Boolean Satisfiability and SAT Solvers

Philippe Suter

SAV, April 16th, 2013

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  - "As hard and any other problem in NP."
- S. Cook, *The complexity of theorem proving procedures*, STOC 1971.

SAT in Practice

- Ubiquitous in hardware/circuit design
  - E.g. equivalence checking.
- Search/AI problems
  - E.g. reduce Sudoku to SAT.
  - Dependency management in Eclipse.
- Software verification
  - By itself, and as part of the SMT stack.

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a	b	c	φ
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- Obviously not very efficient. SAT solving is all about making this enumeration “smart”.

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Note that a truth table is a kind of *disjunctive* normal form (DNF).

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- Instead, we use an encoding based on *introducing new variables*.

Tseitin's Encoding

- Idea: rewrite φ into ψ that is *equisatisfiable*.

$\varphi \equiv (a \wedge (\neg b \vee (c \wedge d))$

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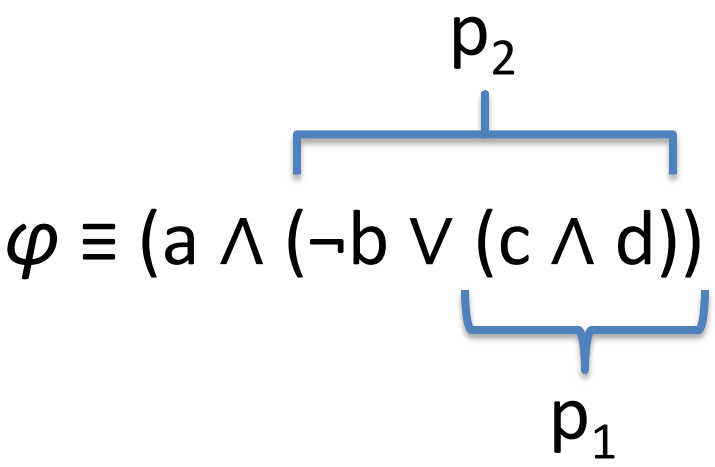

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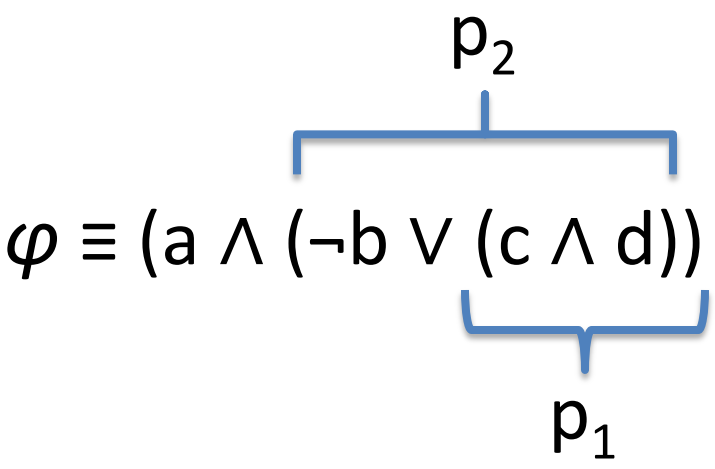

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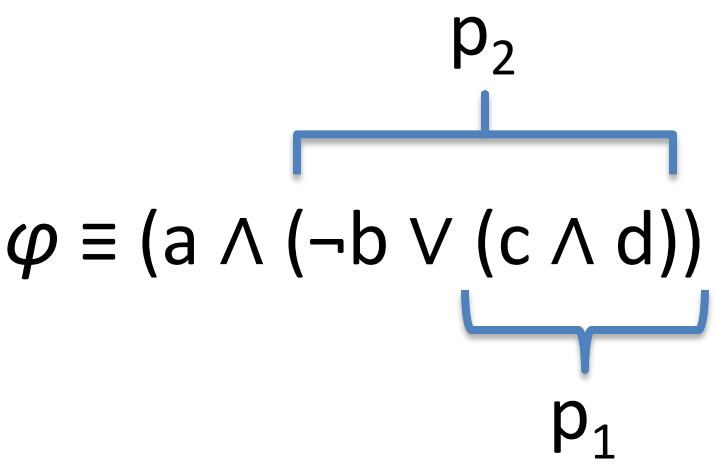

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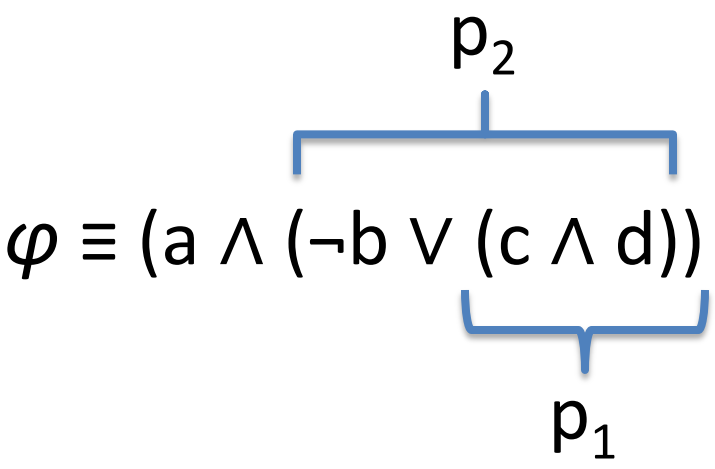

$p_1 \Leftrightarrow c \wedge d$

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$p_2 \Leftrightarrow \neg b \vee p_1$

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$b \vee p_2$

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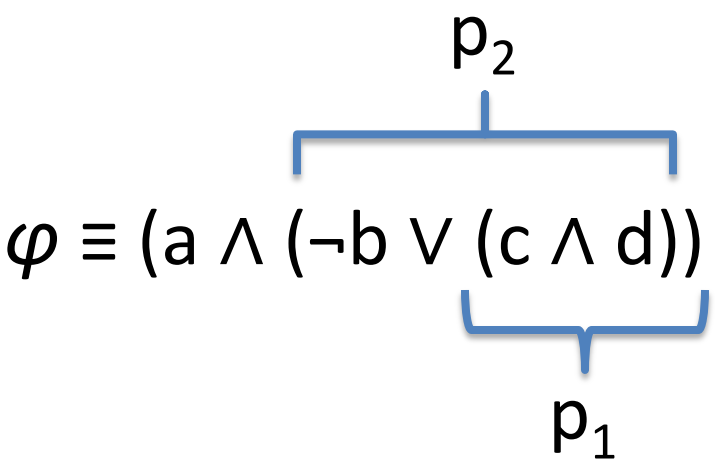

$\psi \equiv a \wedge p_2$

$p_1 \Leftrightarrow c \wedge d$

$\neg p_1 \vee c$

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$\neg c \vee \neg d \vee p_1$

$p_2 \Leftrightarrow \neg b \vee p_1$

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Solving clauses: 2-SAT

- We can assume w.l.o.g. that each clause has at least two literals.
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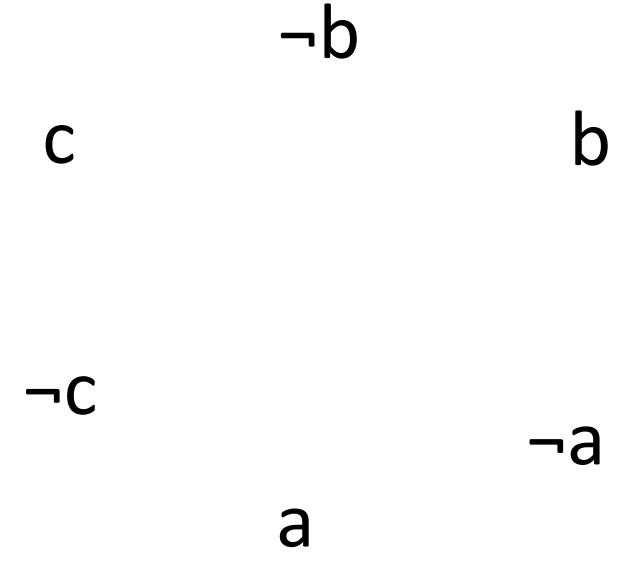

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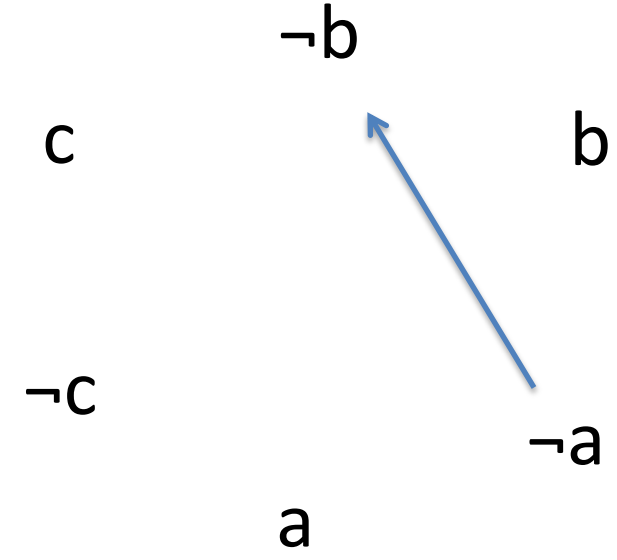

