Synthesis, Analysis, and Verification (SAV)

Lecture 01

http://lara.epfl.ch/w/sav

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SAV in One Slide

We study how to build software
analysis, verification, and synthesis
tools that automatically
answer questions about software systems.

We cover *theory* and *tool building* through *lectures*, *exercises*, and *labs*.

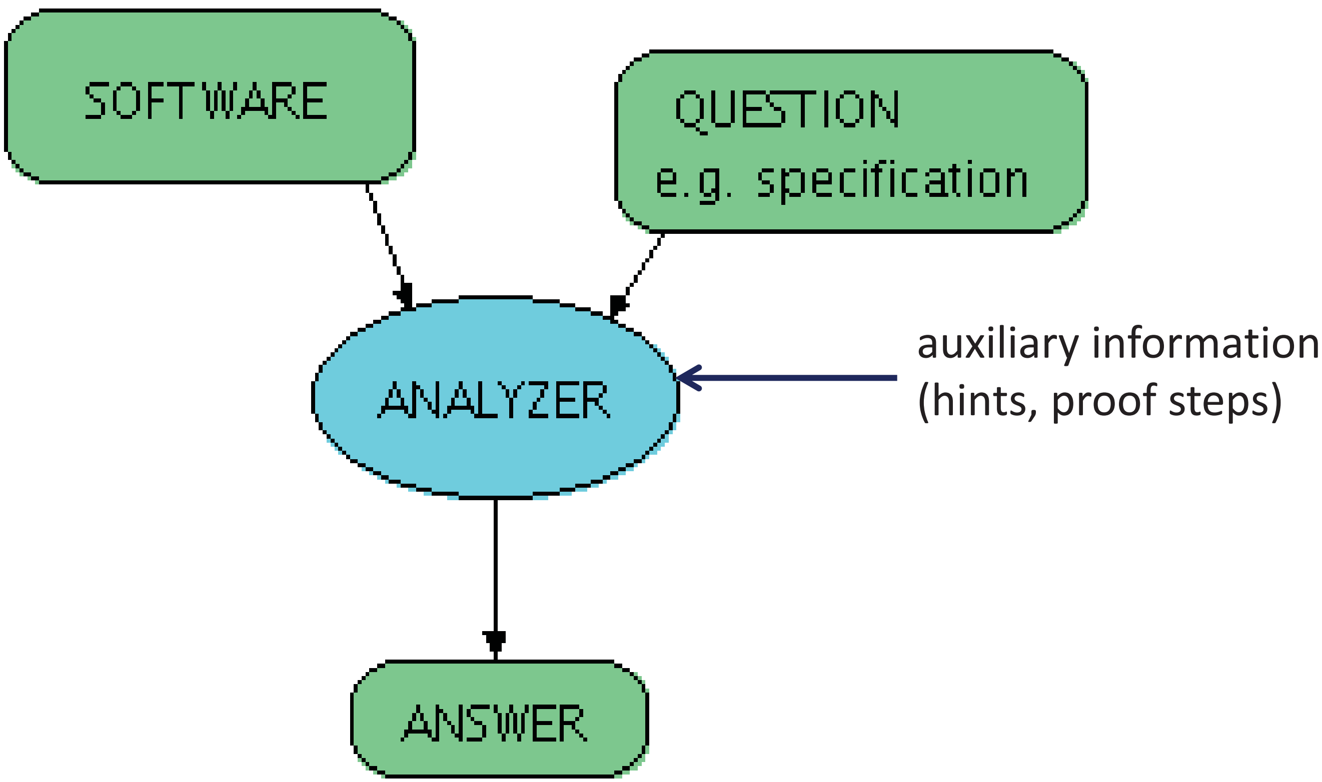
The grading is based on:

- fixed programming project, done in stages: 30%
- midterm (in the second half of the semester): 40%
- personalized project, with writing code (or new proofs), presentation and report: 30%

Suggestion

- Attend all 3 weekly slots
- Always bring a laptop
- Ask questions

Analysis and Verification



Questions of Interest

Example questions in analysis and verification (with sample links to tools or papers):

- [Will the program crash?]
- [Does it compute the correct result?]
- [Does it leak private information?]
- [How long does it take to run?]
- [How much power does it consume?]
- [Will it turn off automated cruise control?]

Activities and Expertise Needed

Modeling: establish precise mathematical meaning for: *software*, *environment*, and *questions* of interest

- discrete mathematics, mathematical logic, algebra

Formalization: formalize this meaning using appropriate representation of *programming languages* and *specification languages*

- program analysis, compilers, theory of formal languages, formal methods

Designing algorithms: derive algorithms that manipulate such formal objects - key technical step

- algorithms, dataflow analysis, abstract interpretation, decision procedures, constraint solving (e.g. SAT), theorem proving

