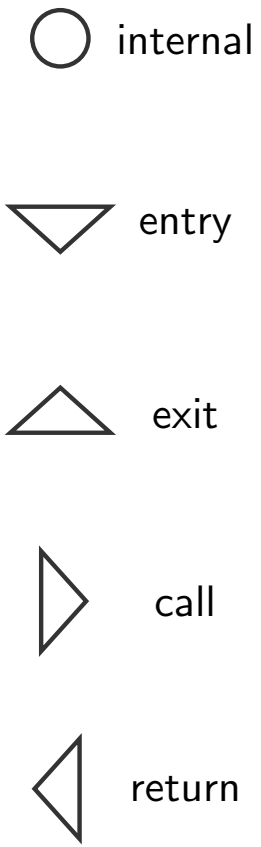
Results

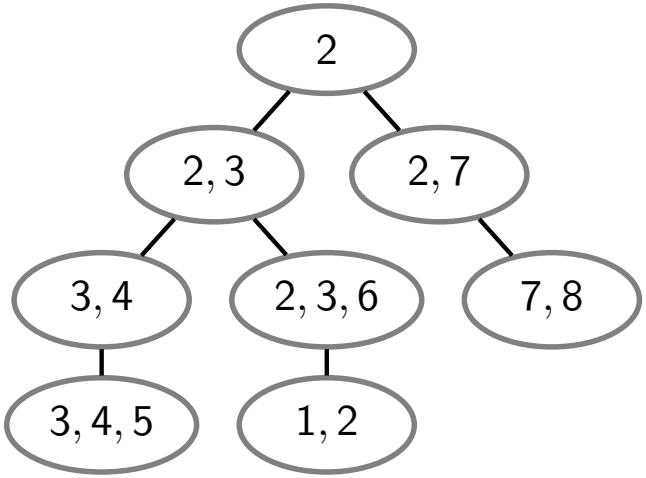
- Algorithms apply to complete semirings
- Improved bounds for IFDS
- Byproduct: Dynamic data-structure

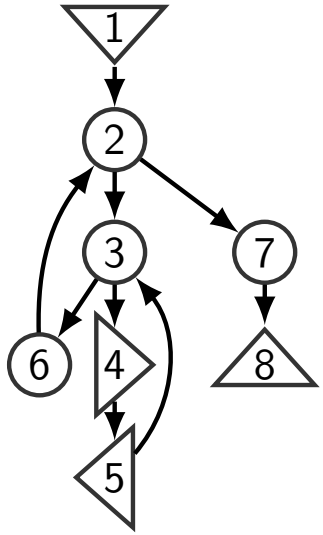
Illustration of Bounded Stack Height Preprocessing



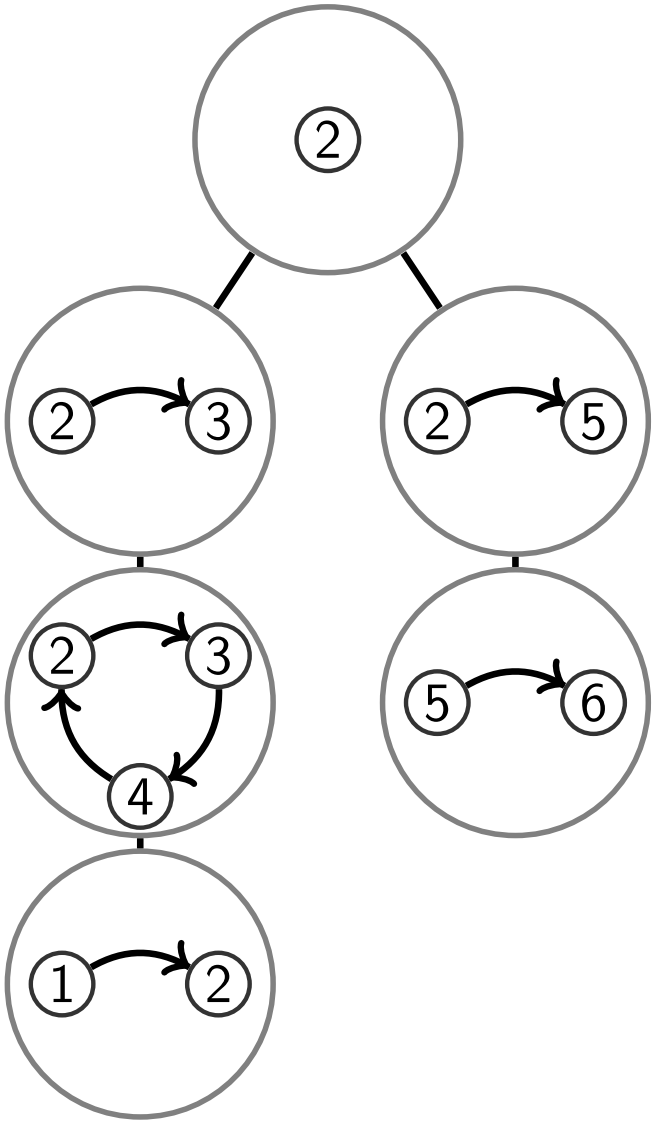


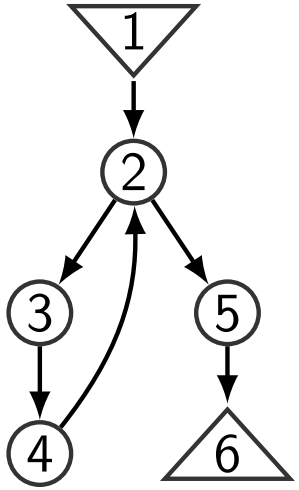






Example: Preprocessing $\mathrm{Tree}(G_1)$





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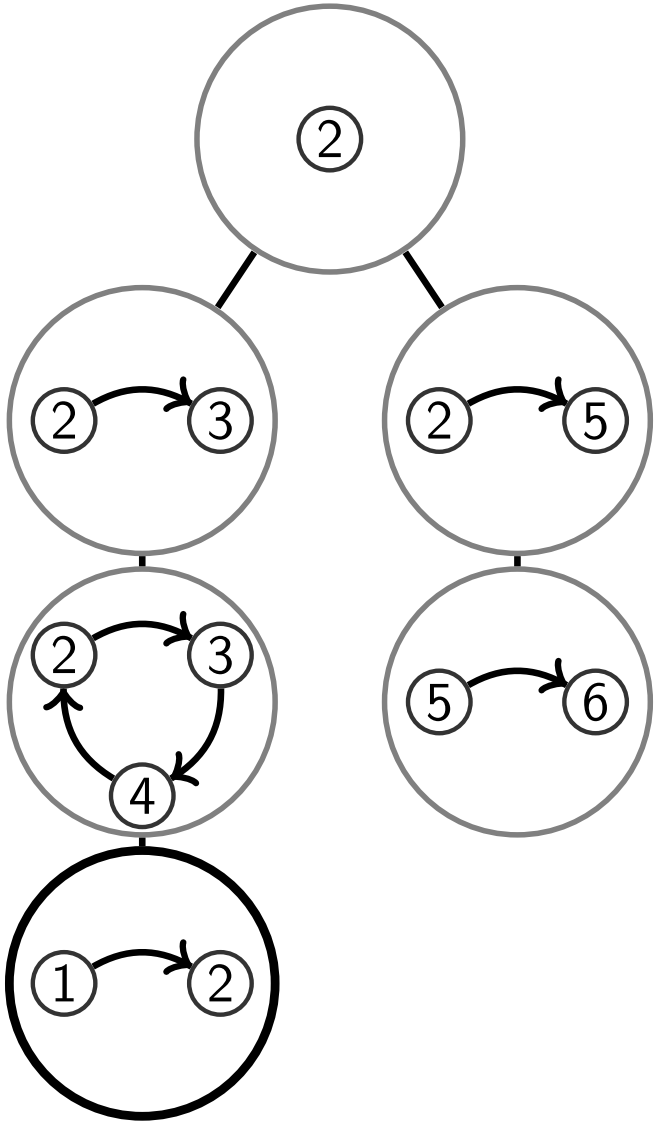

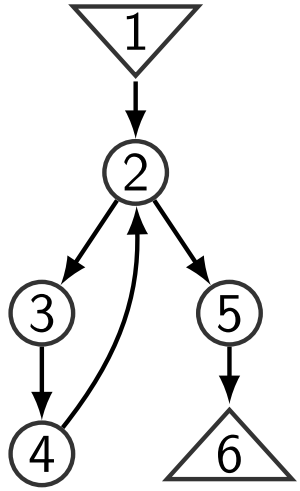

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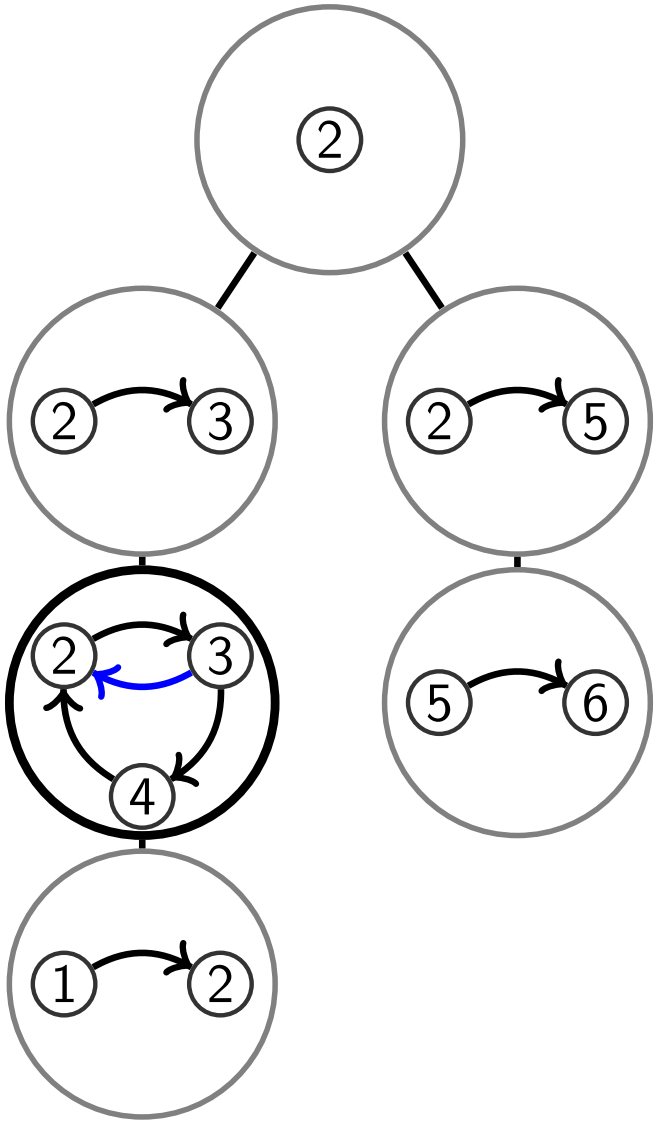

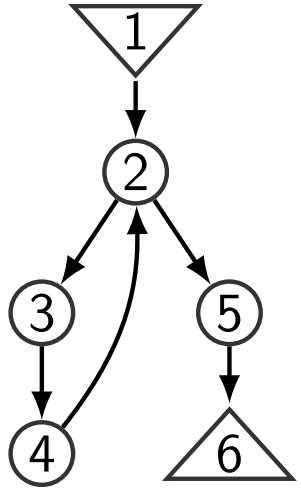

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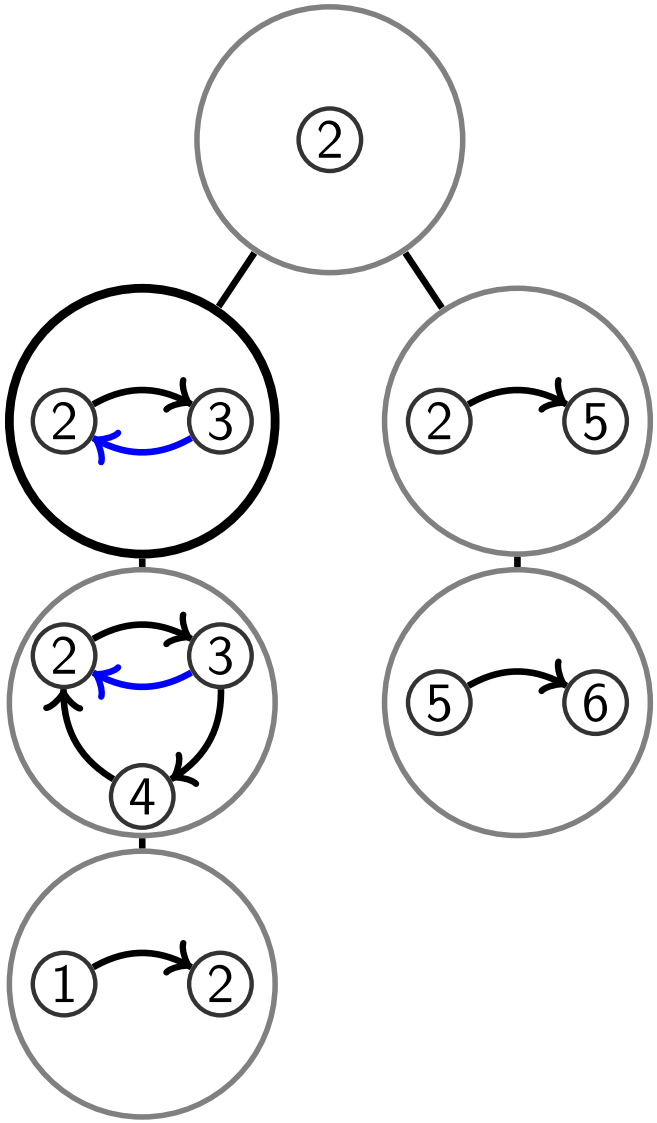