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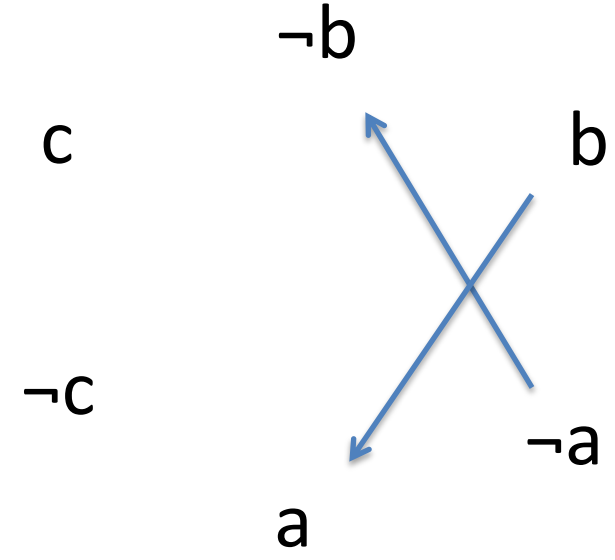

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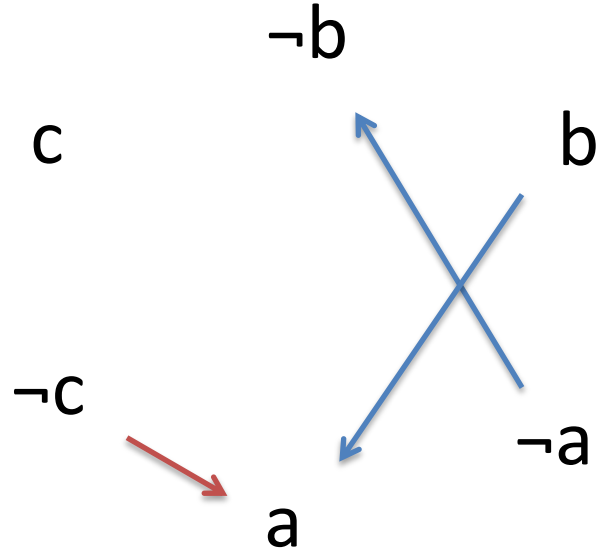

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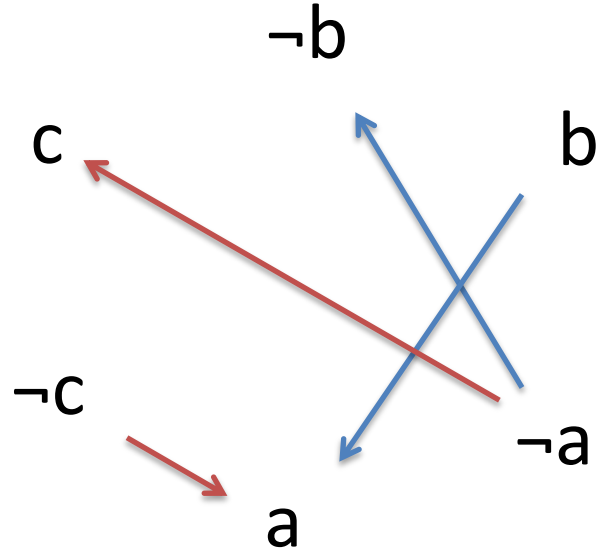

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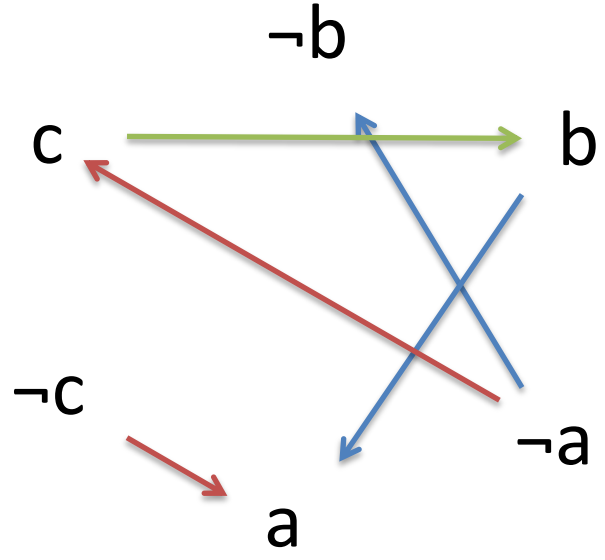

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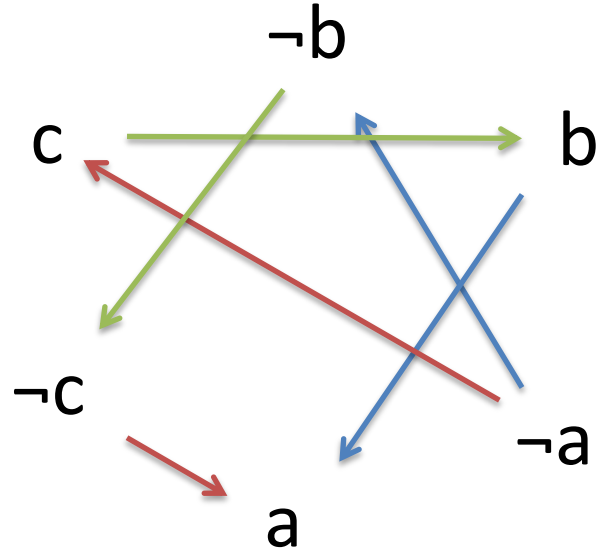

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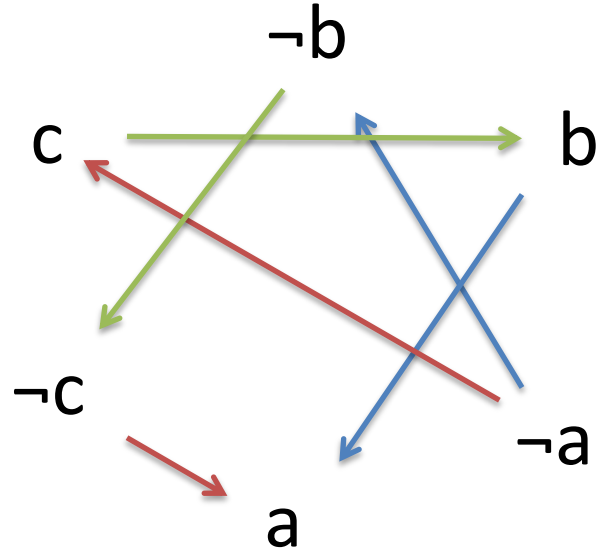

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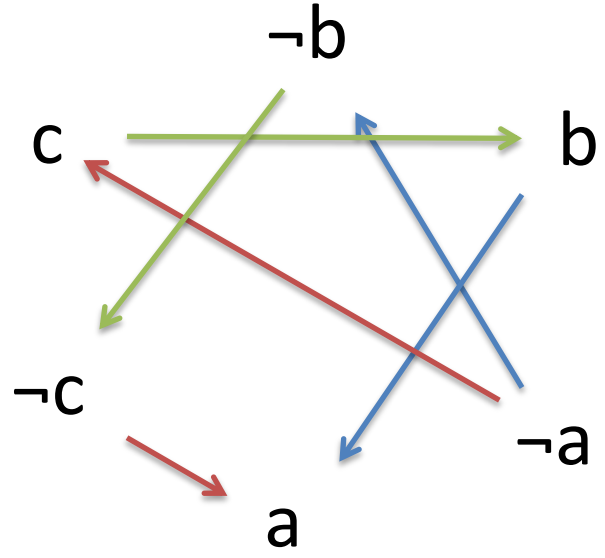

You can solve 2-SAT in polynomial time.
Some of the techniques for 2-SAT are used in general SAT solvers.

3-SAT

- NP-complete.
  - Reduction from SAT: split longer clauses using fresh variables.

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- (Not so relevant to SAT solving technology.)