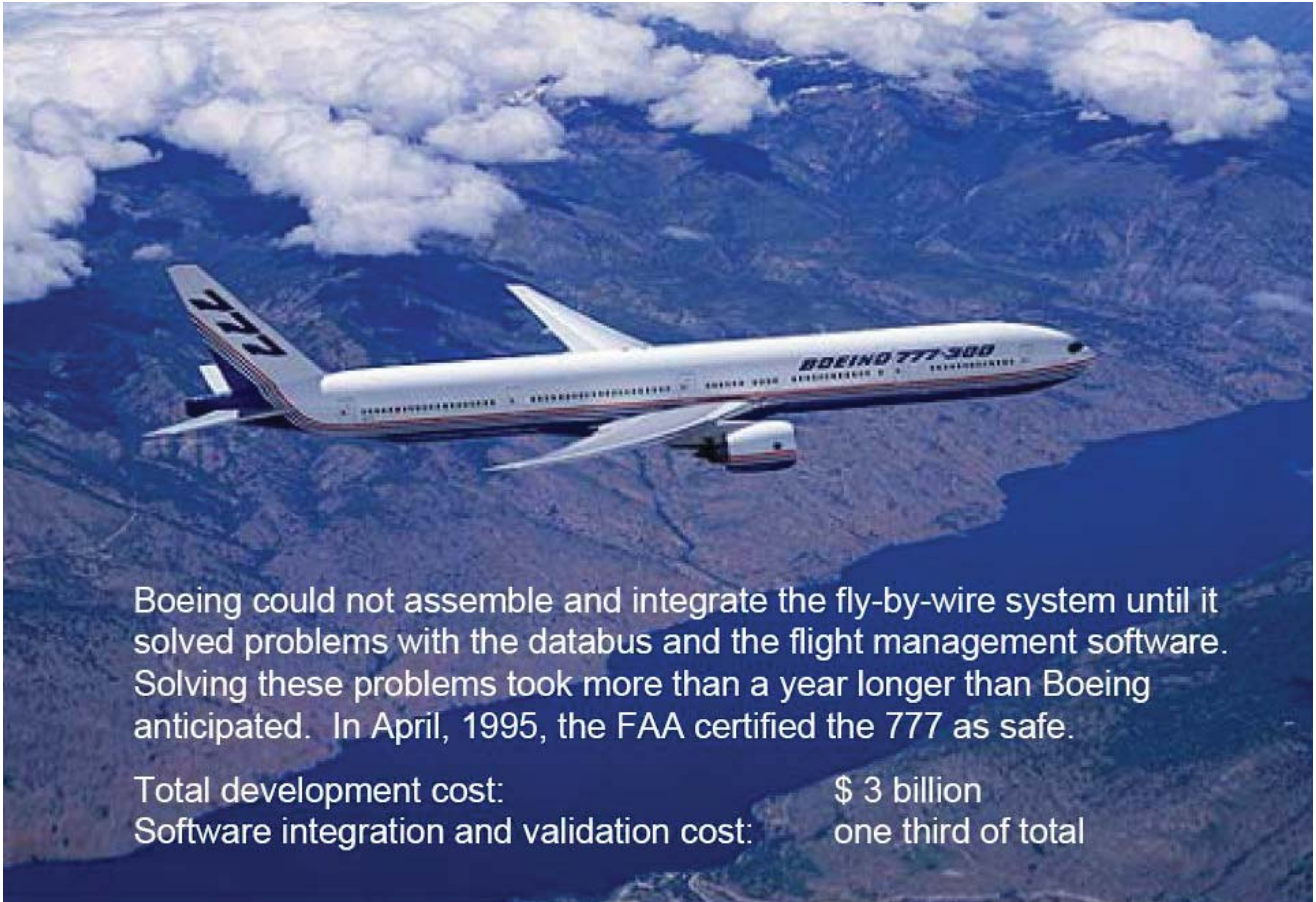
Experimental evaluation: implement these algorithms and apply them to software systems

- developing and using tools and infrastructures, learning lessons to improve and repeat previous steps

Comparison to other Sciences

Specific to SAV is the nature of software as the subject of study, which has several consequences:

- software is an engineering artifact: to an extent we can choose our reality through **programming language design** and **software methodology**
- software has complex **discrete**, **non-linear** structure: **millions of lines** of code, **gigabytes of bits** of state, one condition in if statement can radically change future execution path (non-continuous behavior)
- high standards of correctness: **interest in details** and exceptional behavior (bugs), not just in general trends of software behavior
- high standards along with large the size of software make manual analysis infeasible in most cases, and requires **automation**
- automation requires not just mathematical modeling, where we use everyday mathematical techniques, but also **formal modeling**, which requires us to specify the representation of systems and properties, making techniques from mathematical logic and model theory relevant
- automation means implementing **algorithms** for processing representation of software (e.g. source code) and representation of properties (e.g. formulas expressing desired properties), the study of these algorithms leads to questions of **decidability**, **computational complexity**, and **heuristics** that work in practice.



Boeing could not assemble and integrate the fly-by-wire system until it solved problems with the databus and the flight management software. Solving these problems took more than a year longer than Boeing anticipated. In April, 1995, the FAA certified the 777 as safe.

Total development cost:	$ 3 billion
Software integration and validation cost:	one third of total

Air Transport

August 2005

Gerardo Dominguez/zrh.airlinerpictures.net

As a Malaysia Airlines jetliner cruised from Perth, Australia, to Kuala Lumpur, Malaysia, one evening last August, it suddenly took on a mind of its own and zoomed 3,000 feet upward. The captain disconnected the autopilot and pointed the Boeing 777's nose down to avoid stalling, but was jerked into a steep dive. He throttled back sharply on both engines, trying to slow the plane. Instead, the jet raced into another climb. The crew eventually regained control and manually flew their 177 passengers safely back to Australia.

Investigators quickly discovered the reason for the plane's roller-coaster ride 38,000 feet above the Indian Ocean. A defective software program had provided incorrect data about the aircraft's speed and acceleration, confusing flight computers.