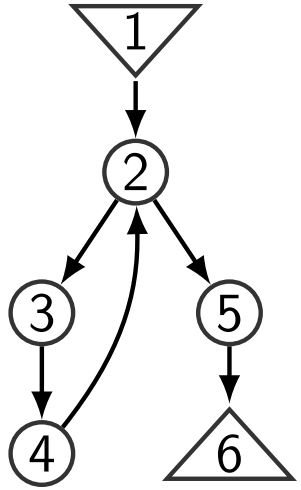

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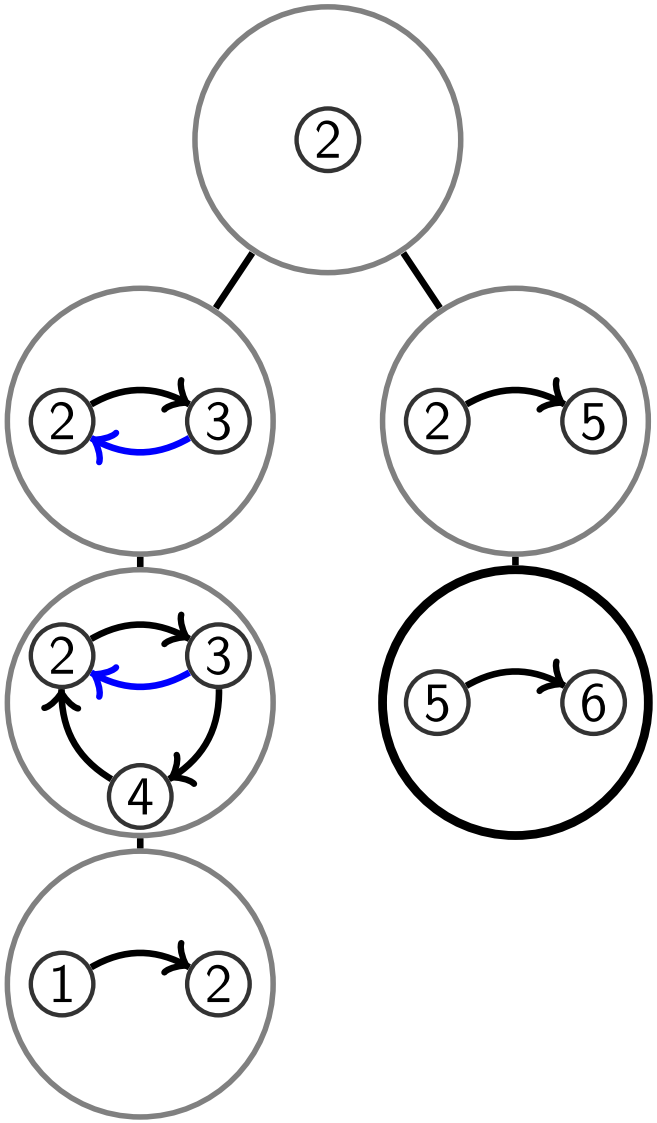

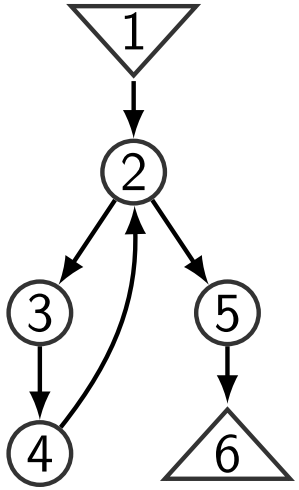

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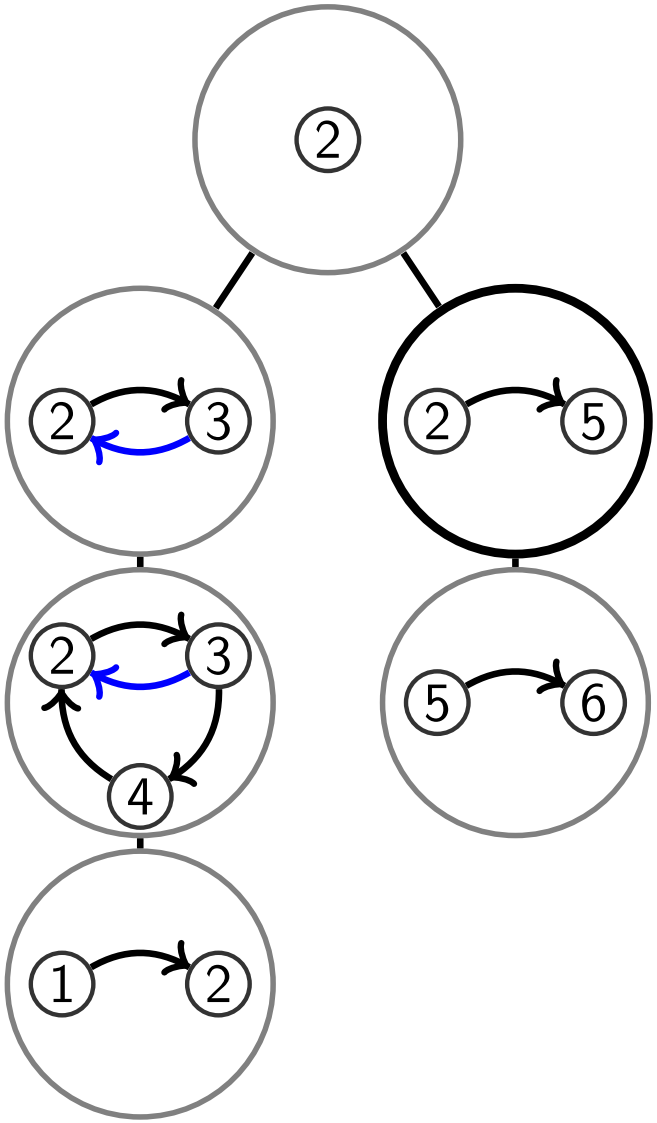

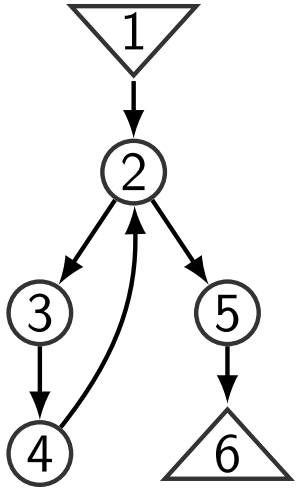

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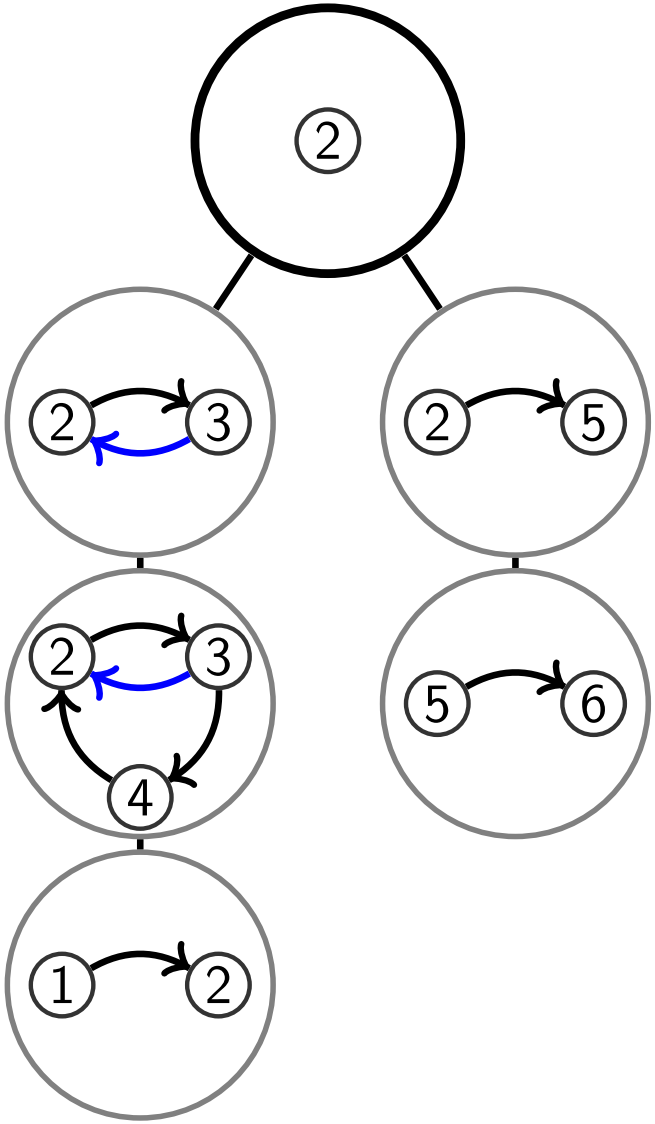
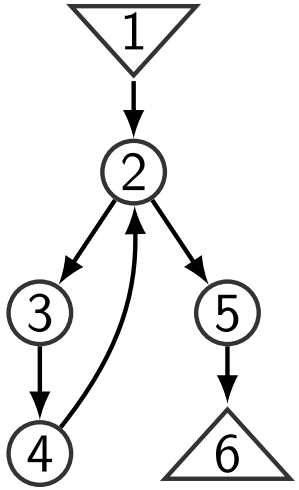
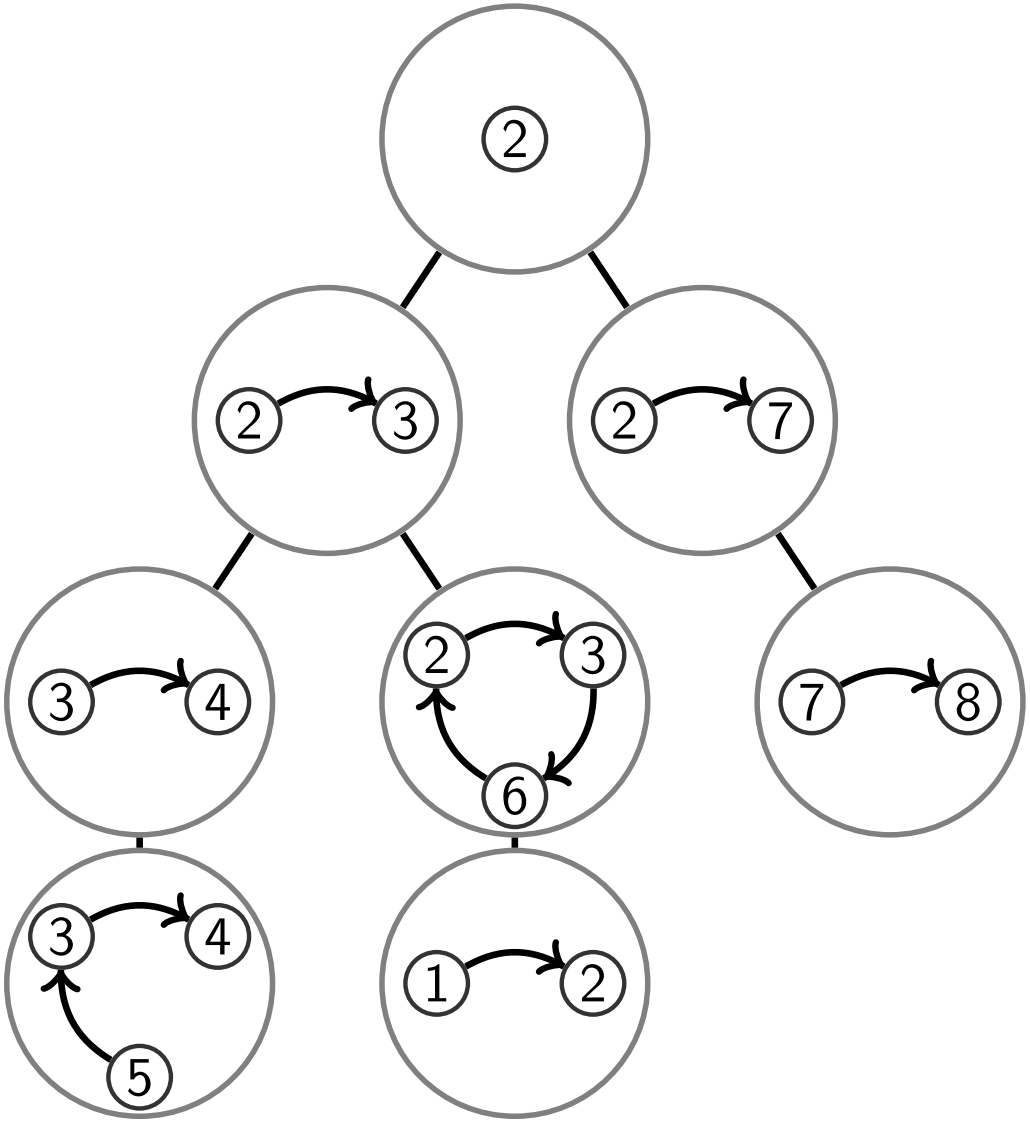
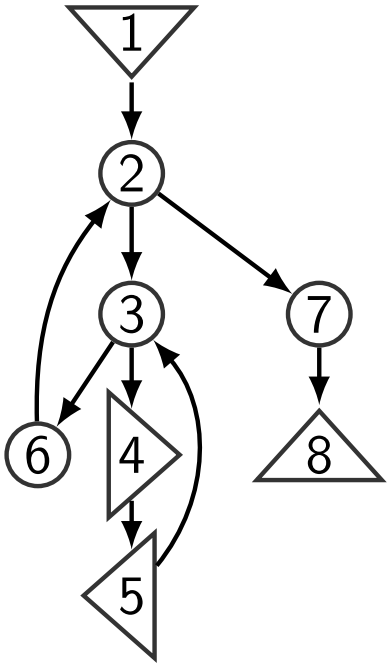