First Approach: Resolution

$a \vee \neg b \vee f$

$\neg a \vee \neg c \vee d \vee \neg e$

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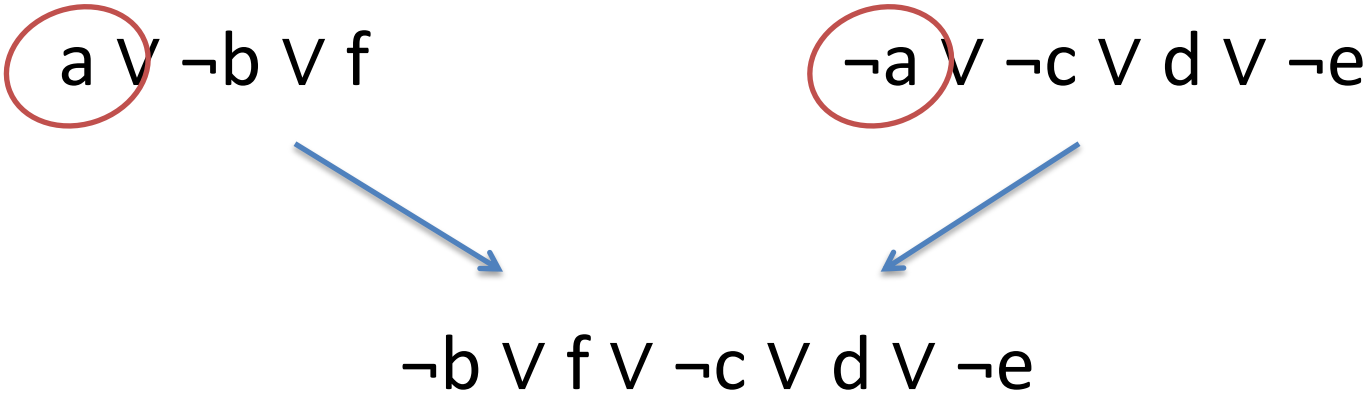
a ∨ ¬b ∨ f

¬a ∨ ¬c ∨ d ∨ ¬e

First Approach: Resolution



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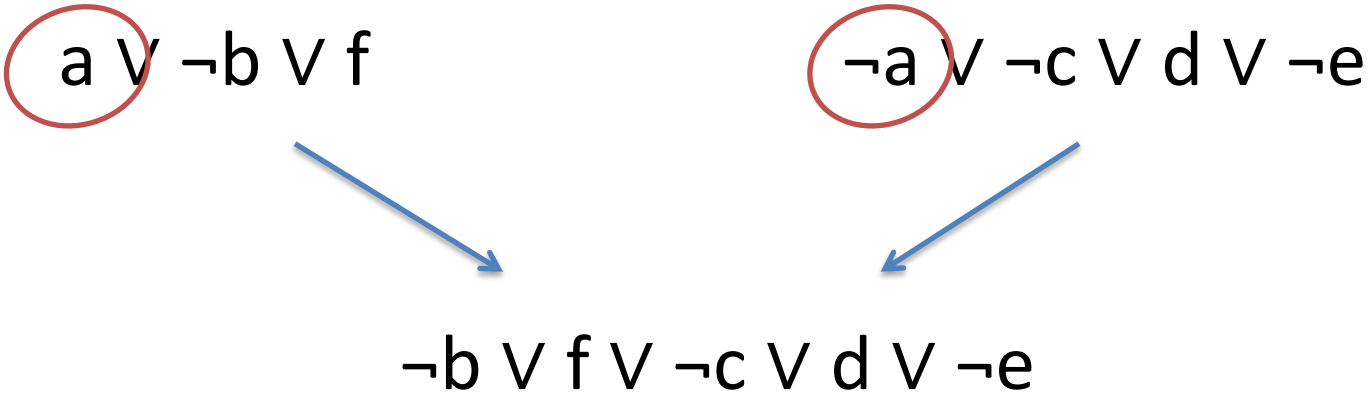

- Resolution eliminates one variable by producing a new clause (*resolvent*) from complementary ones.

Resolution

$(a \vee b) \wedge (a \vee \neg b) \wedge (\neg a \vee c) \wedge (\neg a \vee \neg c)$

Resolution

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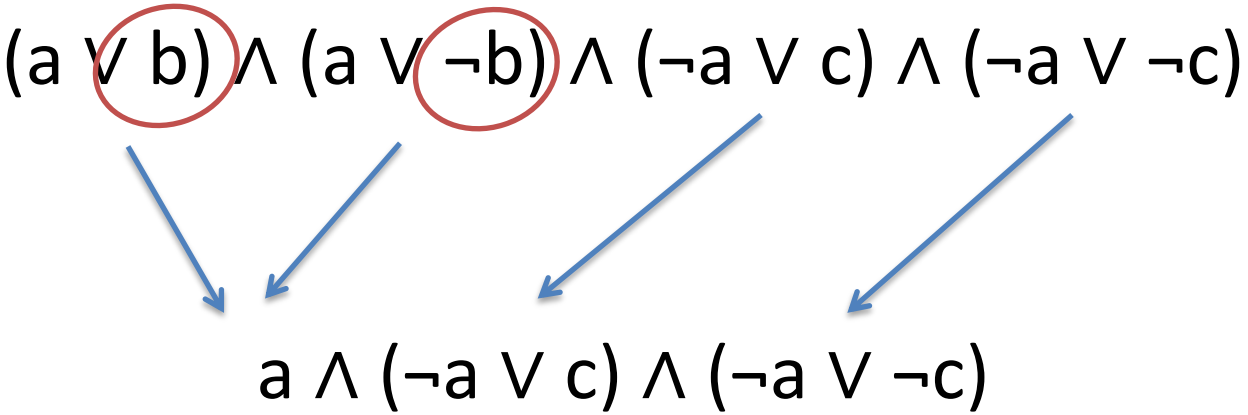

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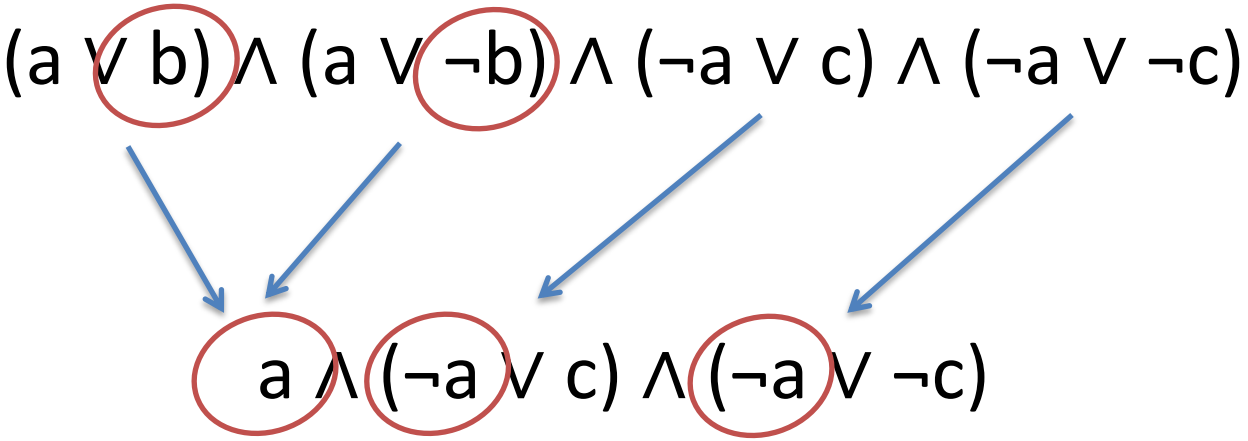

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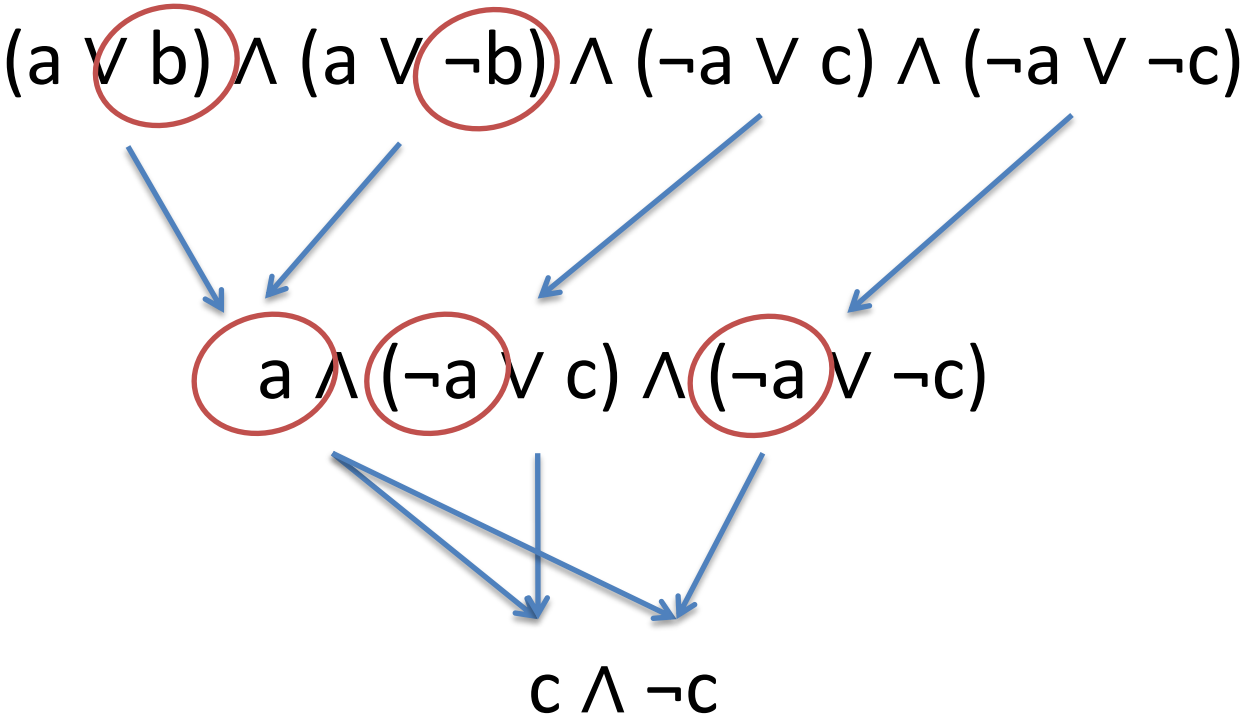