Air Transport

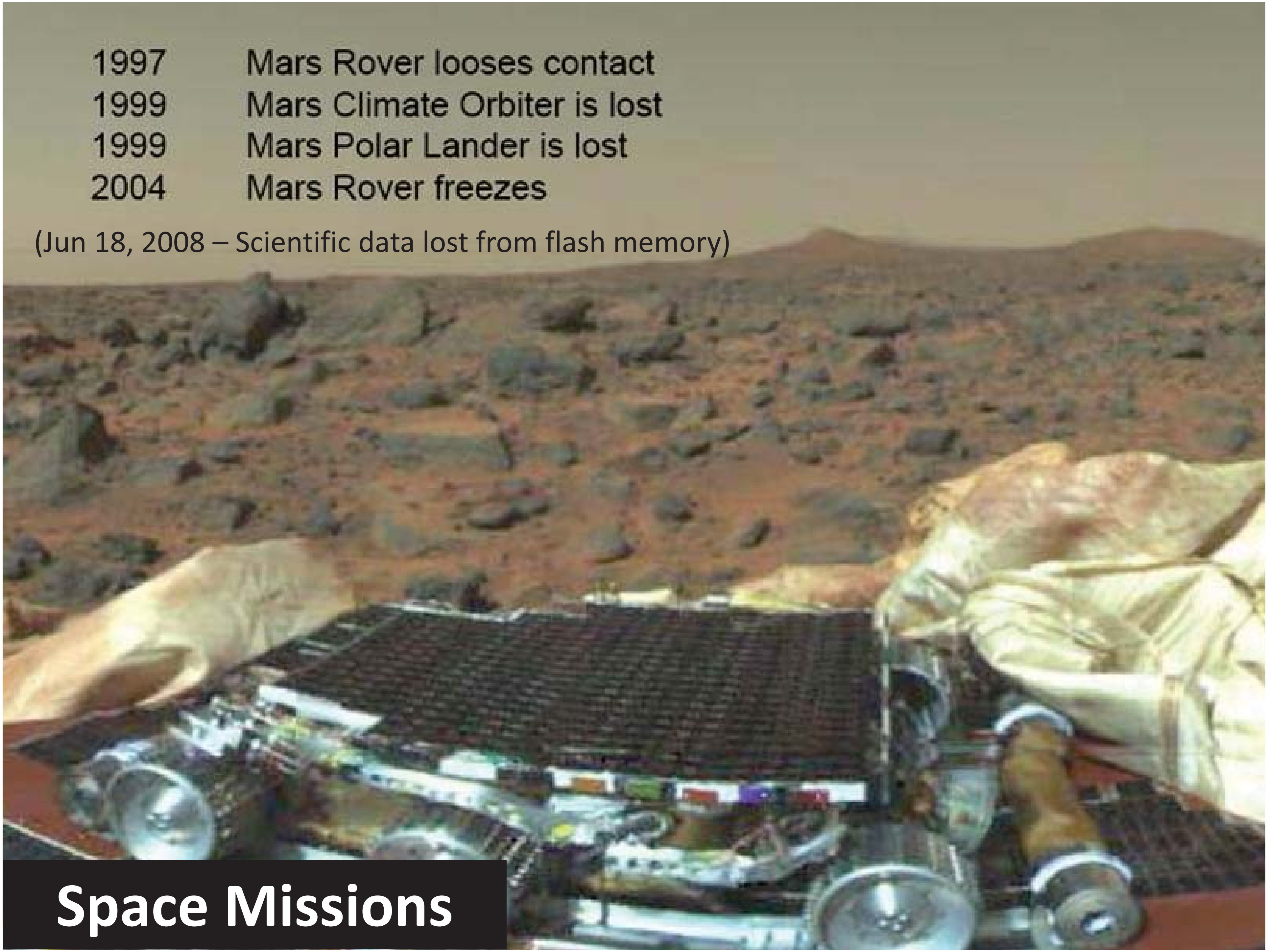
Essential Infrastructure: Northeast Blackout

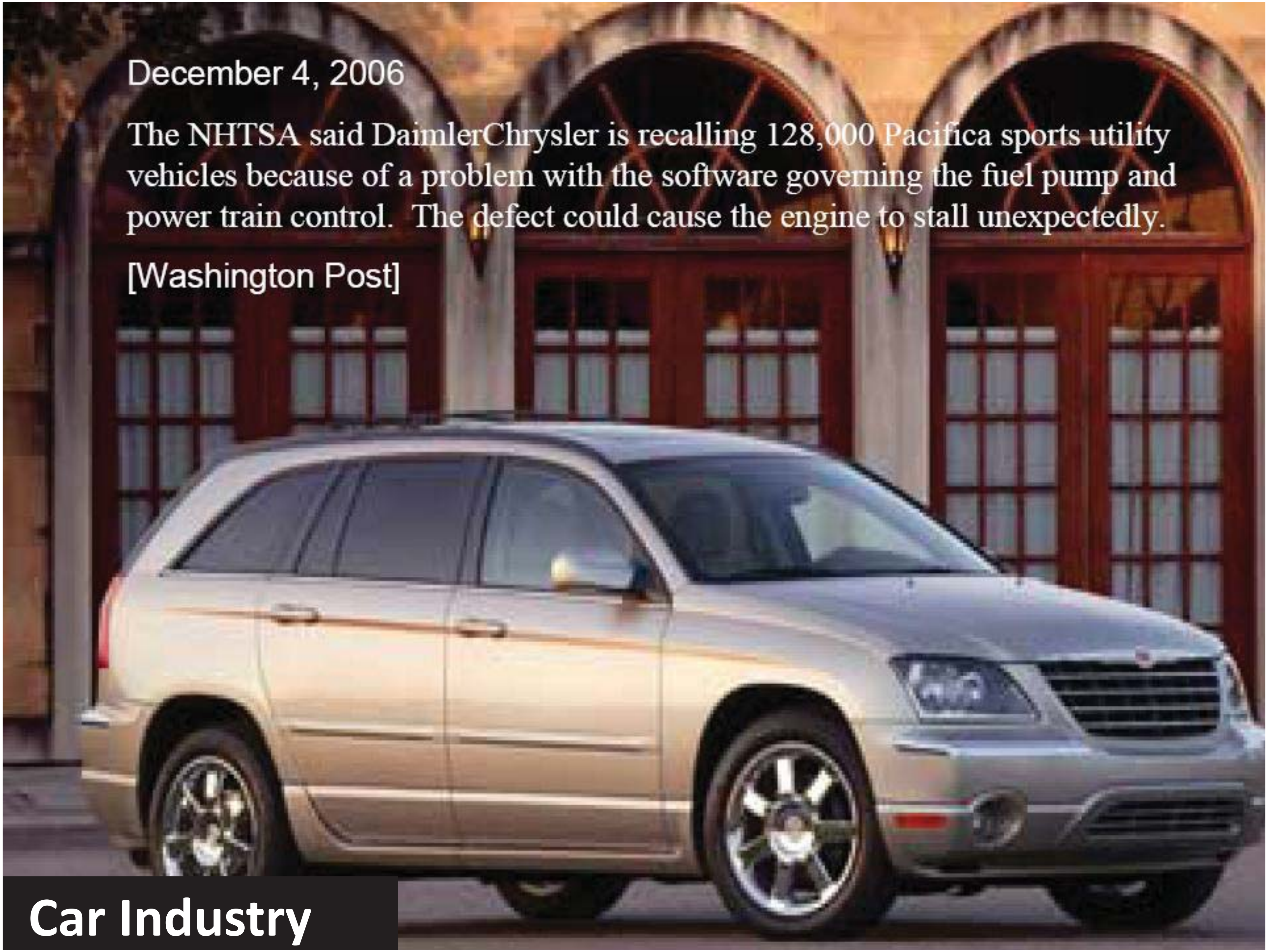
September 14, 2004

Without warning, at about 5 p.m. PDT, air traffic controllers lost contact with about 400 airplanes they were tracking over the southwestern US. A backup system that was supposed to take over in such an event crashed within a minute after it was turned on.