Example: Preprocessing $\mathrm{Tree}(G_2)$





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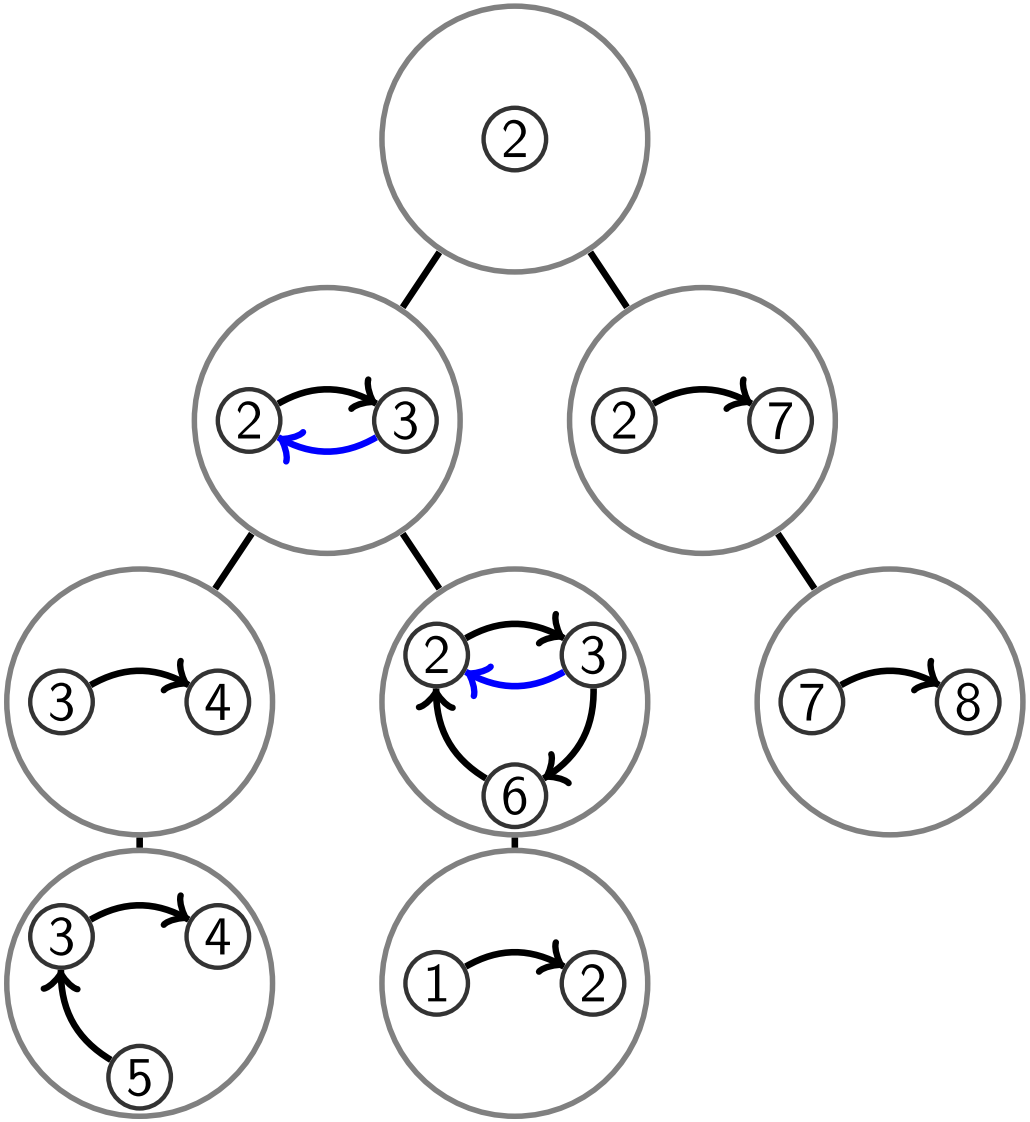
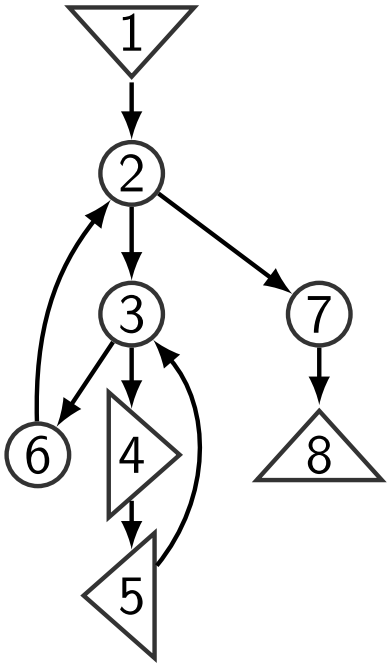
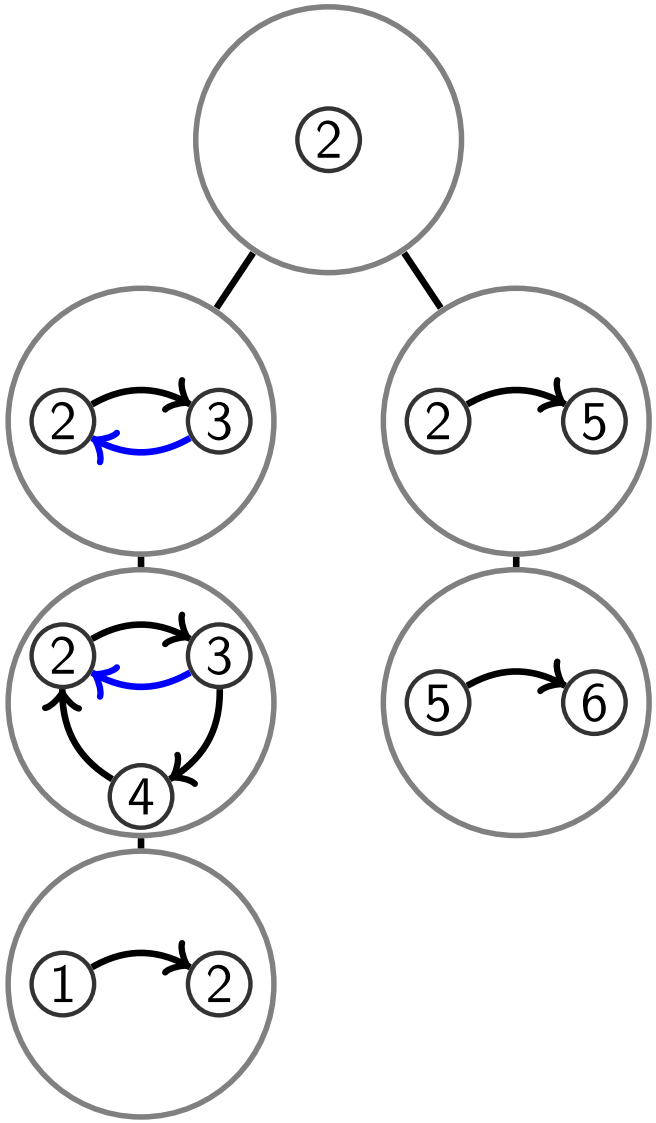
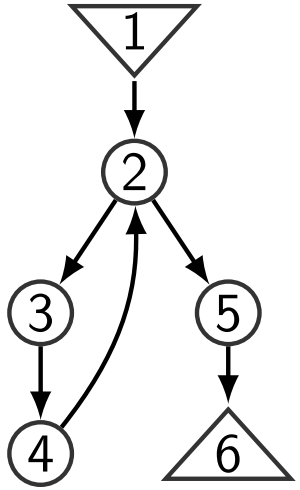




Example: Query $\mathrm{Tree}(G_1)$ for $d(1,6)$





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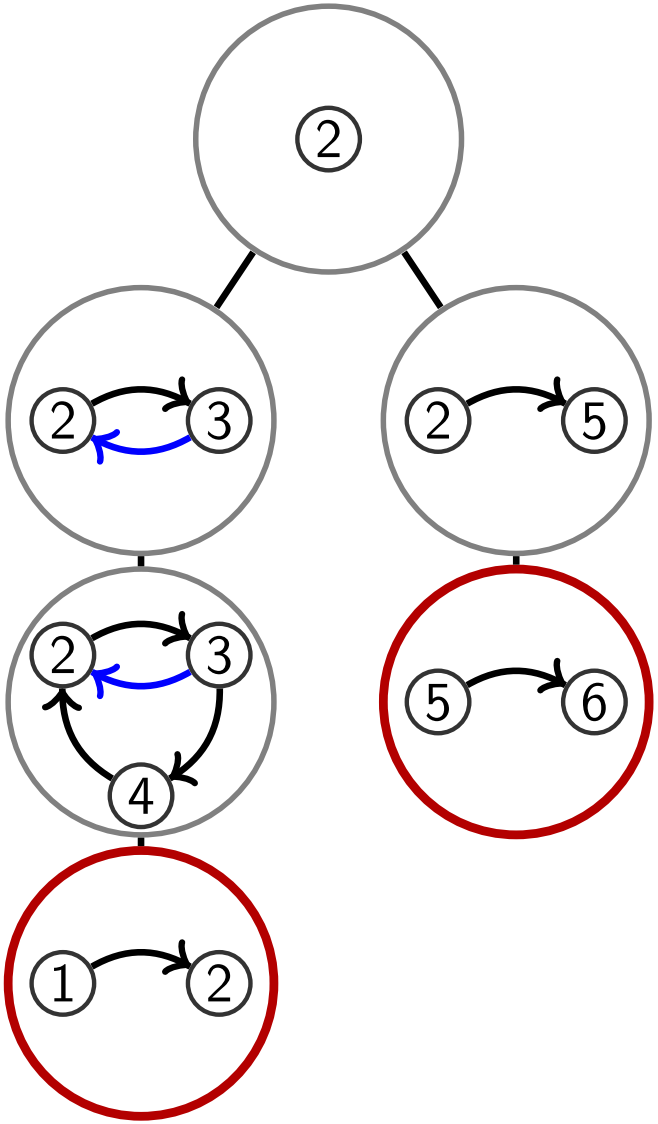

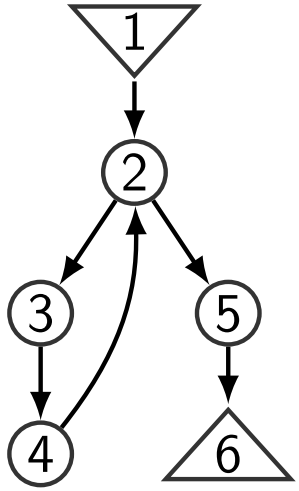

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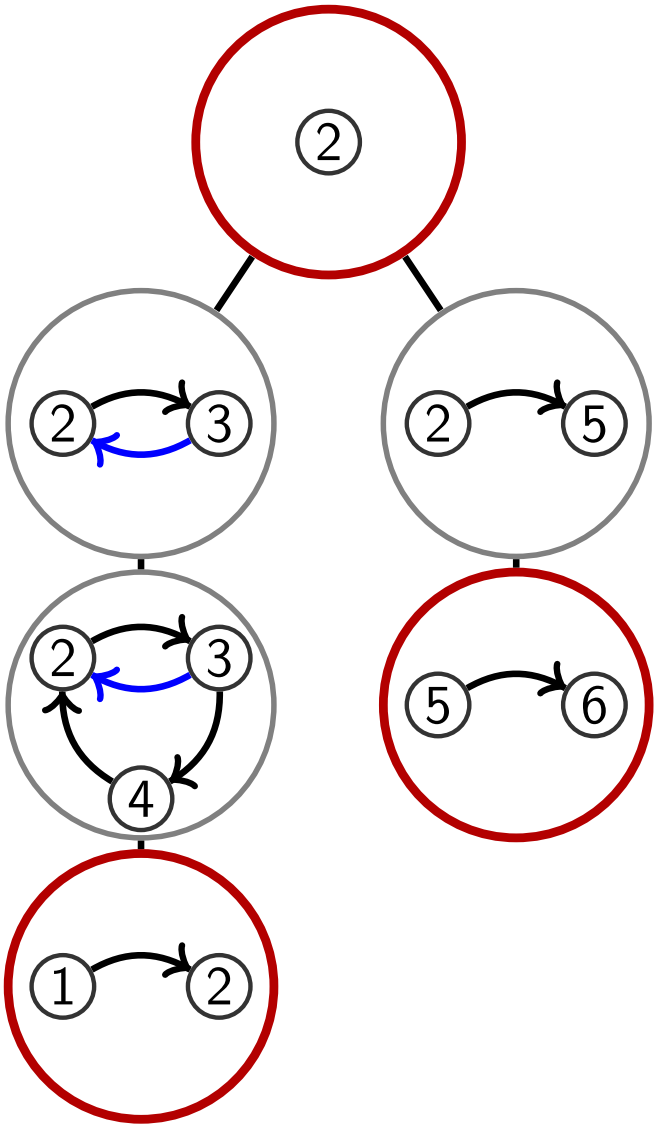