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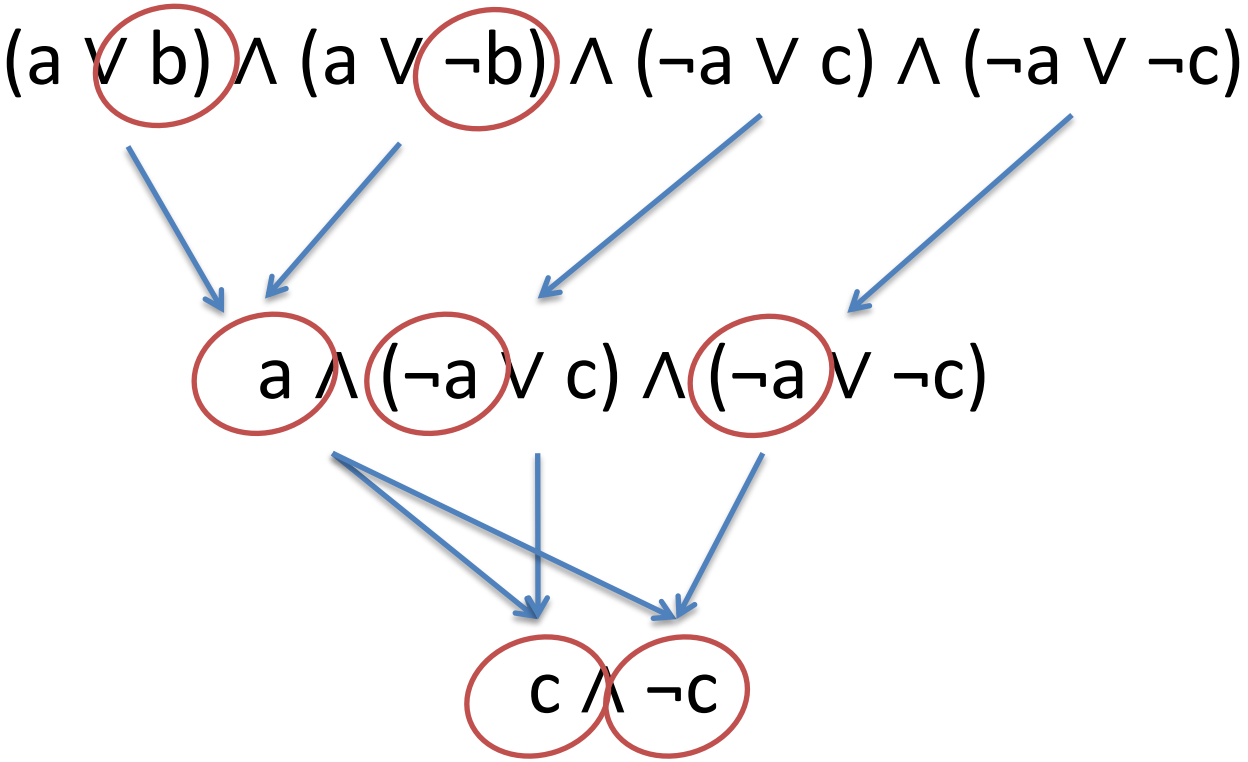

(Part of) Davis Putnam Algorithm

- (Also: when a variable appears in only one polarity, remove all clauses containing it.)
- M. Davis, H. Putnam, *A computing procedure for quantification theory*, JACM, 1960.
- Problem: space explosion!
- DP is *proof-oriented*. Current algorithms are *model-oriented*.

Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
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\end{aligned}
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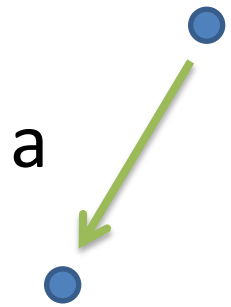

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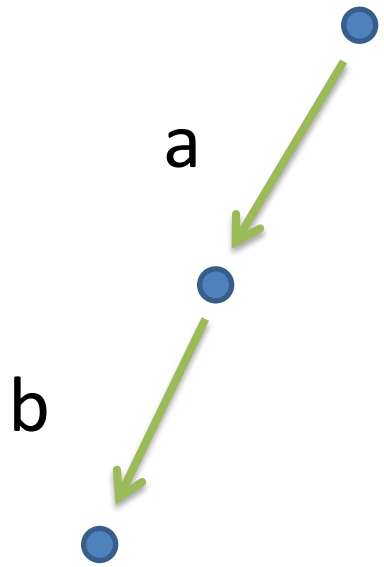

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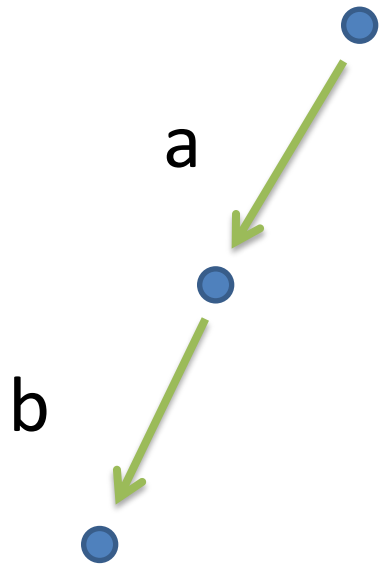

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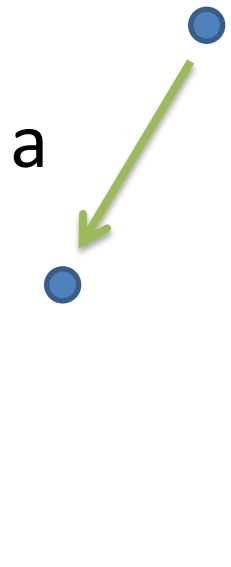

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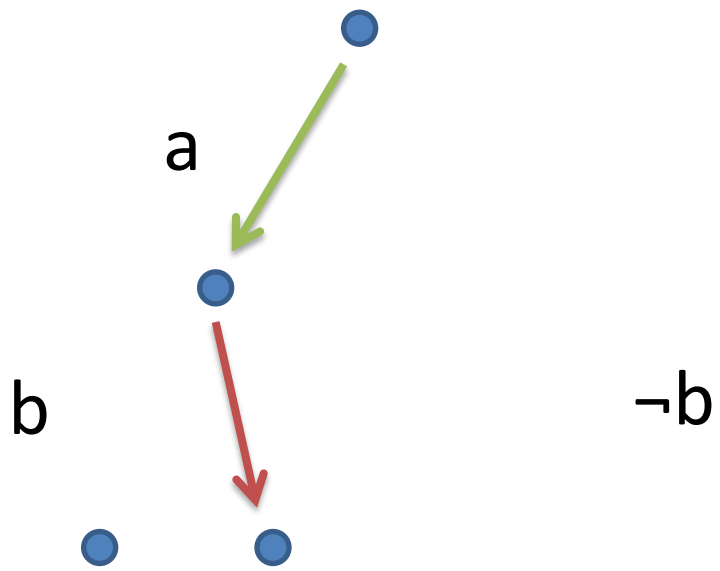