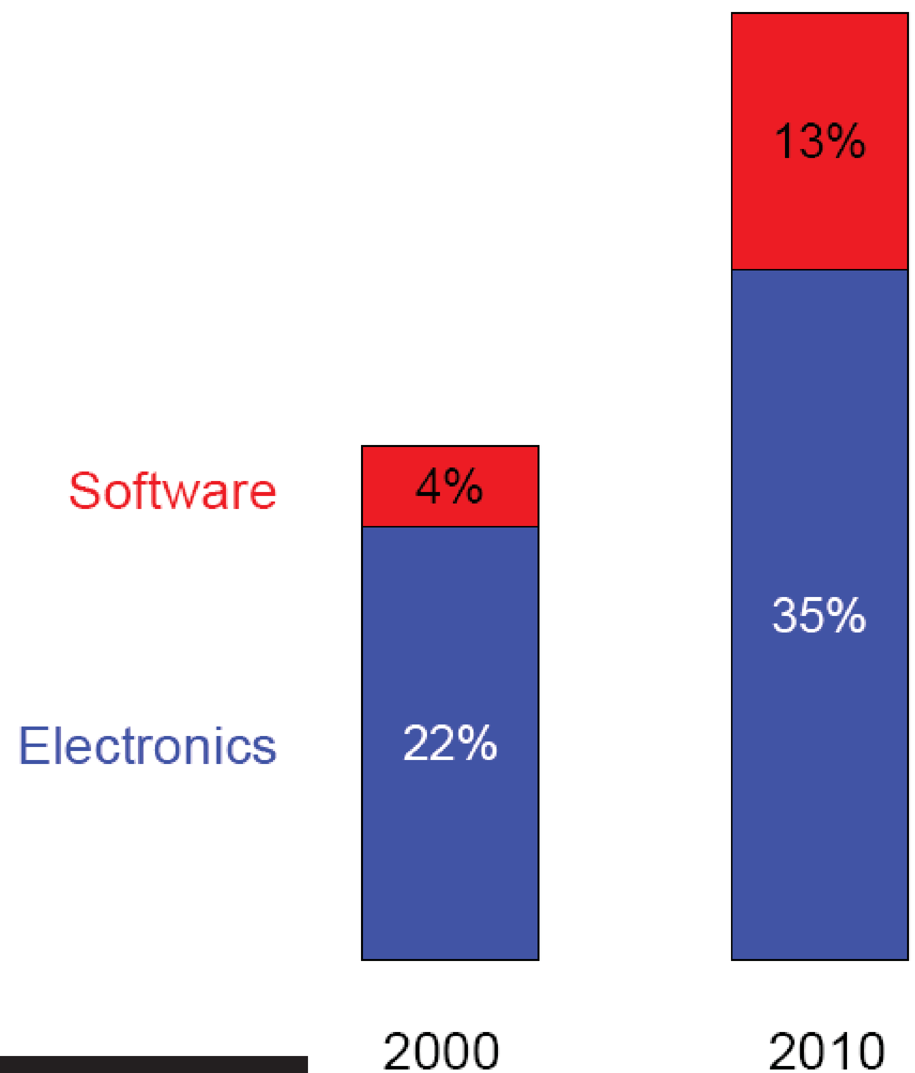
Air Transport


French Guyana, June 4, 1996
t = 0 sec
t = 40 sec
$800 million software failure
Space Missions


1997 Mars Rover looses contact
1999 Mars Climate Orbiter is lost
1999 Mars Polar Lander is lost
2004 Mars Rover freezes
(Jun 18, 2008 – Scientific data lost from flash memory)
Space Missions