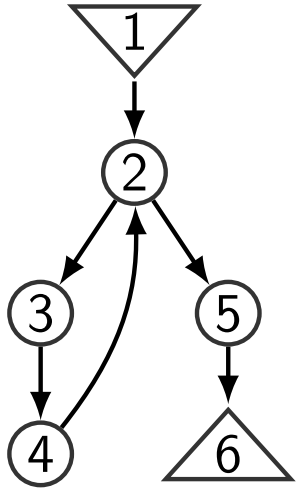

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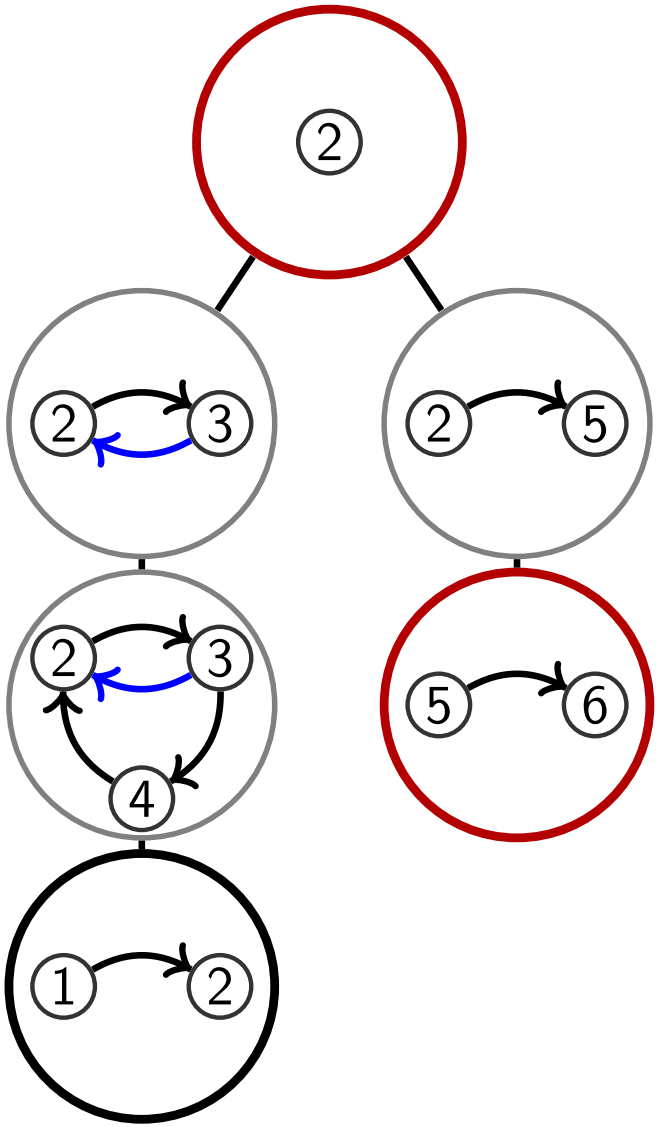

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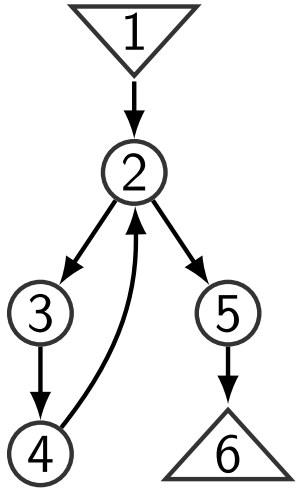

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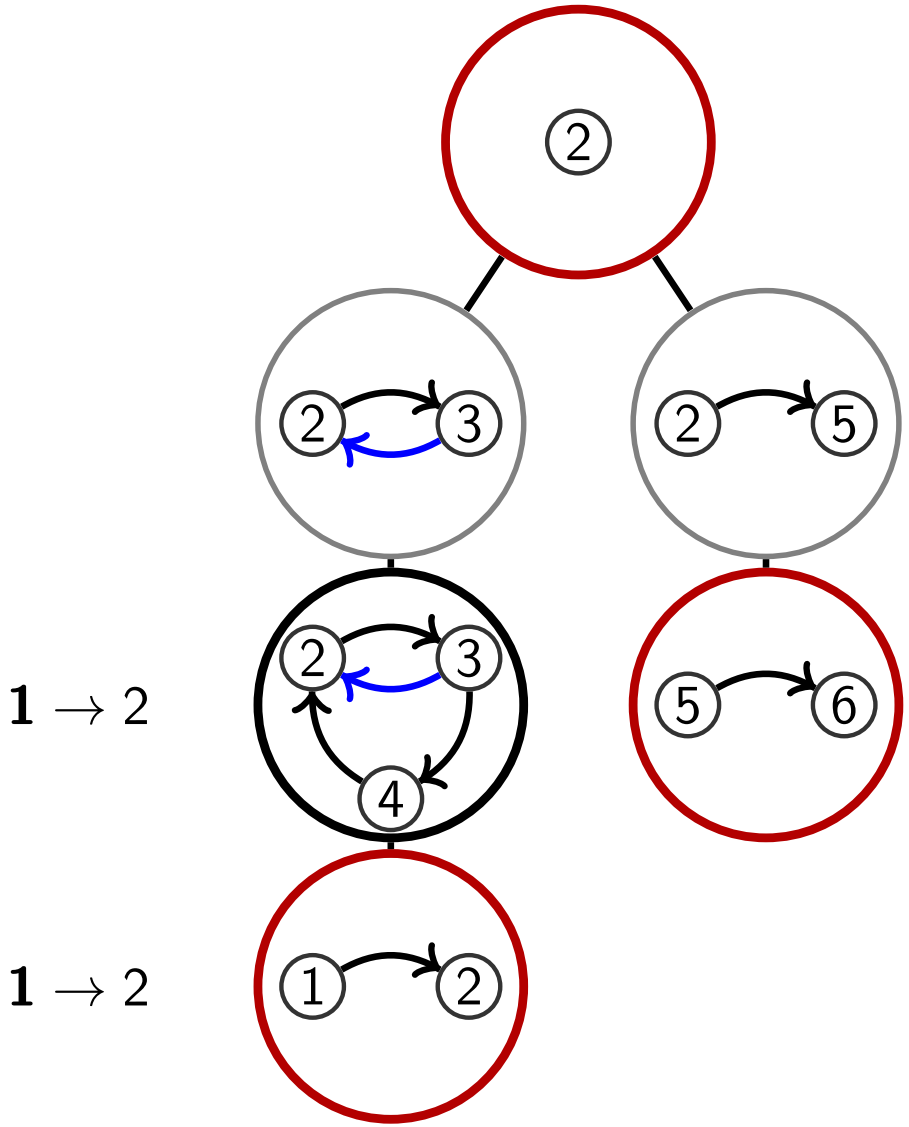

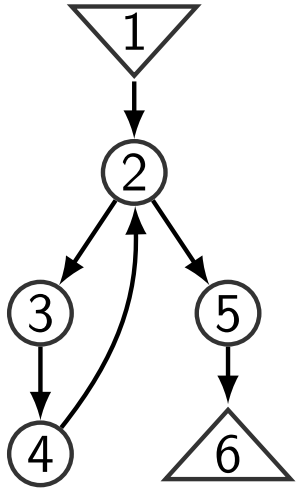

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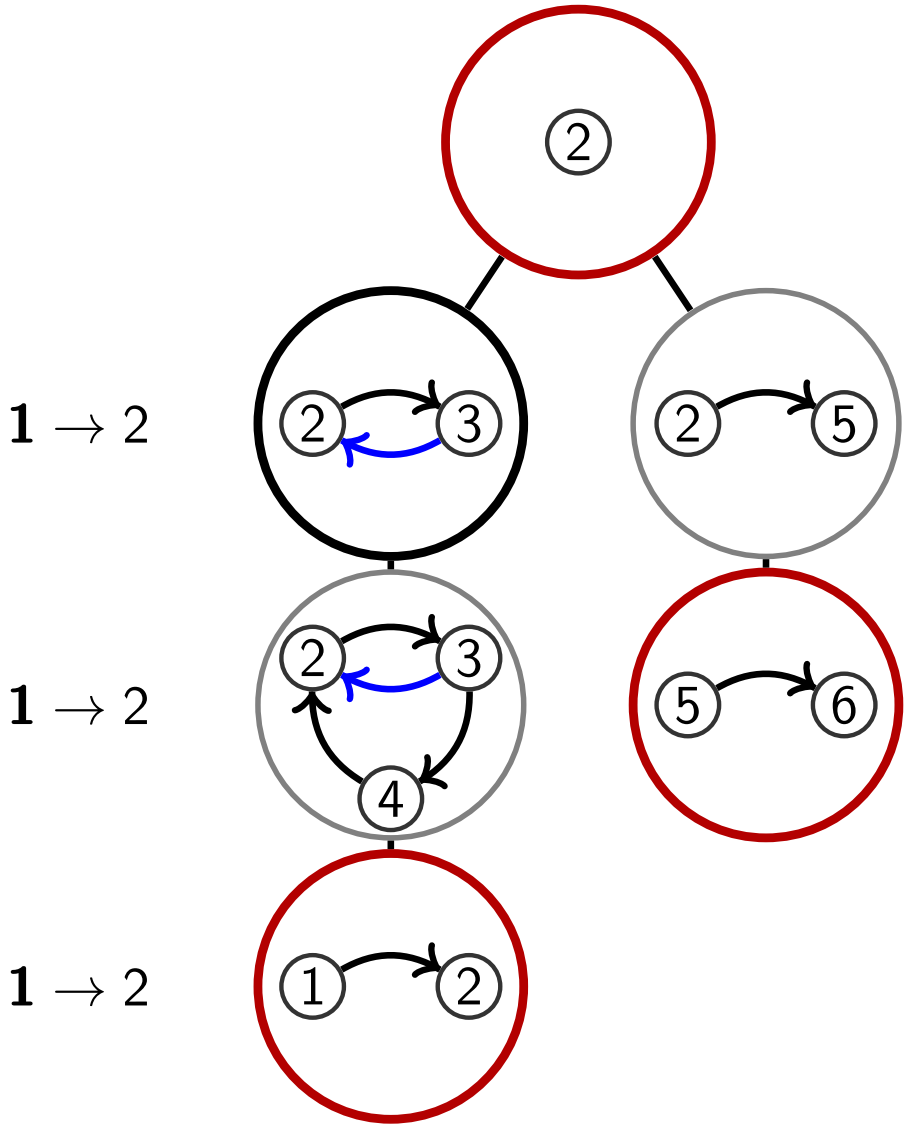