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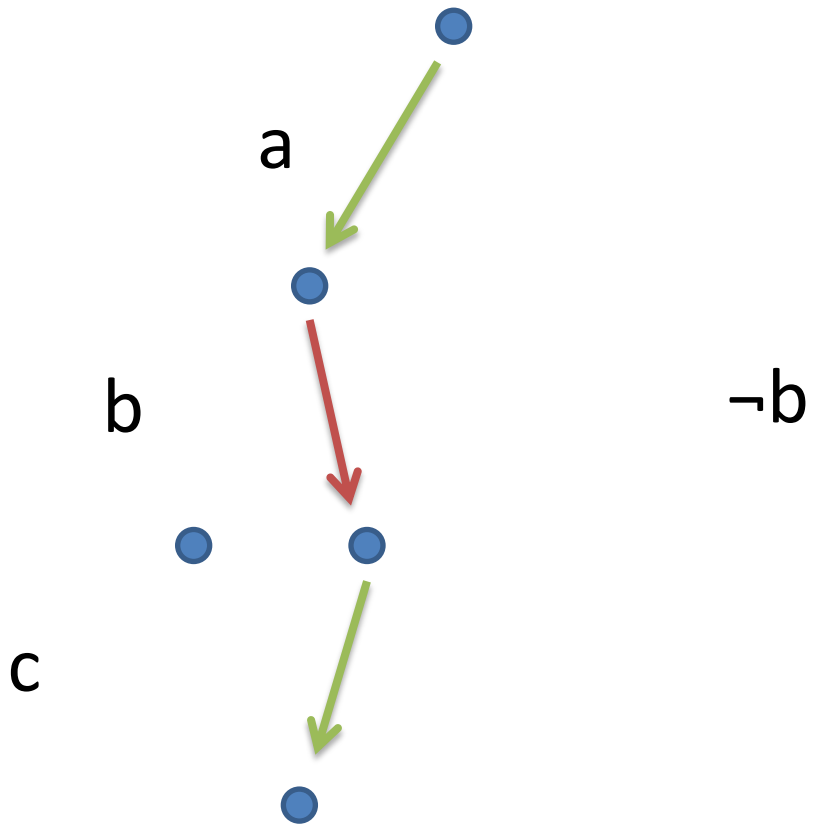
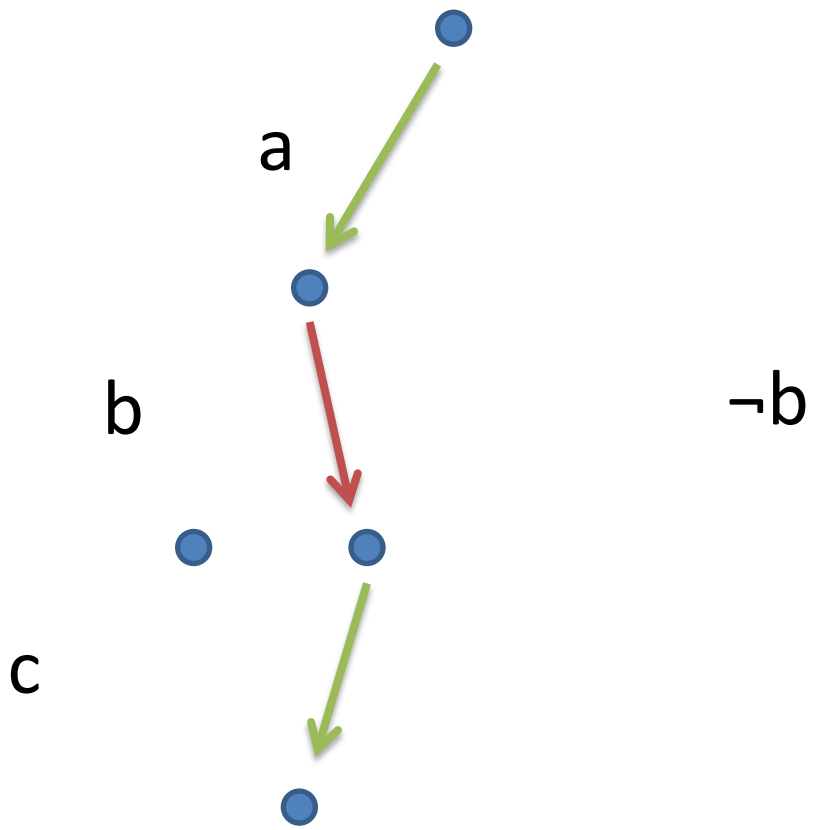


Backtracking Search

(b ∨ ¬c) !
∧ (¬a ∨ b ∨ c)
∧ (¬a ∨ ¬b)



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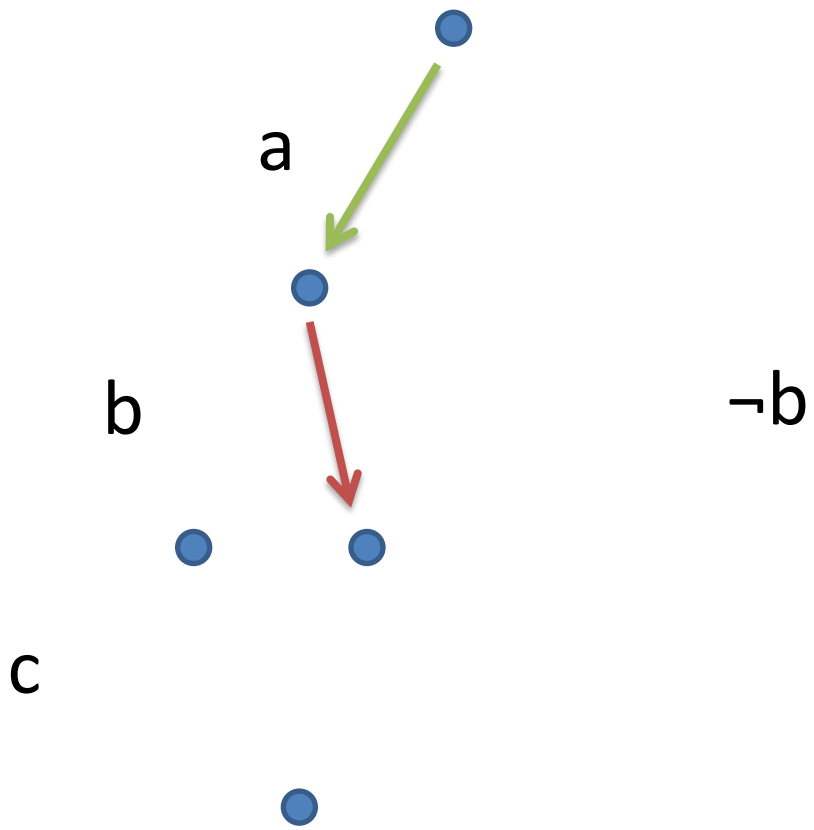

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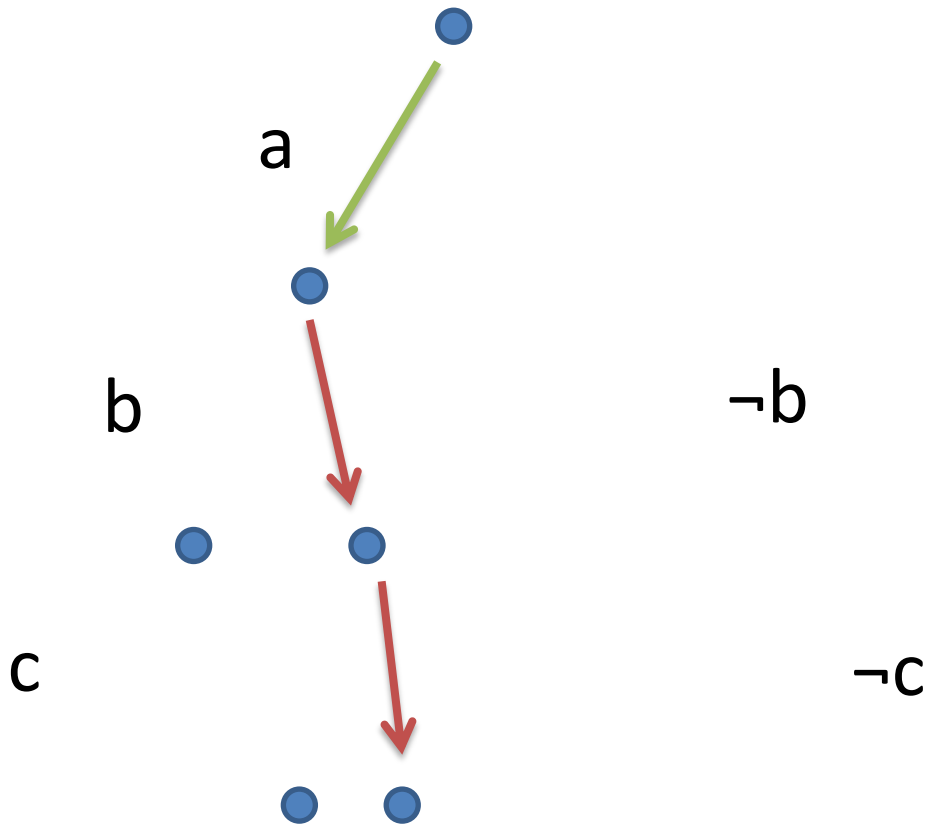