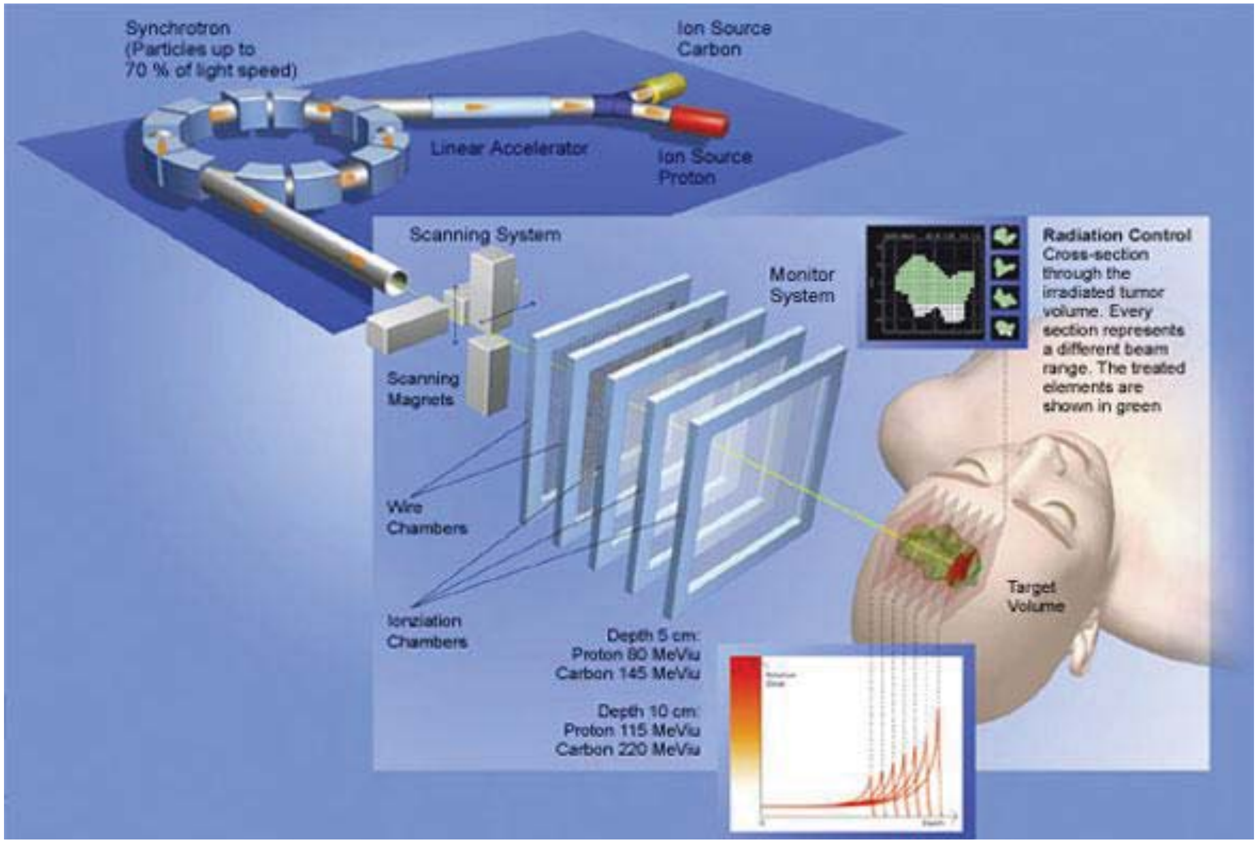

December 4, 2006
The NHTSA said DaimlerChrysler is recalling 128,000 Pacifica sports utility vehicles because of a problem with the software governing the fuel pump and power train control. The defect could cause the engine to stall unexpectedly.
[Washington Post]
Car Industry

Production Cost of Automobiles



[MIT Tech Review]

Car Industry



Radio Therapy

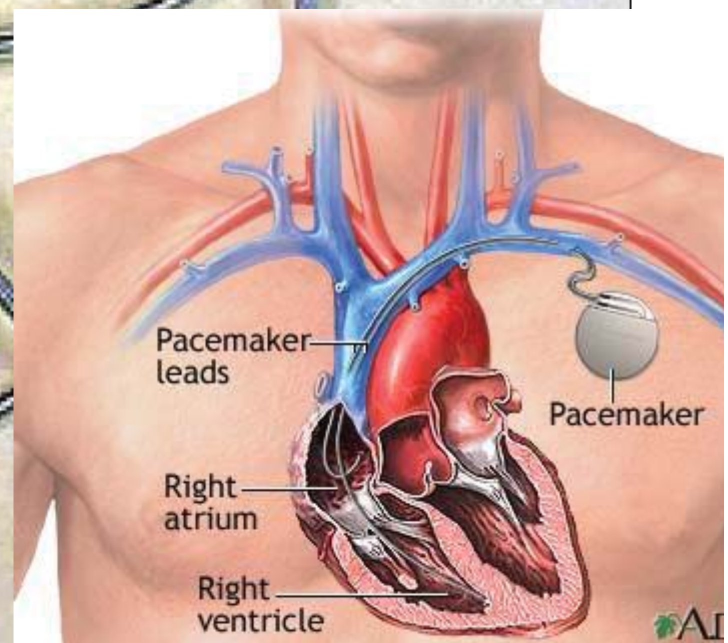
Between June 1985 and January 1987, a computer-controlled radiation therapy machine, called the Therac-25, massively overdosed six people. These accidents have been described as the worst in the 35-year history of medical accelerators [6].

Nancy Leveson
Safeware: System Safety and Computers
Addison-Wesley, 1995

Life-Critical Medical Devices

December 2004

In 1 of every 12,000 settings, the software can cause an error in the programming resulting in the possibility of producing paced rates up to 185 beats/min. It is possible that one or both rate response sensors (i.e., breathing sensor and activity sensor) are switched on, but the timer reset for one or both sensors erroneously remains disabled. In this scenario, the clock timer and the rate response timers can trigger a pace. Of course, with three possible triggers now working independently this can result in high pacing rates.



[Journal of Pacing and Clinical Electrophysiology]

Life-Critical Medical Devices

Zune 30 leapyear problem

- December 31, 2008
- “After doing some poking around in the **source code for the Zune’s clock driver** (available free from the Freescale website), I found the root cause of the now-infamous Zune 30 leapyear issue that struck everyone on New Year’s Eve. The Zune’s real-time clock stores the time in terms of days and seconds since January 1st, 1980. When the Zune’s clock is accessed, the driver turns the number of days into years/months/days and the number of seconds into hours/minutes/seconds. Likewise, when the clock is set, the driver does the opposite.
- The Zune frontend first accesses the clock toward the end of the boot sequence. Doing this triggers the code that reads the clock and converts it to a date and time...”
- “...The function keeps subtracting either 365 or 366 until it gets down to less than a year’s worth of days, which it then turns into the month and day of month. Thing is, in the case of the last day of a leap year, it keeps going until it hits 366. Thanks to the if (days > 366), it stops subtracting anything if the loop happens to be on a leap year. But 366 is too large to break out of the main loop, meaning that the Zune keeps looping forever and doesn’t do anything else.”

http://www.zuneboards.com/forums/zune-news/38143-cause-zune-30-leapyear-problem-isolated.html

More Information

http://mtc.epfl.ch/~tah/Lectures/EPFL-Inaugural-Dec06.pdf

http://www.cse.lehigh.edu/~gtan/bug/softwarebug.html

Success Stories

ASTREE Analyzer

“In Nov. 2003, ASTRÉE was able to prove completely automatically the absence of any RTE in the primary flight control software of the Airbus A340 fly-by-wire system, a program of 132,000 lines of C analyzed in 1h20 on a 2.8 GHz 32-bit PC using 300 Mb of memory (and 50mn on a 64-bit AMD Athlon™ 64 using 580 Mb of memory).”