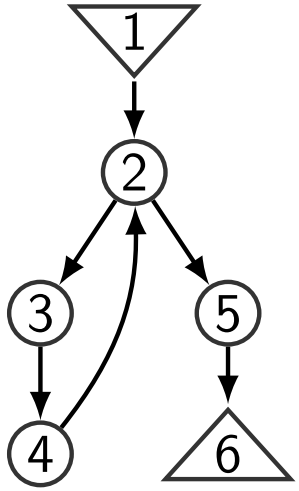

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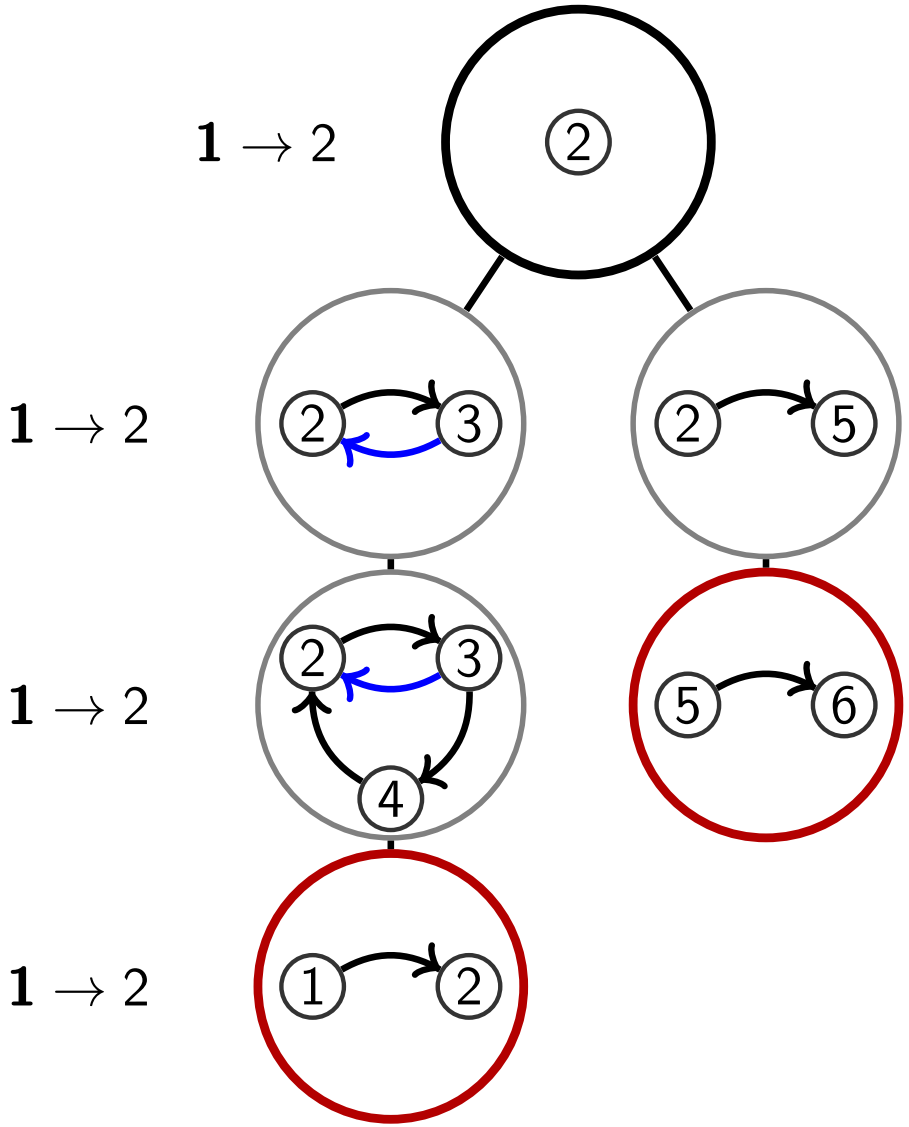

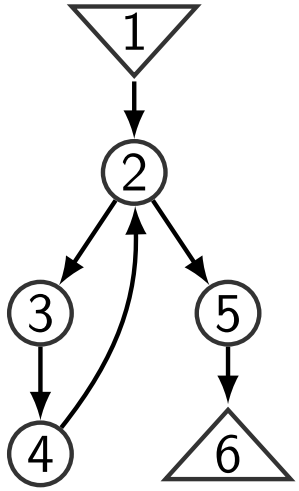

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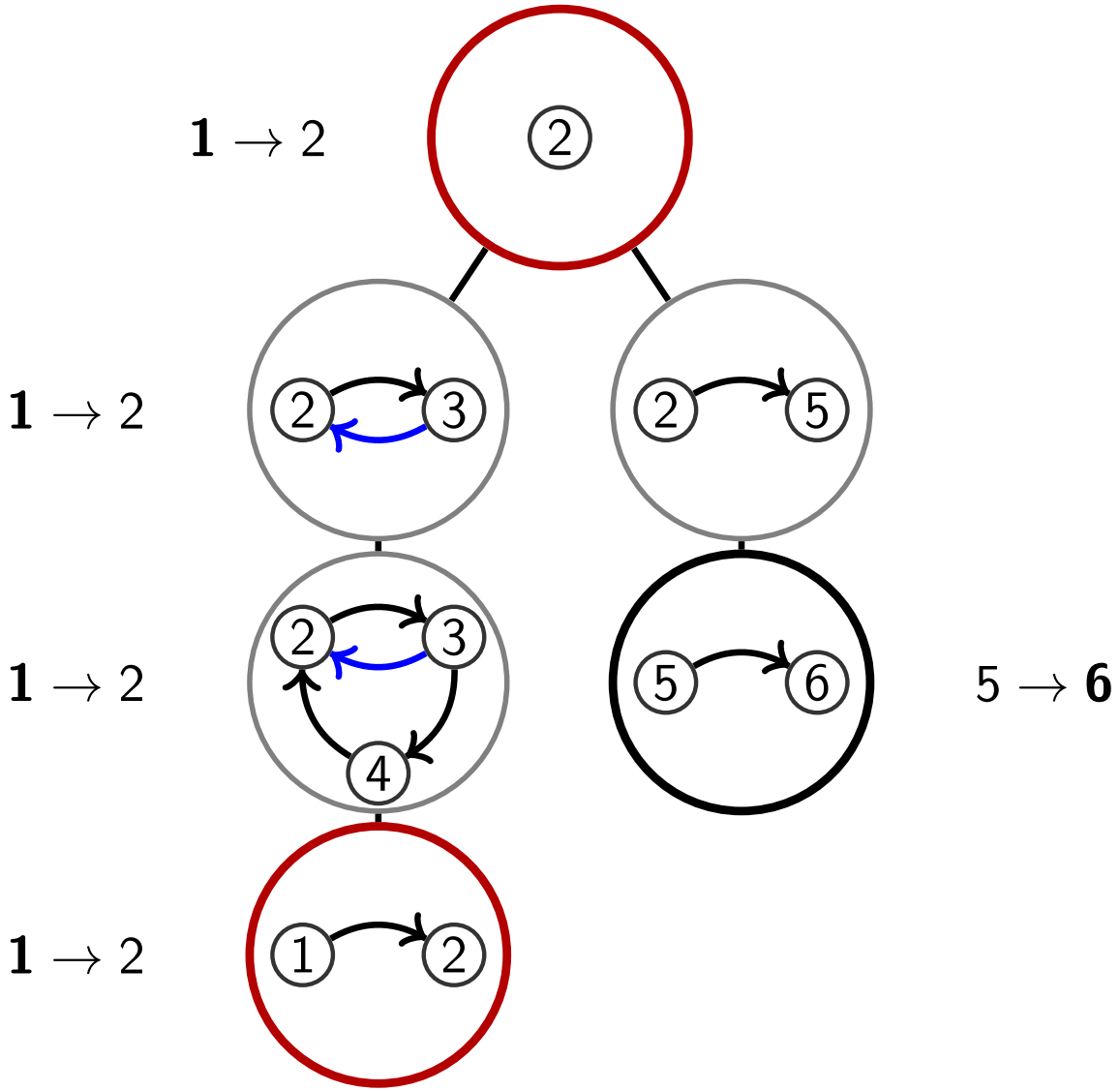

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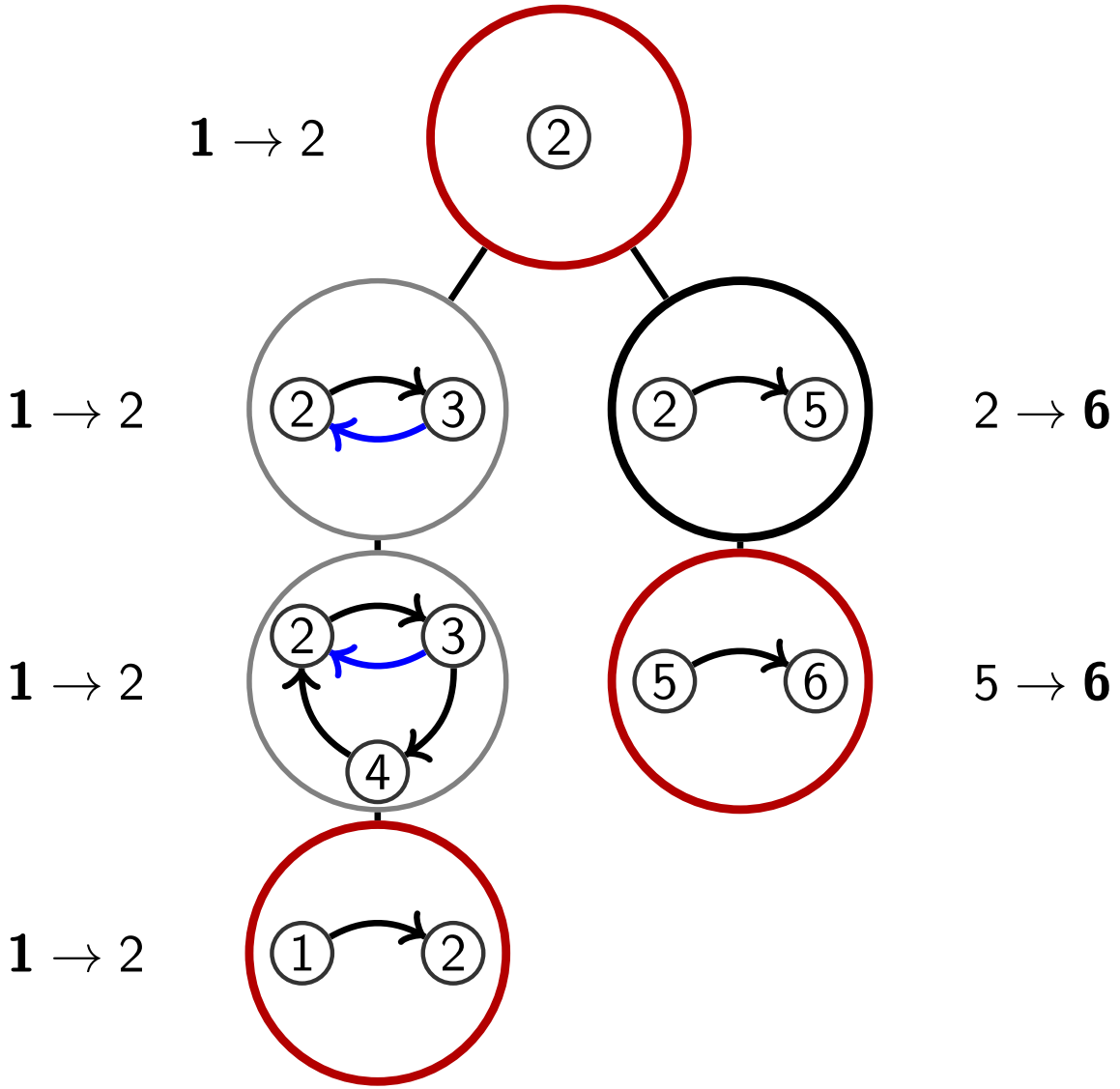