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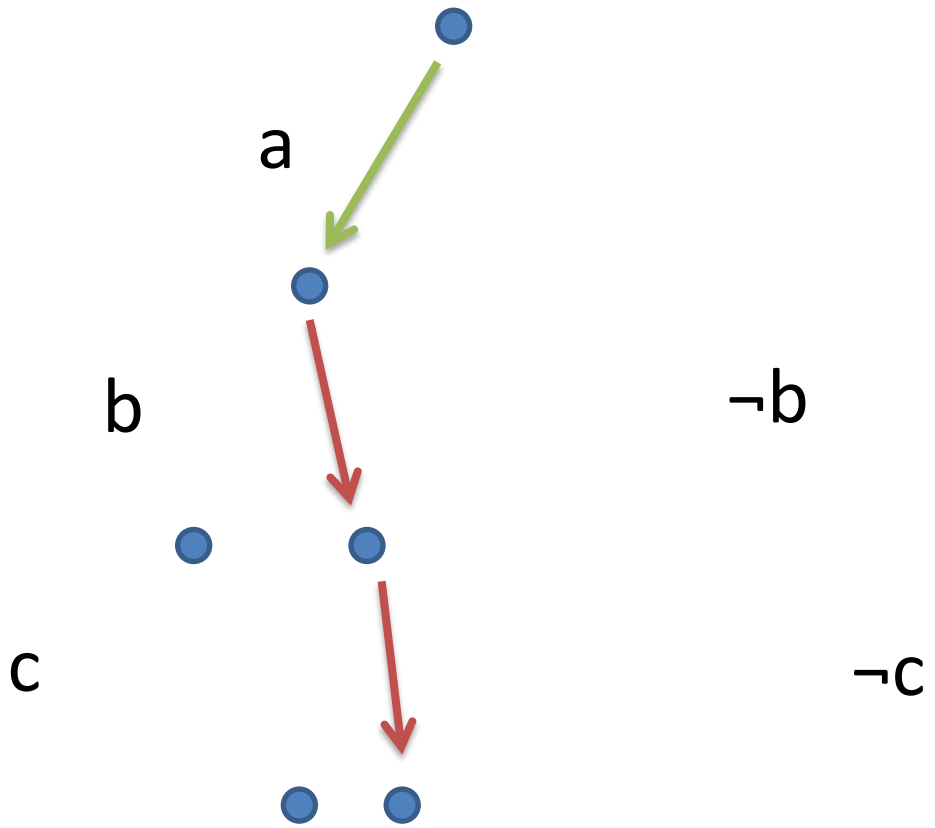
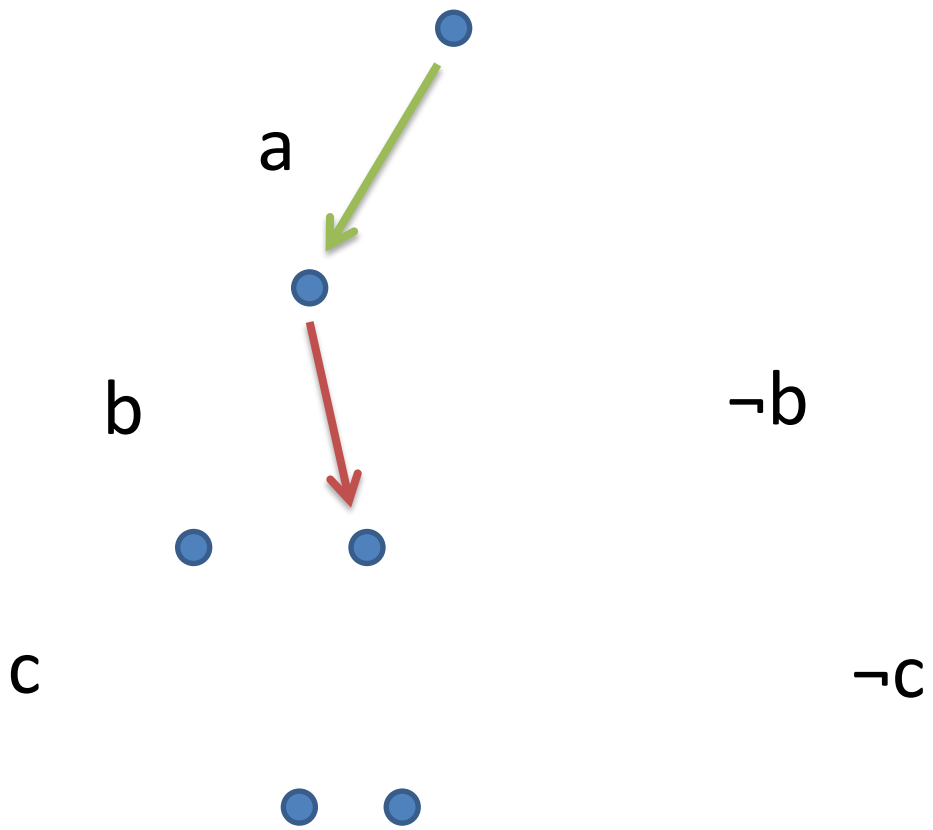
\wedge\ &(\neg a \vee \neg b)
\end{aligned}
$$



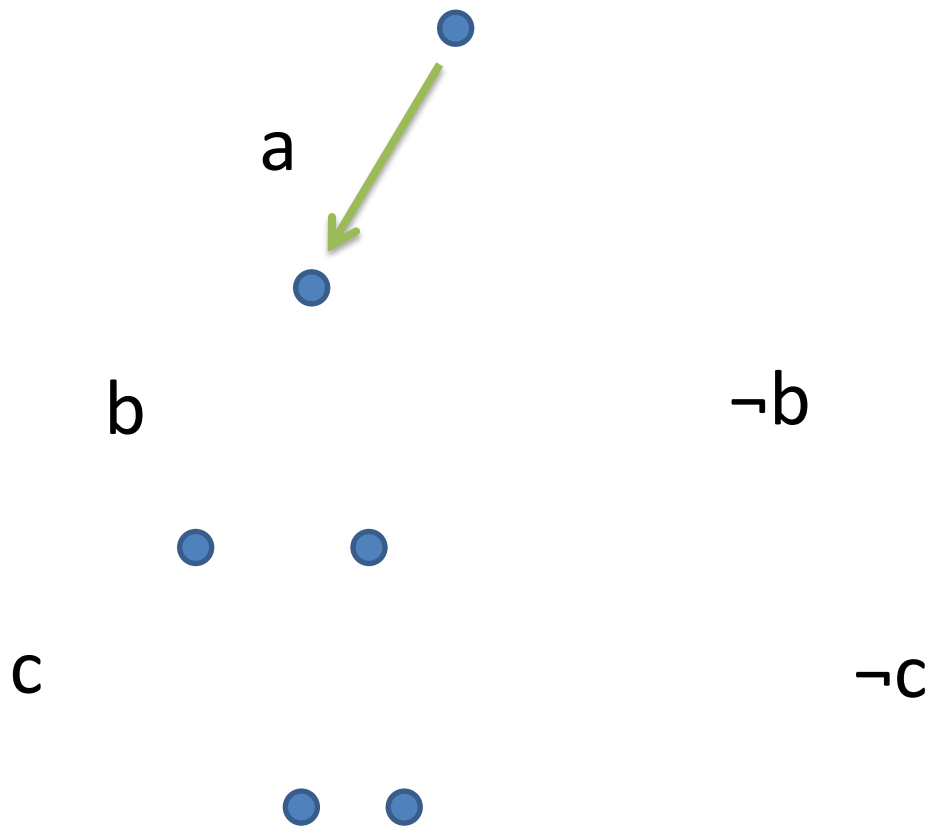
Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



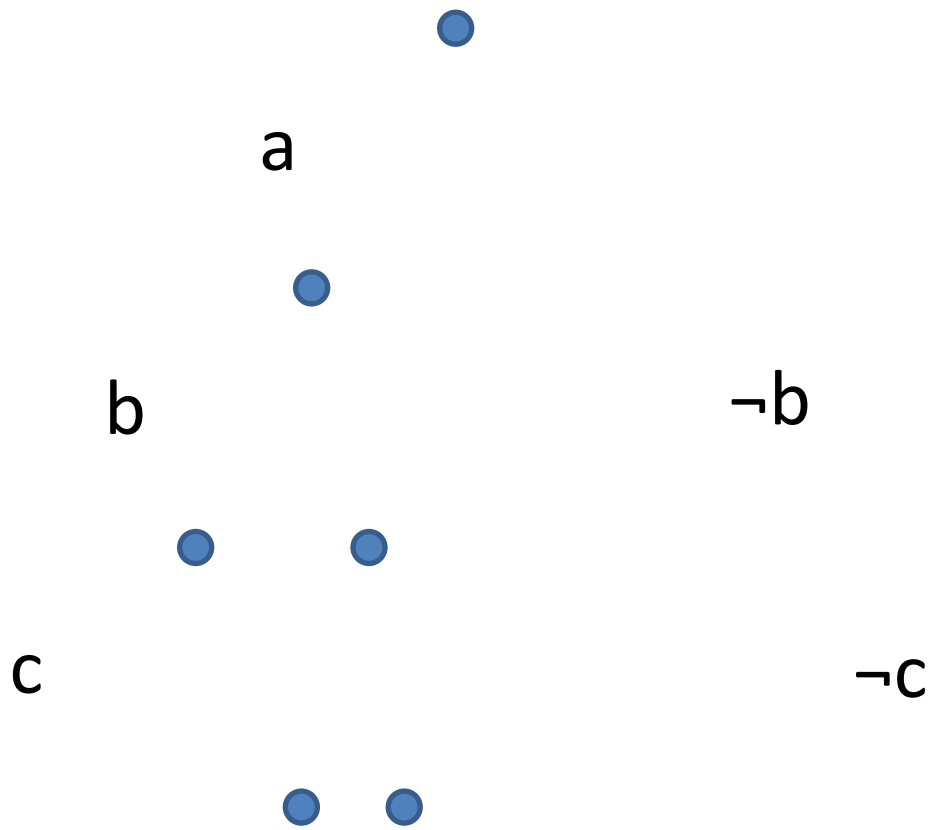
Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