- http://www.astree.ens.fr/

AbsInt

- 7 April 2005. AbsInt contributes to guaranteeing the safety of the A380, the world's largest passenger aircraft. The Analyzer is able to verify the proper response time of the control software of all components by computing the worst-case execution time (WCET) of all tasks in the flight control software. This analysis is performed on the ground as a critical part of the safety certification of the aircraft.

Interactive Theorem Provers

- A Mechanically Checked Proof of IEEE Compliance of a Register-Transfer-Level Specification of the AMD K7 Floating Point Multiplication, Division and Square Root Instructions, doine using ACL2 Prover
- Formal certification of a compiler back-end, or: programming a compiler with a proof assistant. by Xavier Leroy

Coverity Prevent

- SAN FRANCISCO - January 8, 2008 - Coverity®, Inc., the leader in improving software quality and security, today announced that as a result of its contract with US Department of Homeland Security (DHS), **potential security and quality defects** in 11 popular open source software projects were **identified and fixed**. The 11 projects are **Amanda, NTP, OpenPAM, OpenVPN, Overdose, Perl, PHP, Postfix, Python, Samba, and TCL.**

Microsoft's Static Driver Verifier

Static Driver Verifier (SDV) is a thorough, compile-time, static verification tool designed for kernel-mode drivers.

SDV is included in the Windows Driver Kit (WDK)
SDV systematically analyzes the source code of Windows drivers that are written in the C language.

SDV finds serious errors that are unlikely to be encountered even in thorough testing.

SDV uses a set of interface rules and a model of the operating system to determine whether the driver interacts properly with the Windows operating system.

How to prove programs correct

Proving Program Correctness

```
def f(x : Int, y : Int) : Int
{
  if (y == 0)
    0
  } else {
    if (y % 2 == 0) {
      val z = f(x, y / 2);
      2*z
    } else {
      x + f(x, y - 1)
    }
  }
}
```

- What does ‘f’ compute?
- How can we prove it?

Proving Program Correctness

```
def f(x : Int, y : Int) : Int
{   require(y >= 0)
    if (y == 0)
      0
    } else {
      if (y % 2 == 0) {
          val z = f(x, y / 2);
          2*z
      } else {
          x + f(x, y - 1)
      }
    }
} ensuring (result => result == x * y)
```

By translating Java code into math, we obtain the following mathematical definition of f:

$$f(x,y) = \begin{cases} 0, & \text{if } y = 0 \\ 2f(x, \lfloor \frac{y}{2} \rfloor), & \text{if } y > 0, \text{ and } y = 2k \text{ for some } k \\ x + f(x, y-1), & \text{if } y > 0, \text{ and } y = 2k+1 \text{ for some } k \end{cases}$$

By induction on y we then prove $f(x,y) = x \cdot y$.

- **Base case.** Let $y = 0$. Then $f(x,y) = 0 = x \cdot 0$
- **Inductive hypothesis.** Assume that the claim holds for all values less than y.
  - Goal: show that it holds for y where $y > 0$.
  - **Case 1**: $y = 2k$. Note $k < y$. By definition and I.H.

$$f(x,y) = f(x,2k) = 2f(x,k) = 2(xk) = x(2k) = xy$$

-
  - **Case 2**: $y = 2k+1$. Note $y - 1 < y$. By definition and I.H.

$$f(x,y) = f(x,2k+1) = x + f(x,2k) = x + x \cdot (2k) = x(2k+1) = xy$$

This completes the proof.

An imperative version

```
def fi(x : Int, y : Int) : Int
{
  val r : Int = 0
  val i : Int = 0
  while (i < y) {
    i = i + 1
    r = r + x
  }
  r
}
```

- What does ‘fi’ compute?
- How can we prove it?

An imperative version

```
def fi(x : Int, y : Int) : Int
{  require (y >= 0)
   val r : Int = 0
   val k : Int = 0
   while invariant (r = x * k && k <= x)
           (k < y) {
      k = k + 1
      r = r + x
   }
   r
} ensuring (res =>  res == x * y)
```

Preconditions, Postconditions, Invariants

```
void p()
/*: requires Pre
    ensures Post */
{
  s1;
  while /*: invariant 𝒥 */  (e) {
   s2;
  }
  s3;
}
```

Loop Invariant

$\mathcal{I}$ is a loop invariant if the following three conditions hold:

- $\mathcal{I}$ **holds initially**: in all states satisfying *Pre*, when execution reaches loop entry, $\mathcal{I}$ holds
- $\mathcal{I}$ is **preserved**: if we assume $\mathcal{I}$ and loop condition *(e)*, we can prove that $\mathcal{I}$ will hold again after executing *s2*
- $\mathcal{I}$ is **strong enough**: if we assume $\mathcal{I}$ and the negation of loop condition *e,* we can prove that *Post* holds after *s3*

Explanation: because $\mathcal{I}$ holds initially, and it is preserved, by induction from **holds initially** and **preserved** follows that $\mathcal{I}$ will hold in every loop iteration. The **strong enough** condition ensures that when loop terminates, the rest of the program will satisfy the desired postcondition.

Membership in Binary Search Tree - theory of data types

```
sealed abstract class BST {
  def contains(key: Int): Boolean = (this : BST) match {
    case Node(left: BST,value: Int, _) if key < value => left.contains(key)
    case Node(_,value: Int, right: BST) if key > value => right.contains(key)
    case Node(_,value: Int, _) if key == value => true
    case e : Empty => false
  }
}
case class Empty extends BST
case class Node(val left: BST, val value: Int, val right: BST) extends BST
```

Leon verifier:

http://lara.epfl.ch/leon/

- see new version this Friday

How can we automate verification?