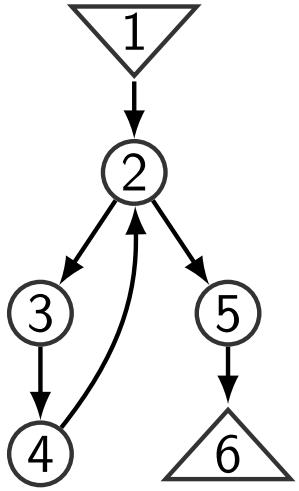

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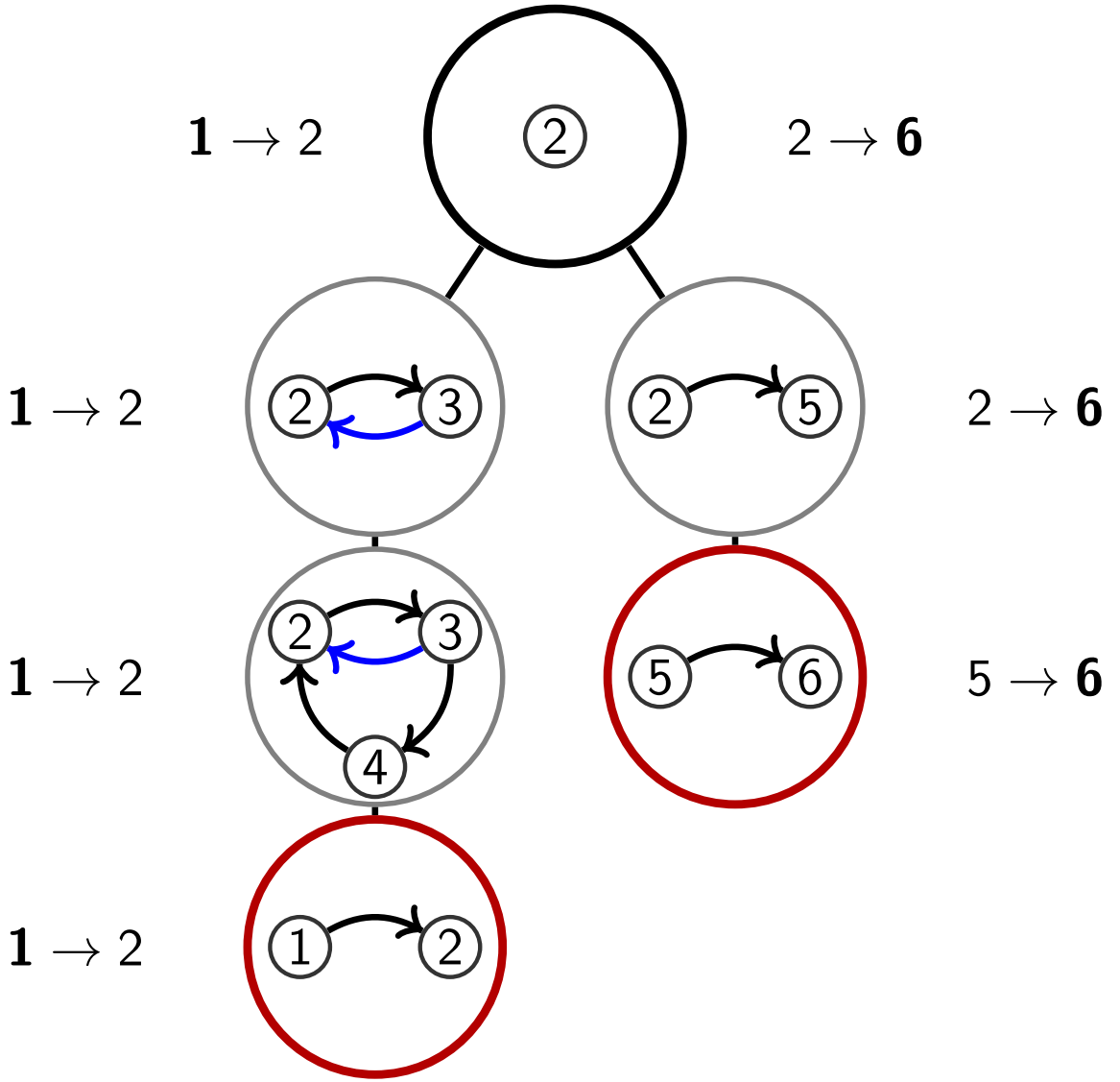
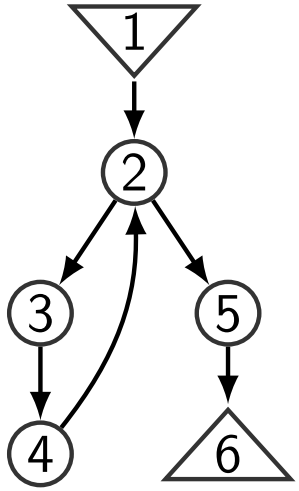
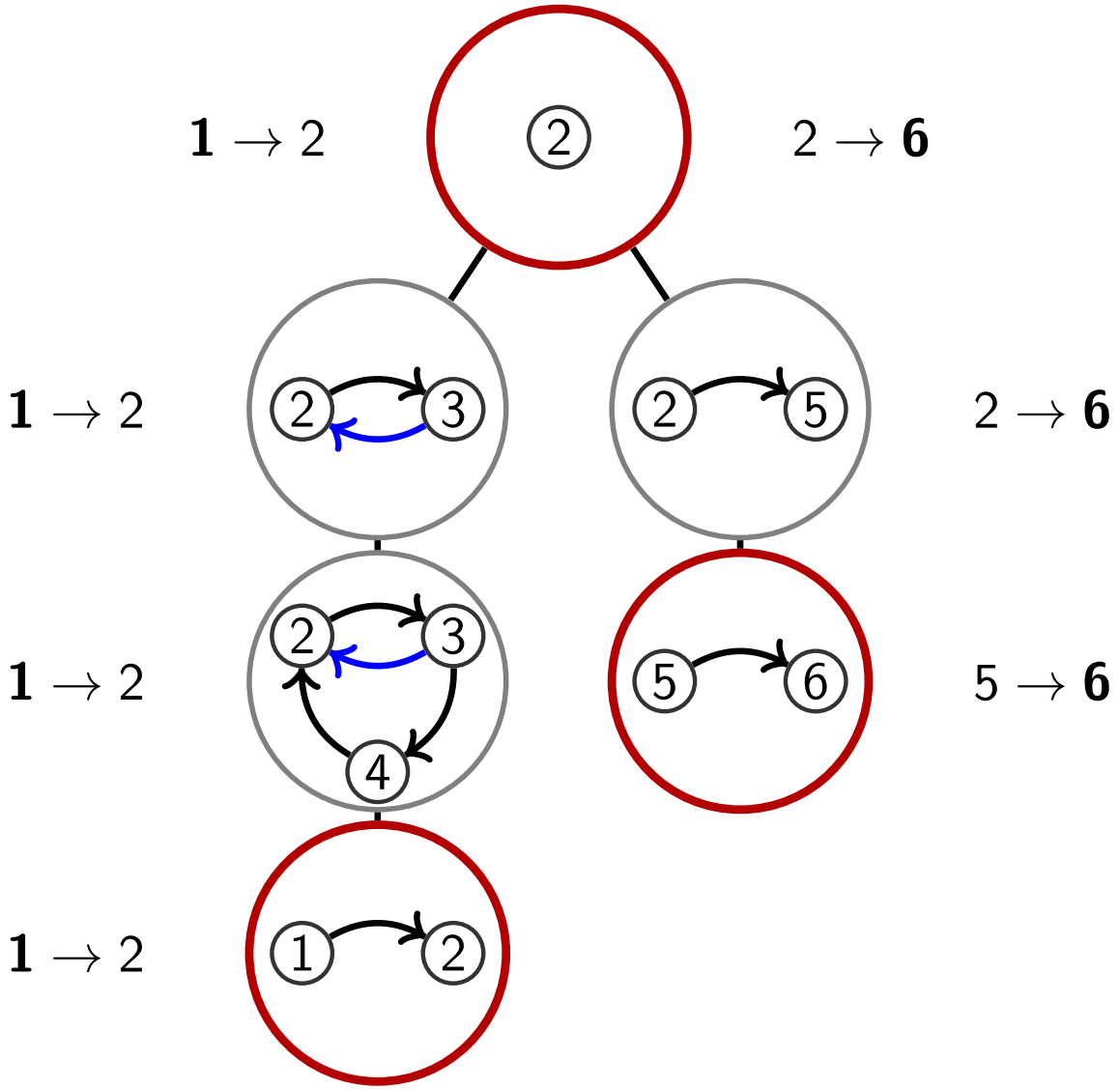
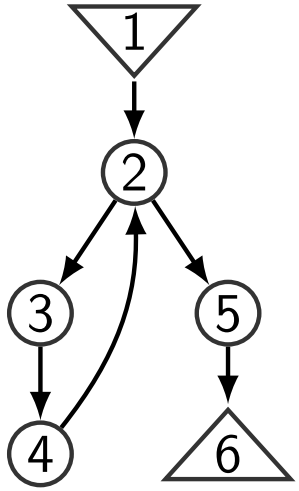




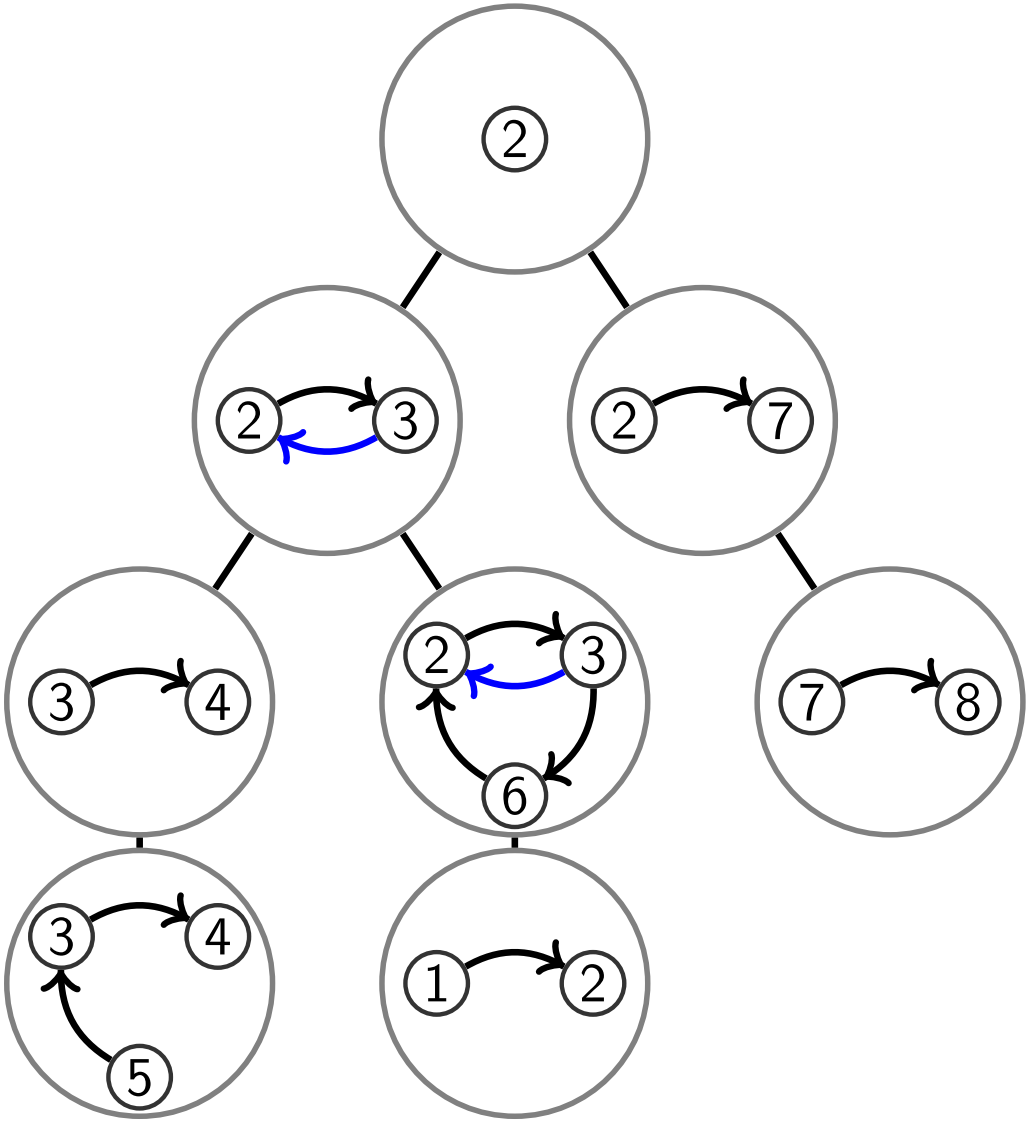
Example: Query $\mathrm{Tree}(G_1)$ for $d(1,6)$

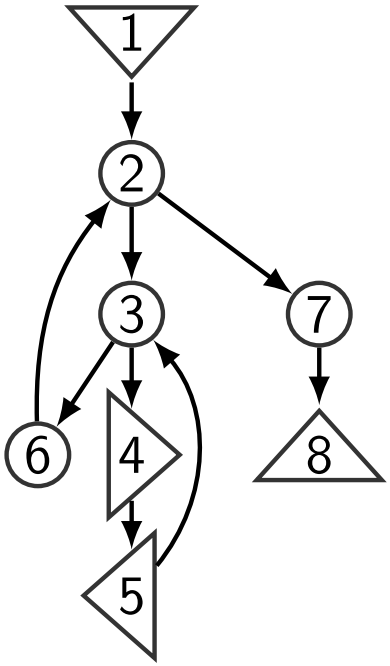




1 → 6

Example: Update $\mathrm{Tree}(G_2)$ with $d(4,5) = \mathrm{True}$





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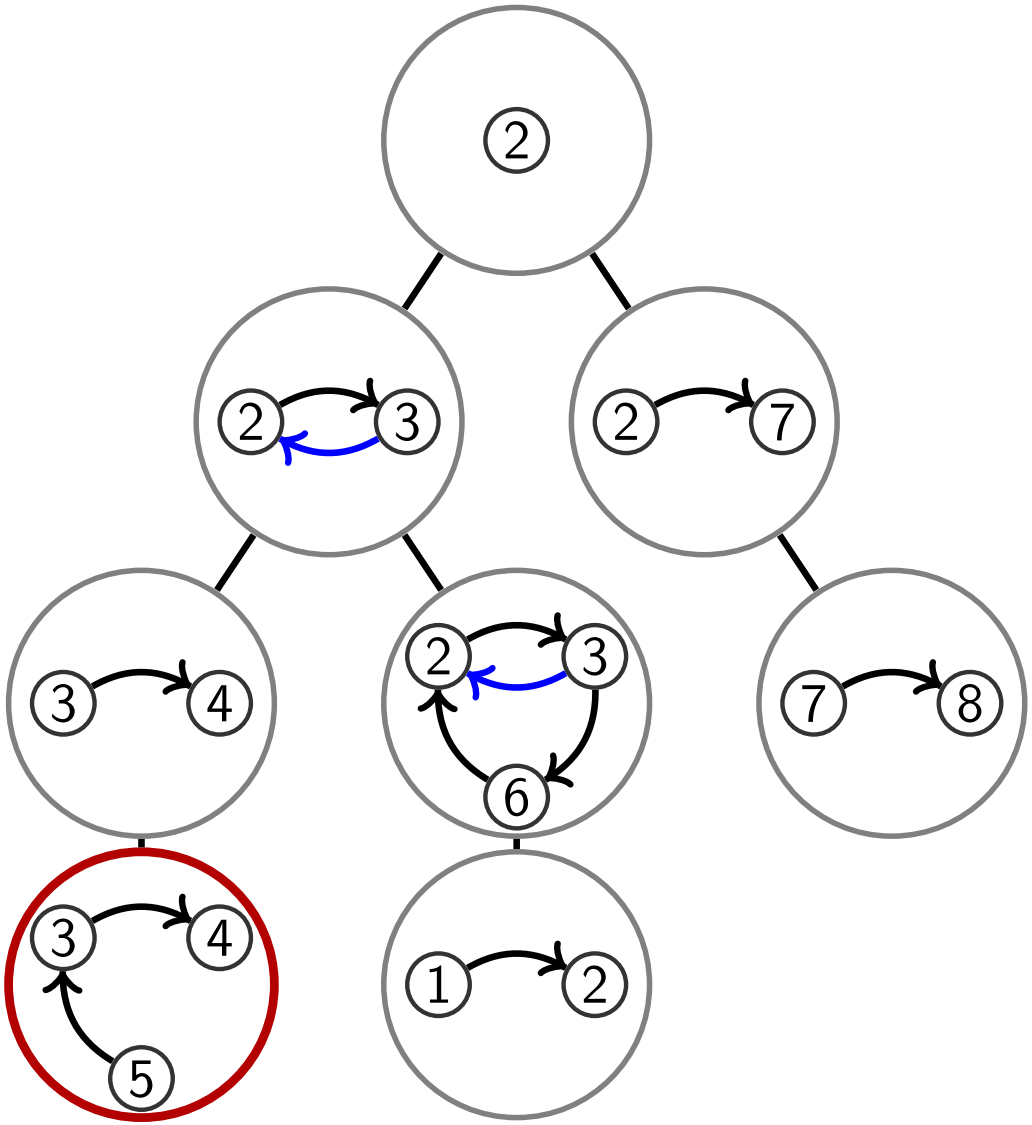