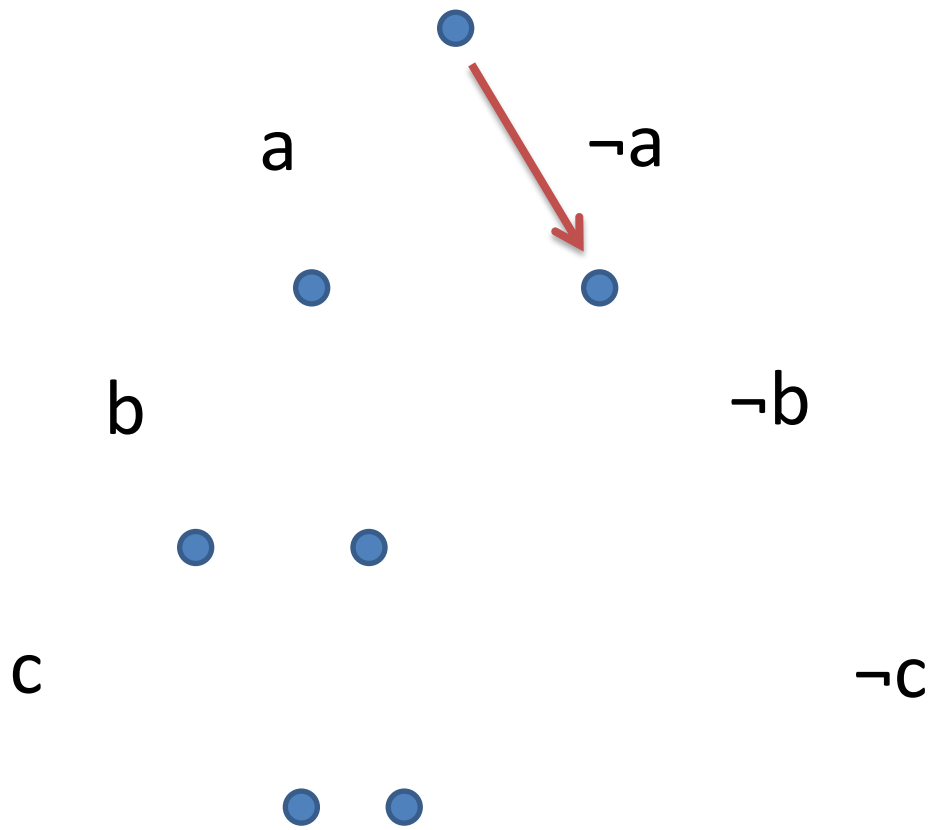
Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



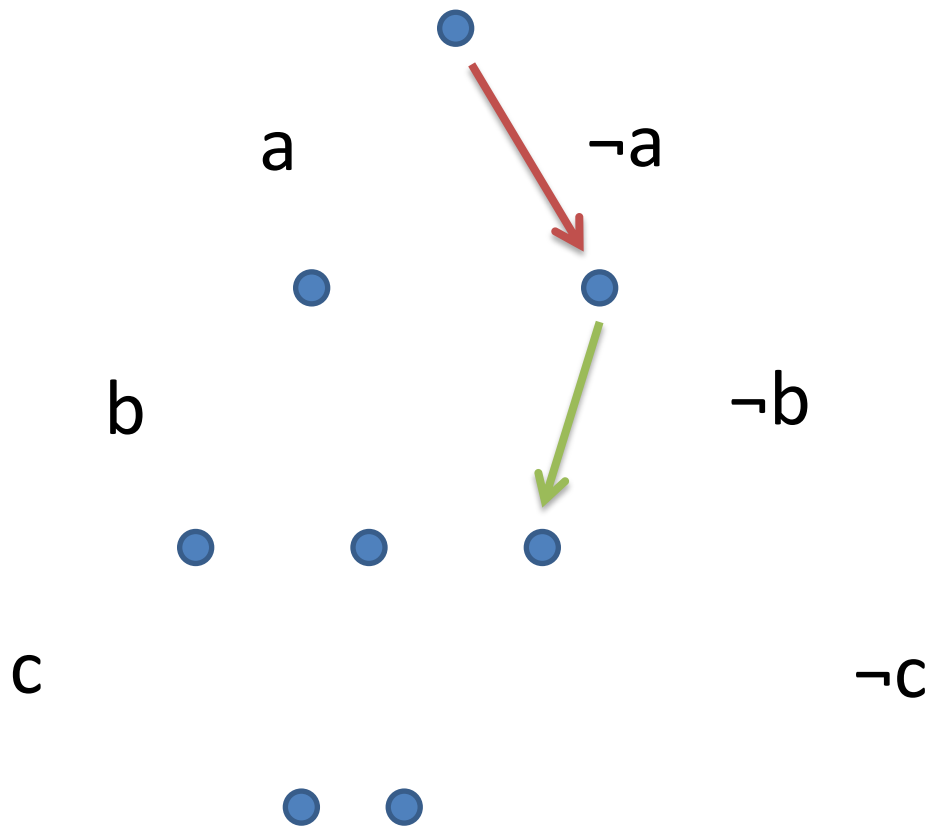
Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



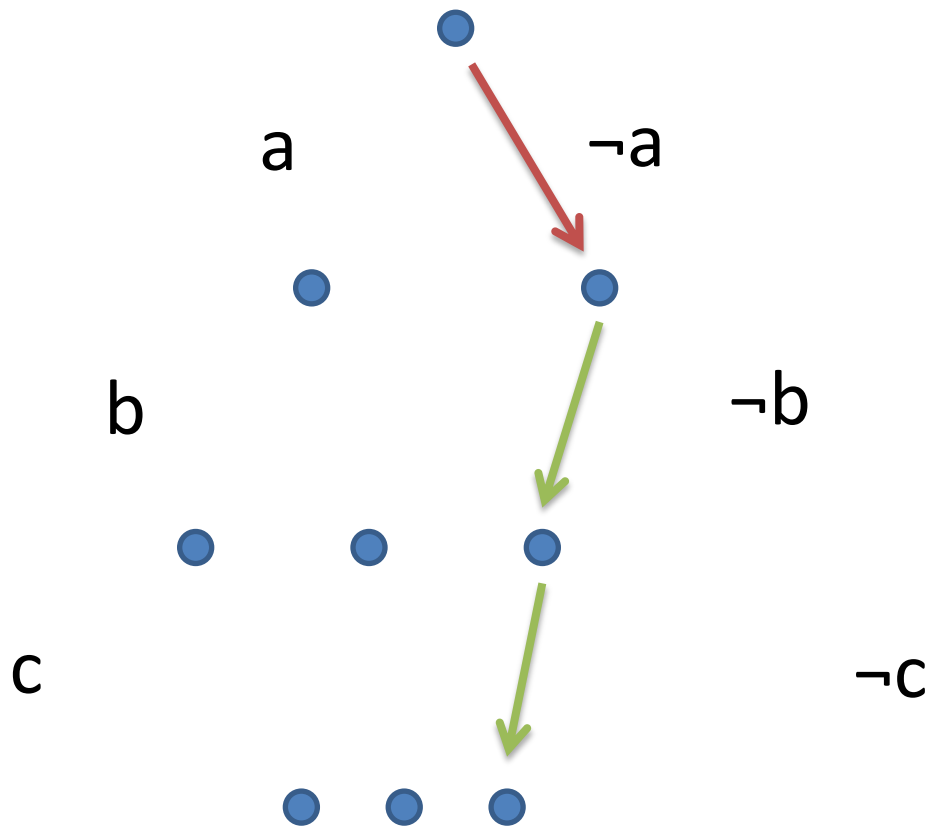
Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Backtracking Search

$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

$$
\begin{aligned}
&(\quad b \vee \neg c\ )\\
\wedge\ &(\neg a \vee b \vee c\ )\\
\wedge\ &(\neg a \vee \neg b\ )
\end{aligned}
$$

Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

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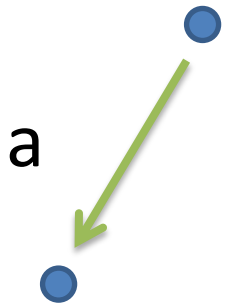
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

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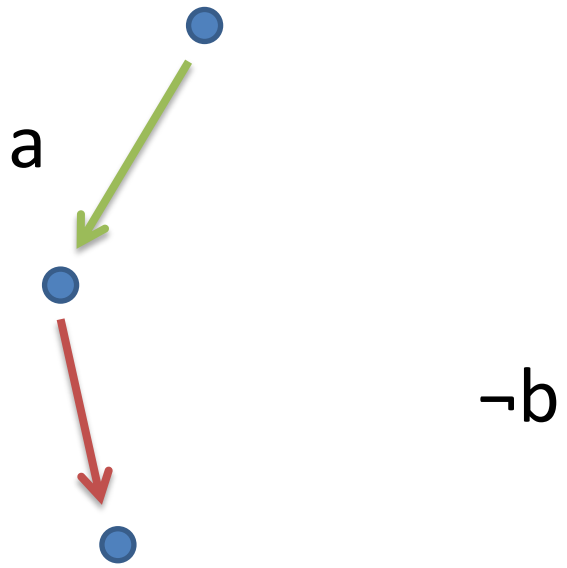
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