Important algorithmic questions:

- **verification condition generation**: compute formulas expressing program correctness
  - Hoare logic, weakest precondition, strongest postcondition
- **theorem proving: prove verification conditions**
  - proof search, counterexample search
  - decision procedures
- **loop invariant inference**
  - predicate abstraction
  - abstract interpretation and data-flow analysis
  - pointer analysis, typestate
- reasoning about numerical computation
- pre-condition and post-condition inference
- ranking error reports and warnings
- finding error causes from counterexample traces

Recommended Reading

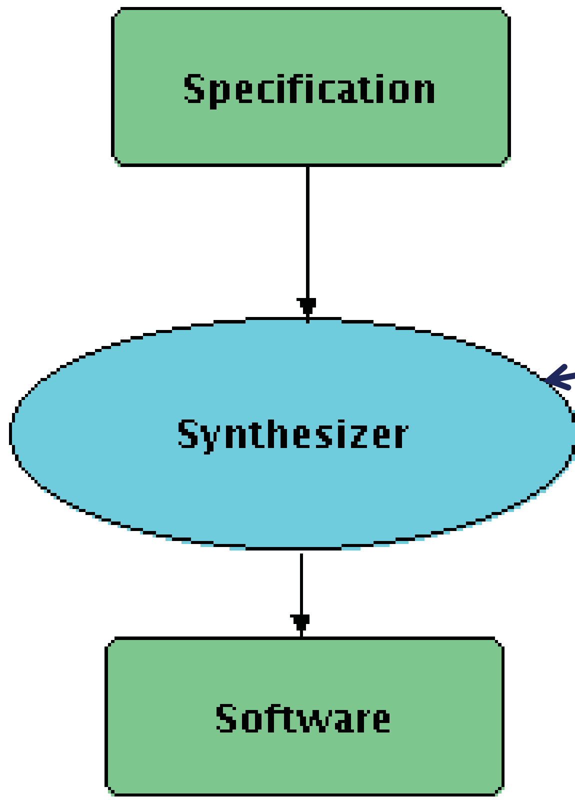
- Recent *Research Highlights* from the **Communications of the ACM**
  - A Few Billion Lines of Code Later: Using Static Analysis to Find Bugs in the Real World

A Great Video

Talk by Professor J Strother Moore

http://slideshot.epfl.ch/play/suri_moore

Synthesis



auxiliary information (structure of expected program)

An example

```
def secondsToTime(totalSeconds: Int) : (Int, Int, Int) =
  choose((h: Int, m: Int, s: Int) ⇒ (
          h * 3600 + m * 60 + s == totalSeconds
     && h ≥ 0
     && m ≥ 0 && m < 60
     && s ≥ 0 && s < 60    ))
```

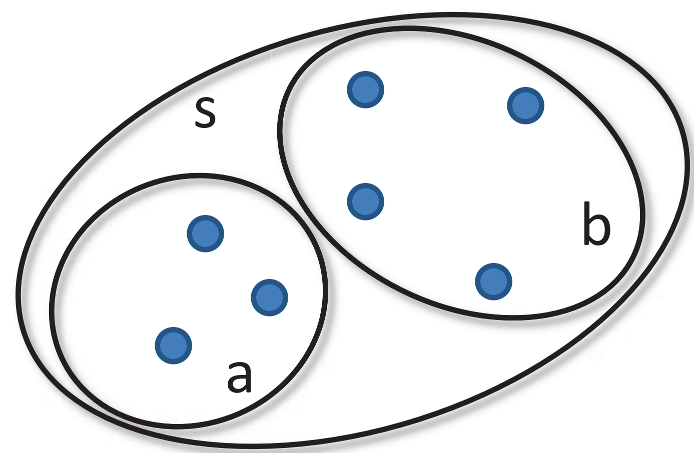
3787 seconds ⟶ 1 hour, 3 mins. and 7 secs.

```
def secondsToTime(totalSeconds: Int) : (Int, Int, Int) =
  val t1 = totalSeconds div 3600
  val t2 = totalSeconds + ((-3600) * t1)
  val t3 = min(t2 div 60, 59)
  val t4 = totalSeconds + ((-3600) * t1) + (-60 * t3)
  (t1, t3, t4)
```

Synthesis for sets

```
def splitBalanced[T](s: Set[T]) : (Set[T], Set[T]) =
  choose((a: Set[T], b: Set[T]) ⇒ (
           a union b == s && a intersect b == empty
     && a.size – b.size ≤ 1
     && b.size – a.size ≤ 1
  ))
```

```
def splitBalanced[T](s: Set[T]) : (Set[T], Set[T]) =
  val k = ((s.size + 1)/2).floor
  val t1 = k
  val t2 = s.size – k
  val s1 = take(t1, s)
  val s2 = take(t2, s minus s1)
  (s1, s2)
```



choose$((x, y) \Rightarrow 5 * x + 7 * y == a \ \&\& \ x \leq y)$

Use extended Euclid's algorithm to find particular solution to $5x + 7y = a$:

(5,7 are mutually prime, else we get divisibility pre.)

$x = 3a$
$y = -2a$

Express general solution of *equations* for x, y using a new variable z:

$x = -7z + 3a$
$y = 5z - 2a$

Rewrite *inequations* $x \leq y$ in terms of z:

$5a \leq 12z$

$\rightarrow z \geq \text{ceil}(5a/12)$

Obtain synthesized program:

```
val z = ceil(5*a/12)
val x = -7*z + 3*a
val y = 5*z + -2*a
```

For a = 31:

$z = \text{ceil}(5*31/12) = 13$
$x = -7*13 + 3*31 = 2$
$y = 5*13 – 2*31 = 3$

choose((x, y) ⇒ 5 * x + 7 * y == a && x ≤ y && x ≥ 0)

Express general solution of *equations* for x, y using a new variable z:

$x = -7z + 3a$
$y = 5z - 2a$

Rewrite *inequations* x ≤ y in terms of z: $z \geq \mathrm{ceil}(5a/12)$

Rewrite x ≥ 0: $z \leq \mathrm{floor}(3a/7)$

Precondition on a: $\boxed{\mathrm{ceil}(5a/12) \leq \mathrm{floor}(3a/7)}$

(exact precondition)

Obtain synthesized program:

```
assert(ceil(5*a/12) ≤ floor(3*a/7))
val z = ceil(5*a/12)
val x = -7*z + 3*a
val y = 5*z + -2*a
```