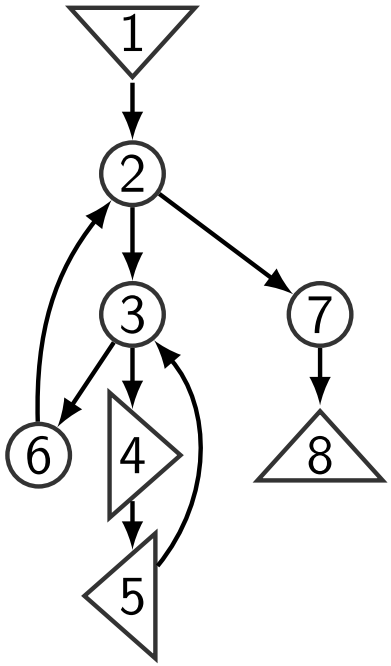

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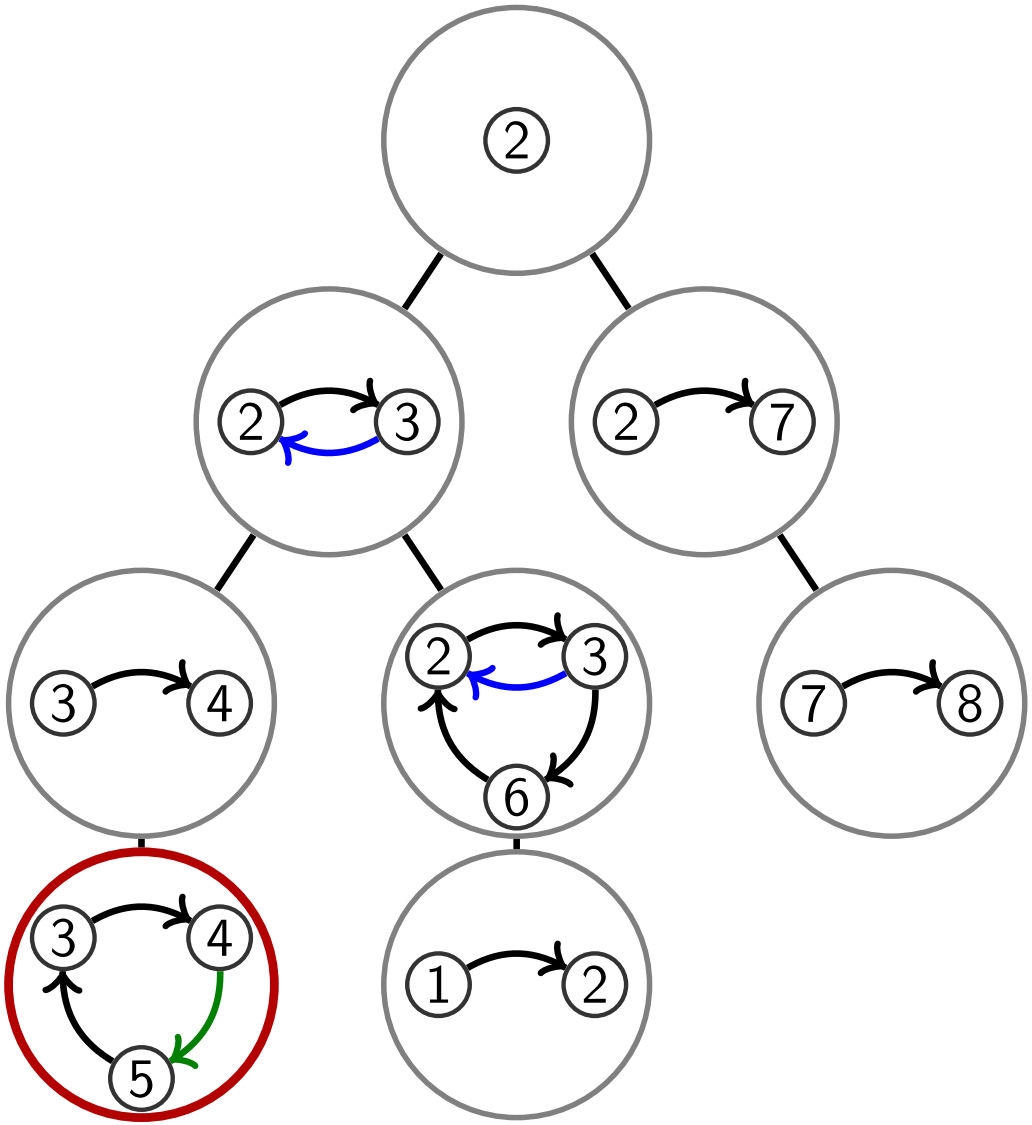

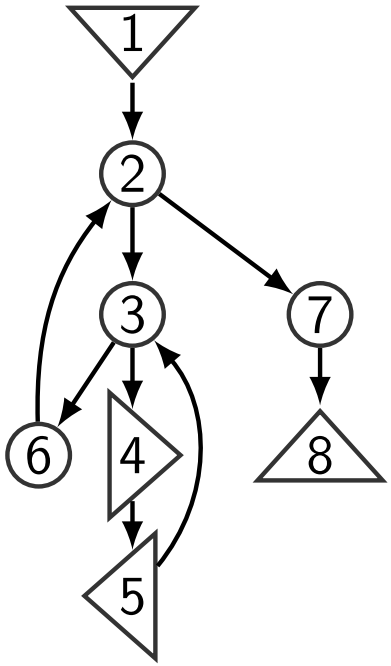

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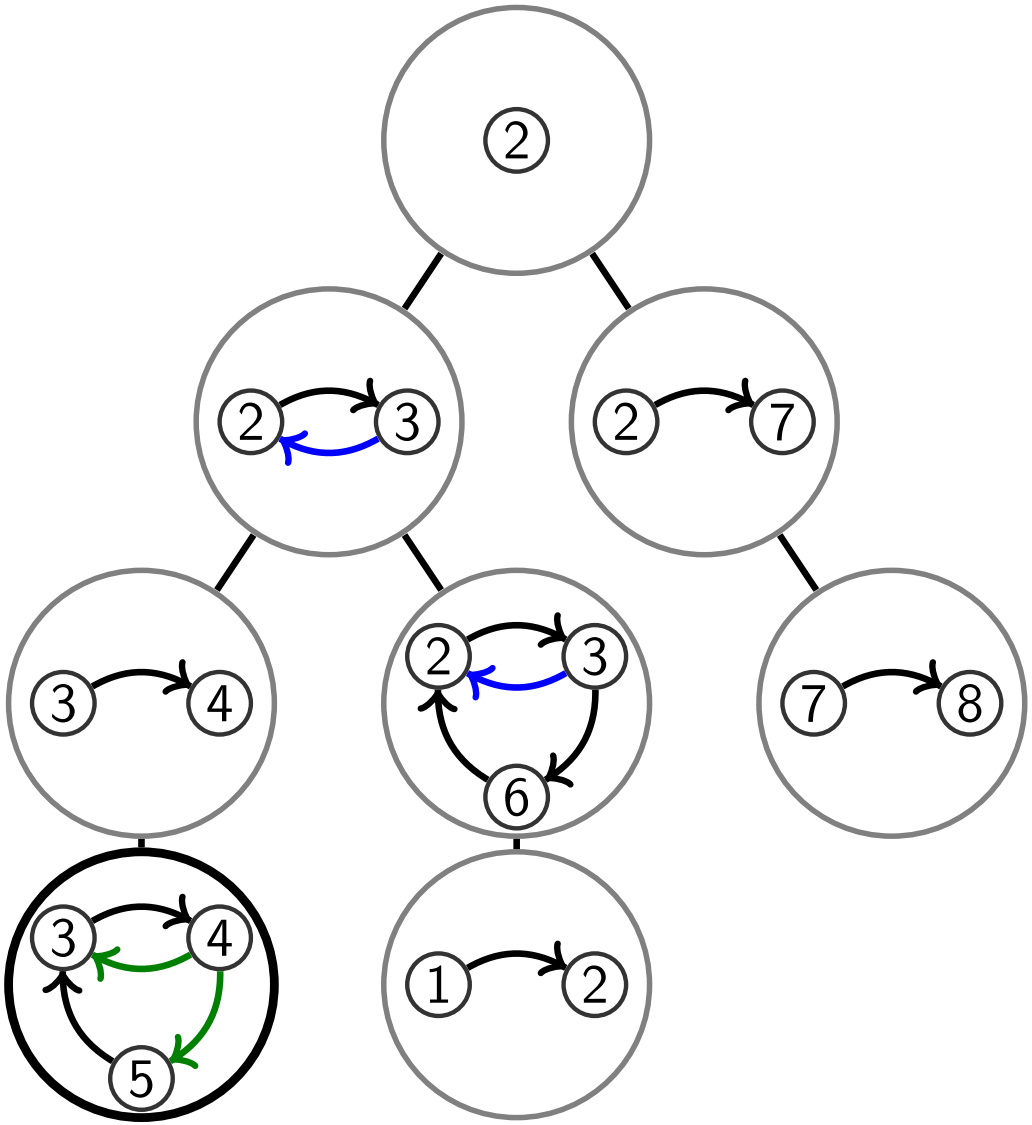

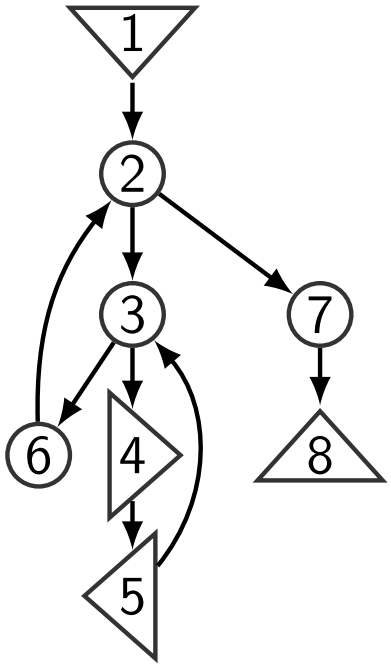

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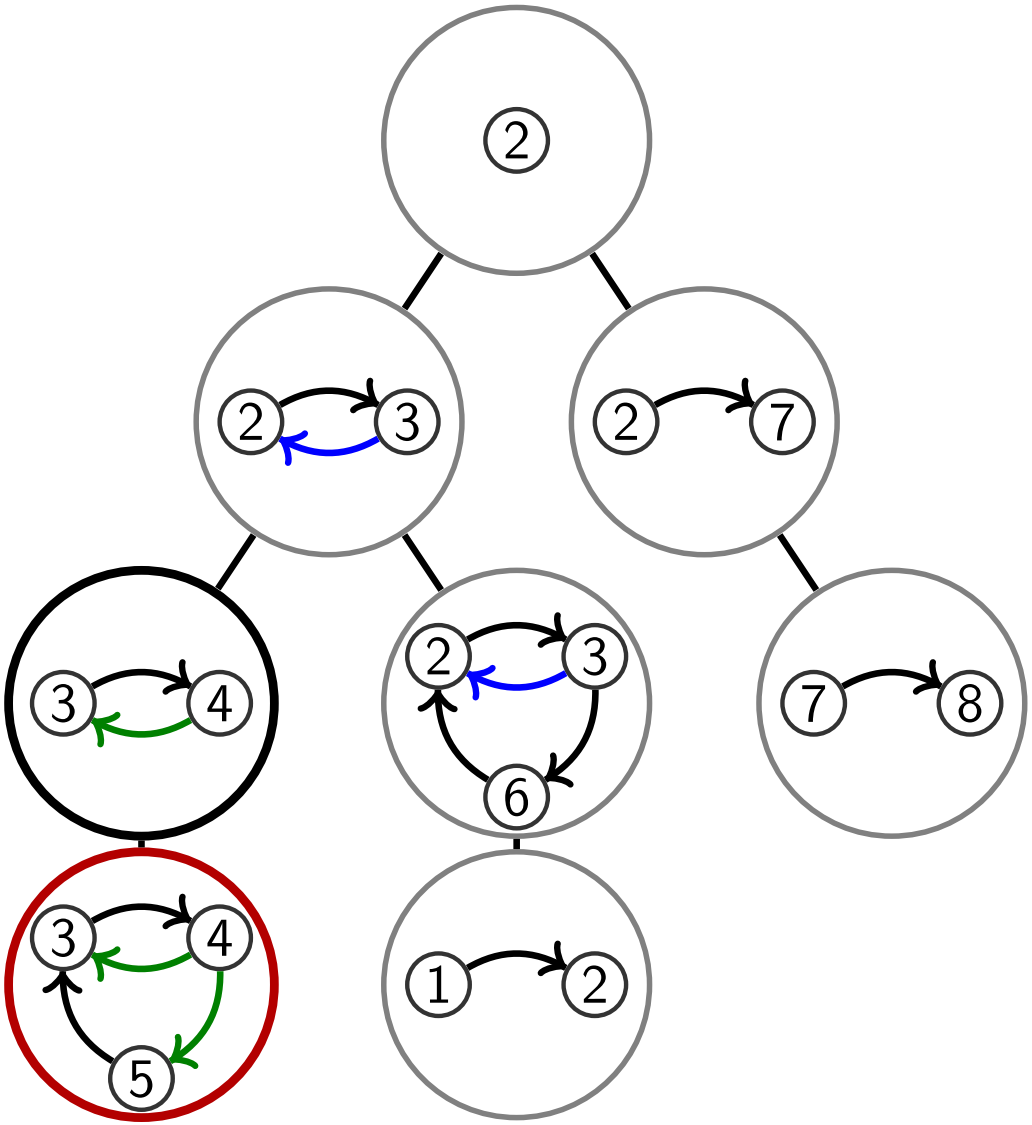

