Lecturecise 21

2013

Example Formula in First-Order Logic

model of a formula = interpretation (structure) that makes a formula true

$$
\begin{aligned}
\neg(\quad & (\forall x.\exists y.\ R(x,y)) \land \\
& (\forall x.\forall y.\ (R(x,y) \Rightarrow \forall z.\ R(x, f(y,z)))) \land \\
& (\forall x.\ (P(x) \lor P(f(x,a)))) \\
& \Rightarrow \forall x.\exists y.\ (R(x,y) \land P(y)))
\end{aligned}
$$

After normal form and Skolemization we obtain these first-order clauses:

$$
\begin{aligned}
& R(x, g_1(x)) \\
& \neg R(x,y) \lor R(x, f(y,z)) \\
& P(x) \lor P(f(x,a)) \\
& \neg R(c_0, y) \lor \neg P(y)
\end{aligned}
$$

- variables are implicitly $\forall$ quantified; there are no $\exists$ quantifiers
- each clause is disjunction of literals (atomic formulas or their negation)
- from any model of these clauses we can obtain model for the original formula (just ignore interpretation of Skolem constants g_1, c_0)

Applying Resolution

1. $R(x, g_1(x))$
2. $\neg R(x, y) \vee R(x, f(y, z))$
3. $P(x) \vee P(f(x, a))$
4. $\neg R(c_0, y) \vee \neg P(y)$

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3. $P(x) \vee P(f(x, a))$
4. $\neg R(c_0, y) \vee \neg P(y)$
5. (1,2): $R(x, f(g_1(x), z))$

Applying Resolution

1 $R(x, g_1(x))$

2 $\neg R(x, y) \vee R(x, f(y, z))$

3 $P(x) \vee P(f(x, a))$

4 $\neg R(c_0, y) \vee \neg P(y)$

5 (1,2): $R(x, f(g_1(x), z))$

6 (1,4): $\neg P(g_1(c_0))$

Applying Resolution

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8 : $\neg R(c_0, f(g_1(c_0), a))$

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9 : $\emptyset$

Applying Resolution

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2 $\neg R(x, y) \vee R(x, f(y, z))$

3 $P(x) \vee P(f(x, a))$

4 $\neg R(c_0, y) \vee \neg P(y)$

5 (1,2): $R(x, f(g_1(x), z))$

6 (1,4): $\neg P(g_1(c_0))$

7 (3,6): $P(f(g_1(c_0), a))$

8 : $\neg R(c_0, f(g_1(c_0), a))$

9 : $\emptyset$

Proof found!