With more inequalities we may generate a for loop

Other Forms of Synthesis

Synthesis within IDEs

Compiling declarative constructs

Automata-Theoretic Synthesis

- reactive synthesis
- regular synthesis over unbounded domains

Synthesis of Synchronization Constructs

Quantitative Synthesis

Synthesis from examples

- **Sumit Gulwani**: *Automating String Processing in Spreadsheets using Input-Output Examples (video available in the ACM Digital Library)*

Presburger Arithmetic

Motivation

```
res = 0
i = x
while invariant res + 2*i == 2*x
  (i > 0) {
   i = i - 1
   res = res + 2
}
assert(res == 2*x)
```

Verification condition showing loop inv. preserved

$res + 2*i = 2*x \ \wedge\ i_1 = i - 1 \ \wedge\ res_1 = res + 2 \rightarrow$

$res_1 + 2*i_1 = 2*x$

Proving integer linear arithmetic formulas

Verification condition showing loop inv. preserved

$$(res + 2\ i = 2\ x\ \wedge\ i_1 = i - 1\ \wedge\ res_1 = res + 2) \rightarrow$$
$$res_1 + 2\ i_1 = 2\ x$$

Need to show it is *true for all* variables

Show: *negation is never true* (unsatisfiable)

$res + 2\ i = 2\ x\ \wedge\ i_1 = i - 1\ \wedge\ res_1 = res + 2\ \wedge$
$\quad res_1 + 2\ i_1 \neq 2\ x$

In this case, it is simple. Substitute variables:

$(res+2) + 2(i - 1) \neq res + 2\ i$

$0 \neq 0$ group coefficients to obtain "false"

A More Difficult Example

$\exists$ x,y,k,p.

$(x < y + 2 \wedge y < x + 1 \wedge x = 3k \wedge$

$(y = 6p+1 \vee y = 6p-1))$

Is this statement true?

General question:

is a formula of **Presburger arithmetic** satisfiable?

$$\mathbf{F ::= A \mid F_1 \wedge F_2 \mid F_1 \vee F_2 \mid \neg F \mid \exists k.F \mid \forall k.F}$$

$$\mathbf{A ::= T_1 = T_2 \mid T_1 < T_2}$$

$$\mathbf{T ::= k \mid C \mid T_1 + T_2 \mid T_1 - T_2 \mid C * T \mid T \,\%\, C}$$

Presburger Arithmetic

$F ::= A \mid F_1 \wedge F_2 \mid F_1 \vee F_2 \mid \neg F \mid \exists k.F \mid \forall k.F$
$A ::= T_1 = T_2 \mid T_1 < T_2$
$T ::= k \mid C \mid T_1 + T_2 \mid T_1 - T_2 \mid C * T \mid T \,\%\, C$

t%C - the reminder in division by C

Formula $\exists x.\ x < y$ has

- one bound variable: x
- one free variable: y

If we have free variables we cannot ask if formula is true, but only if it is satisfiable (true for some values of free variables), valid (always true), unsatisfiable (always false)

Presburger arithmetic is decidable

There is an algorithm that, given arbitrary formula in the syntax of Presburger arithmetic, detects whether this formulas is satisfiable.

Thus also decidable are:

unsatisfiability, validity, equivalence, entailment.

Mojzesz Presburger. *Über die Vollstandigkeit eines gewissen Systems der Arithmetik*. Comptes rendus du I Congrès des Pays Slaves, Warsaw 1929.

Mojżesz Presburger (1904–1943) was student of Alfred Tarski and is known for, among other things, having invented Presburger arithmetic.

Method used: **quantifier elimination**

Quantifier Elimination

Take a formula of the form

$\exists$ y. F(x,y)

replace it with an **equivalent** formula

G(x)

without introducing new variables.

Idea: eliminate quantified variables. E.g.

$\exists \mathbf{k}. \ (\mathbf{x + k = 2 \land k < 10})$

$\exists \mathbf{k}. \ (\mathbf{k = 2 - x \land k < 10})$ (one-point rule)

$\mathbf{2 - x < 10}$

Arithmetic with only multiplication

$$x = y * z * p * z \;\wedge\; (x * y = u * z \;\vee\; u*u = x)$$

Decidable. Use prime factor representation

$$x = 2^{p1}\, 3^{p2}\, 5^{p3}\, 7^{p4}\, 11^{p5} \ldots$$

$$y = 2^{q1}\, 3^{q2}\, 5^{q3}\, 7^{q4}\, 11^{q5} \ldots$$

$$xy = 2^{(p1+q1)}\, 3^{(p2+q2)}\, 5^{(p3+q3)}\, 7^{(p4+q4)}\, 11^{(p5+q5)} \ldots$$

Feferman-Vaught theorem: if we can decide logic of elements, we can decide logic of sequences of elements with point-wise relations on them.

Solomon Feferman (born 13 December 1928) is an American philosopher and mathematician with major works in mathematical logic. He was born in New York City, New York, and received his Ph.D. in 1957 from the University of California, Berkeley under **Alfred Tarski**. He is a Stanford University professor.

Alfred Tarski (January 14, 1901, Warsaw, Russian-ruled Poland – October 26, 1983, Berkeley, California) was a Polish logician and mathematician. Educated in the Warsaw School of Mathematics and philosophy, he emigrated to the USA in 1939, and taught and carried out research in mathematics at the University of California, Berkeley, from 1942 until his death.

… He is regarded as perhaps one of the four greatest logicians of all time, matched only by Aristotle, Kurt Gödel, and Gottlob Frege.

Formulas with both plus and times over integers

- Posed as a big open problem at the beginning of 20th century to find decision procedure (Hilbert's 10th Problem)

Yuri Matiyasevich. *Enumerable sets are diophantine.* Journal of Sovietic Mathematics, (11):354 358, 1970.

Undecidability of Hilbert's Tenth Problem:

Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers.

Formulas over plus and times over real numbers

- Decidable!
  - Also over complex numbers
- Shown by Alfred Tarski before WW II
- First implementation by Collins
  - we have a Scala implementation available

Summary

- Programs can be converted to formulas
- To prove program correct, we prove formula valid (true in all models)
- For some classes (e.g. Presburger arithmetic) we understand how to prove them
  - other classes – future research
  - such research can lead to tools that make software reliable