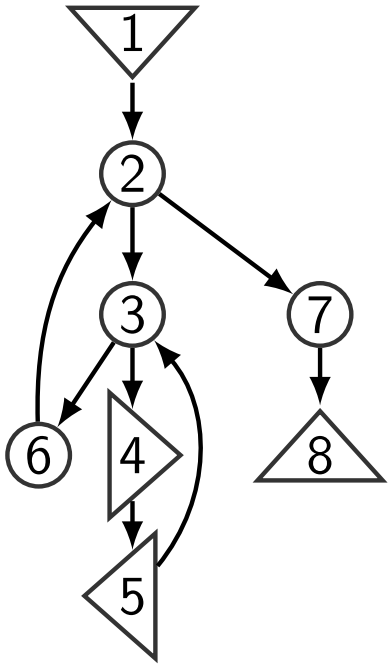

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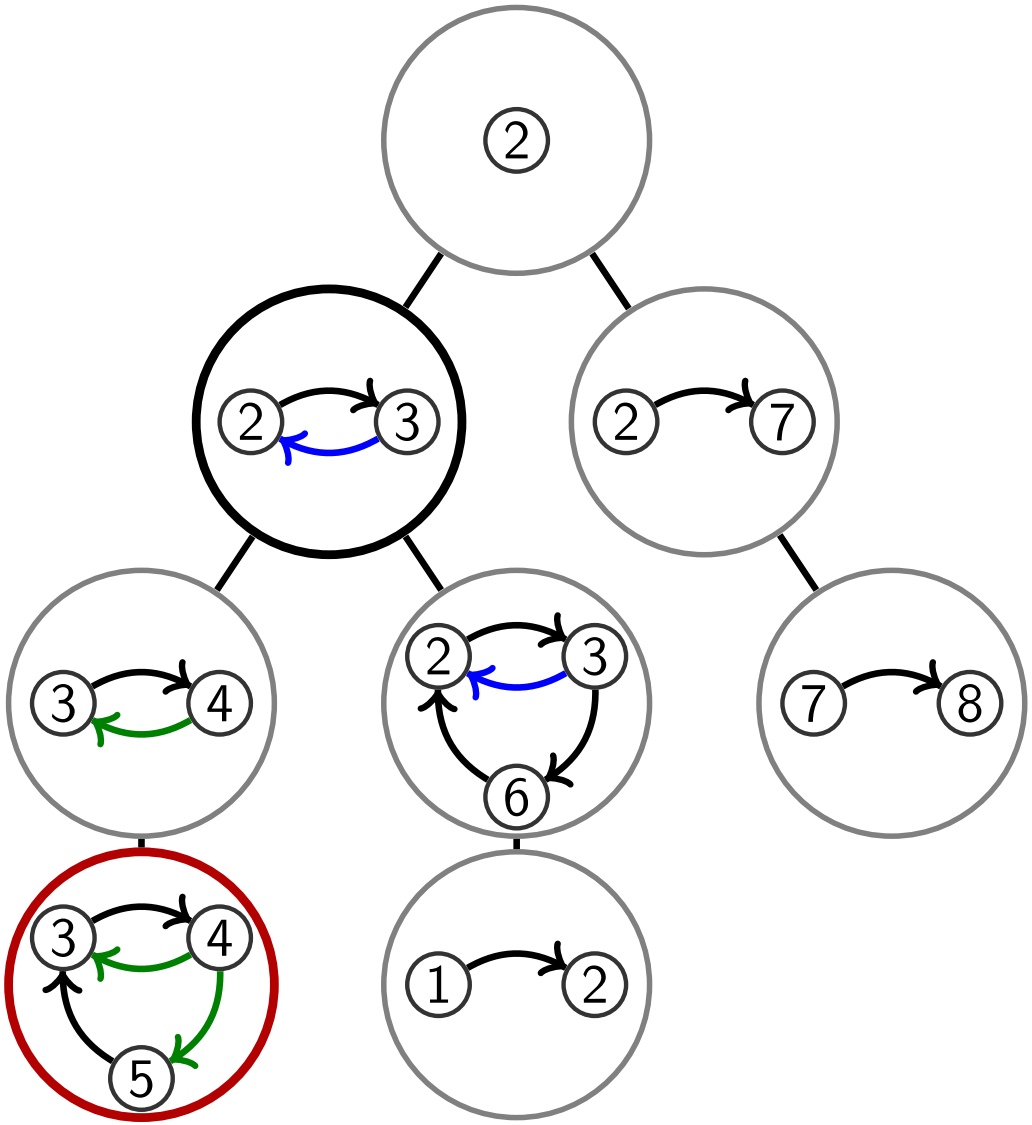

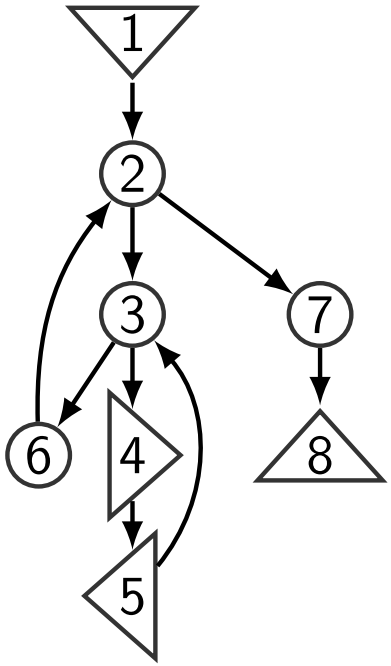

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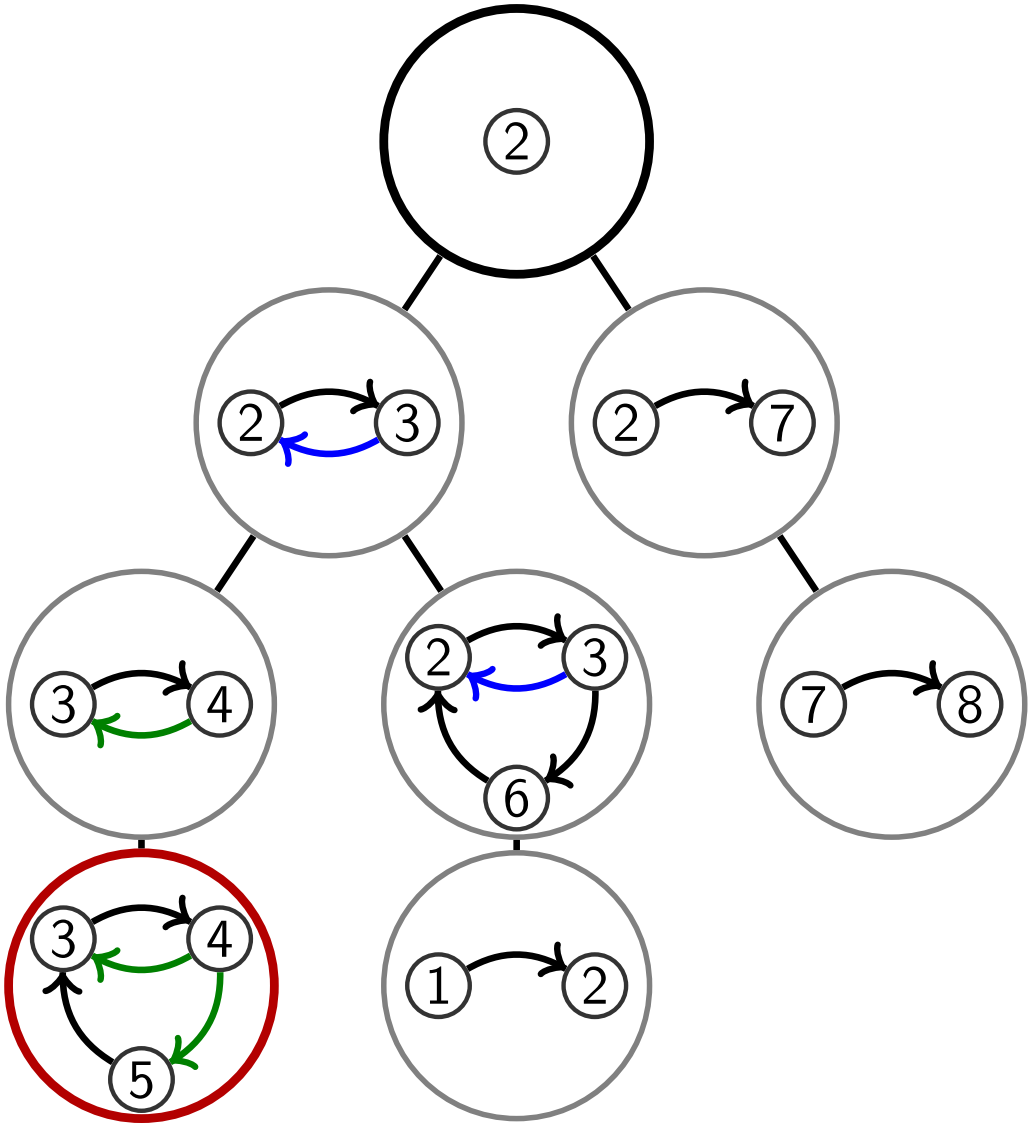

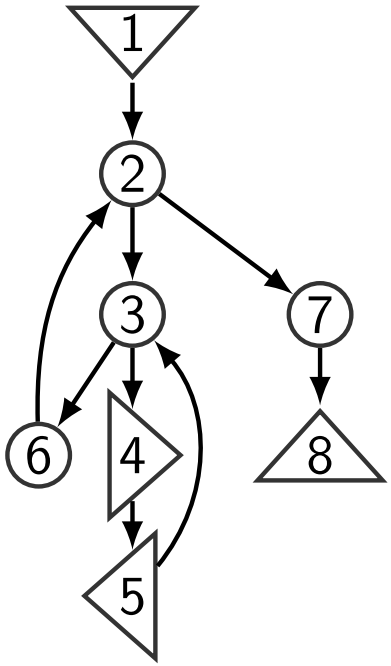

Experimental Results

- Prototype implementation (Java & Soot)
- Intraprocedural reachability and shortest path
- Compared with standard algorithms
- Reachability:
  - Preprocessing: All pairs reachability DFS
  - Query: DFS
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Speedup table

Benchmarks			Reachability			Shortest path		
			Preproc.	Query		Preproc.	Query	
	n	t		*Single*	*Pair*		*Single*	*Pair*
antlr	698	1.0	1.78	10	95	51	97	$6 \cdot 10^4$
bloat	696	2.3	1.97	18	143	130	98	$6 \cdot 10^4$
chart	1159	1.5	4.09	35	171	988	223	$2 \cdot 10^5$
eclipse	656	1.6	3.75	35	82	70	44	$5 \cdot 10^4$
fop	1209	1.7	3.04	20	358	2562	59	$4 \cdot 10^4$
hsqldb	698	1.0	3.24	16	99	57	103	$6 \cdot 10^4$
jython	748	1.5	1.58	11	116	89	77	$9 \cdot 10^4$
luindex	885	1.3	3.94	36	165	182	198	$1 \cdot 10^5$
lusearch	885	1.3	3.68	16	200	132	199	$1 \cdot 10^5$
pmd	644	1.4	1.67	33	96	64	69	$6 \cdot 10^4$
xalan	698	1.0	2.40	29	75	58	63	$5 \cdot 10^4$
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Thank you!
Questions?