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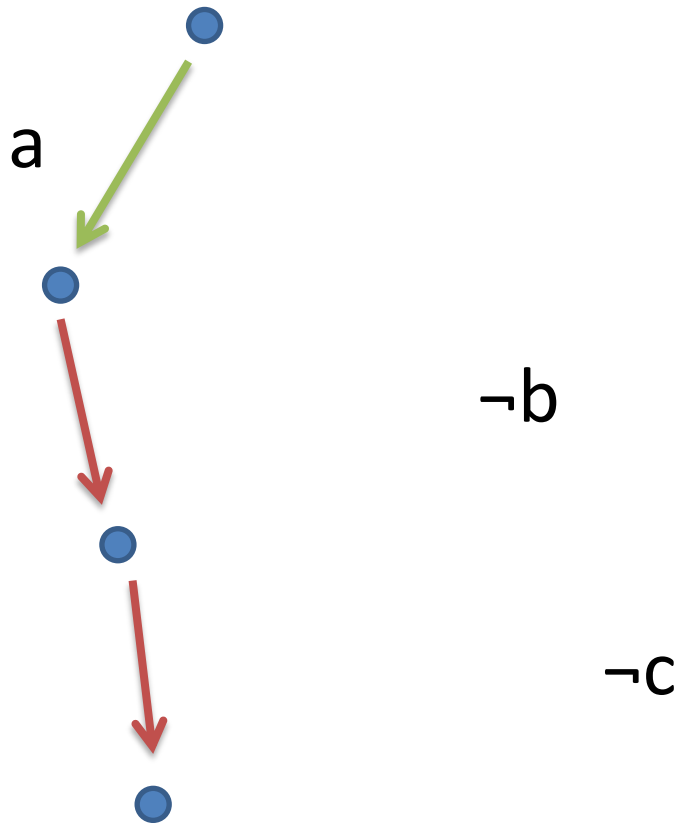
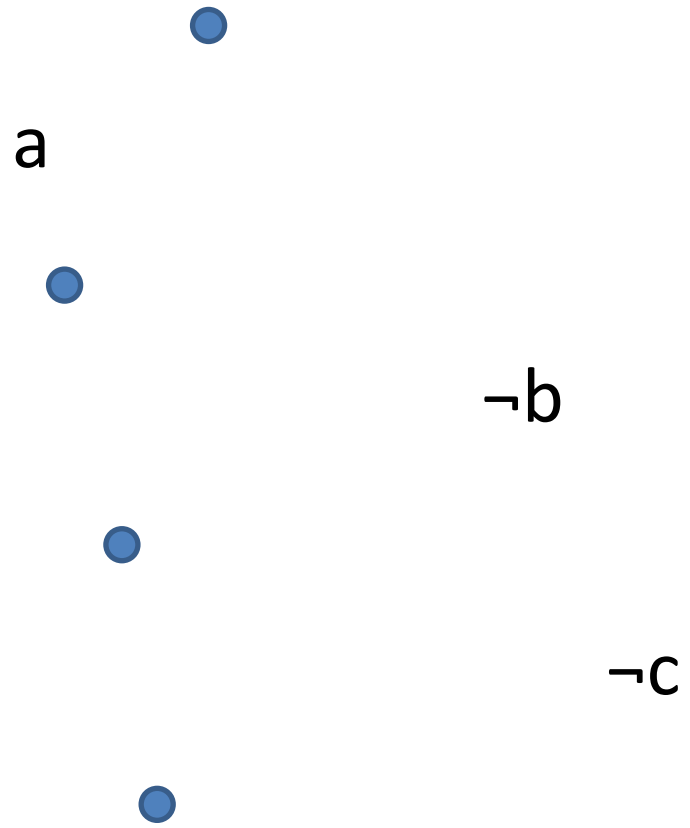
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

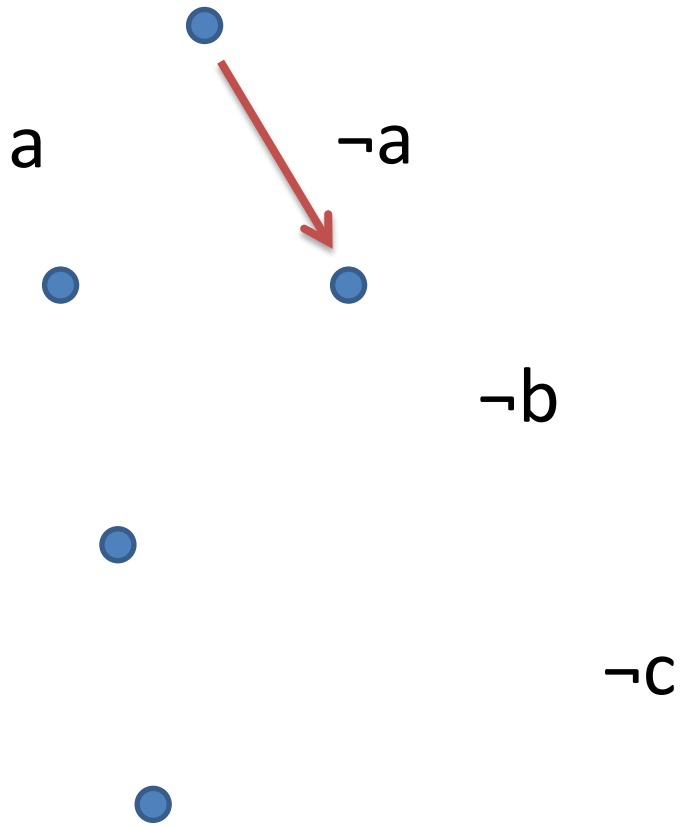
$$
\begin{aligned}
&(\quad\quad\; b \vee \neg c\,) \\
\wedge\,&(\,\neg a \vee b \vee c\,) \\
\wedge\,&(\,\neg a \vee \neg b\quad\;)
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

$$
\begin{aligned}
&(\quad\quad\quad b \vee \neg c\ )\\
\wedge\ &(\ \neg a \vee \quad b \vee \quad c\ )\\
\wedge\ &(\ \neg a \vee \neg b\quad\quad\ )
\end{aligned}
$$



Boolean Constraint Propagation

- *"When all but one literal are falsified, it becomes implied."*

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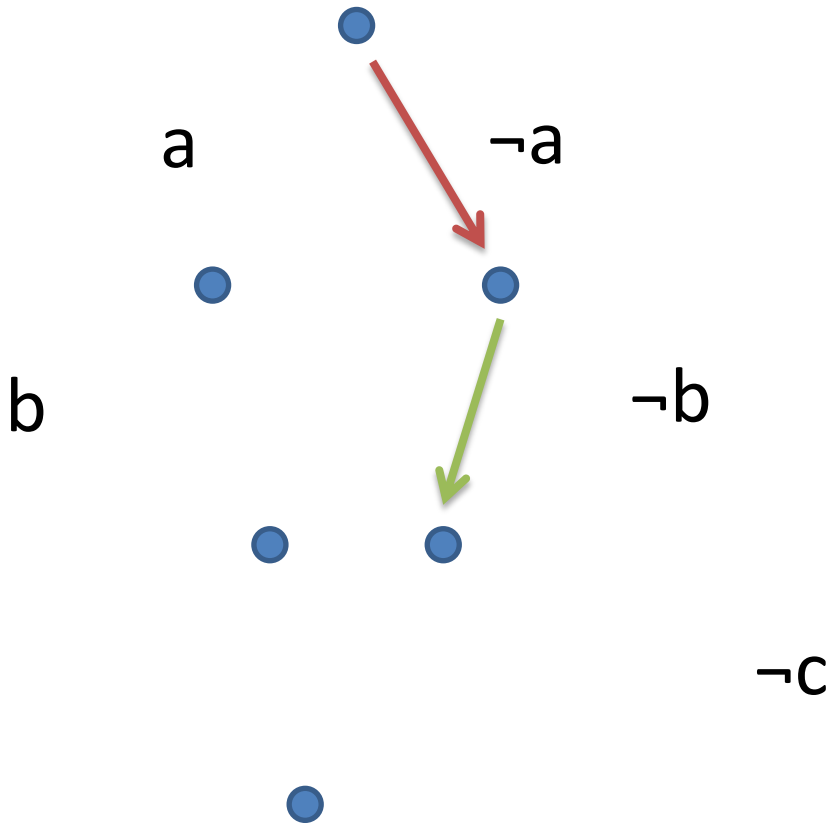
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Boolean Constraint Propagation

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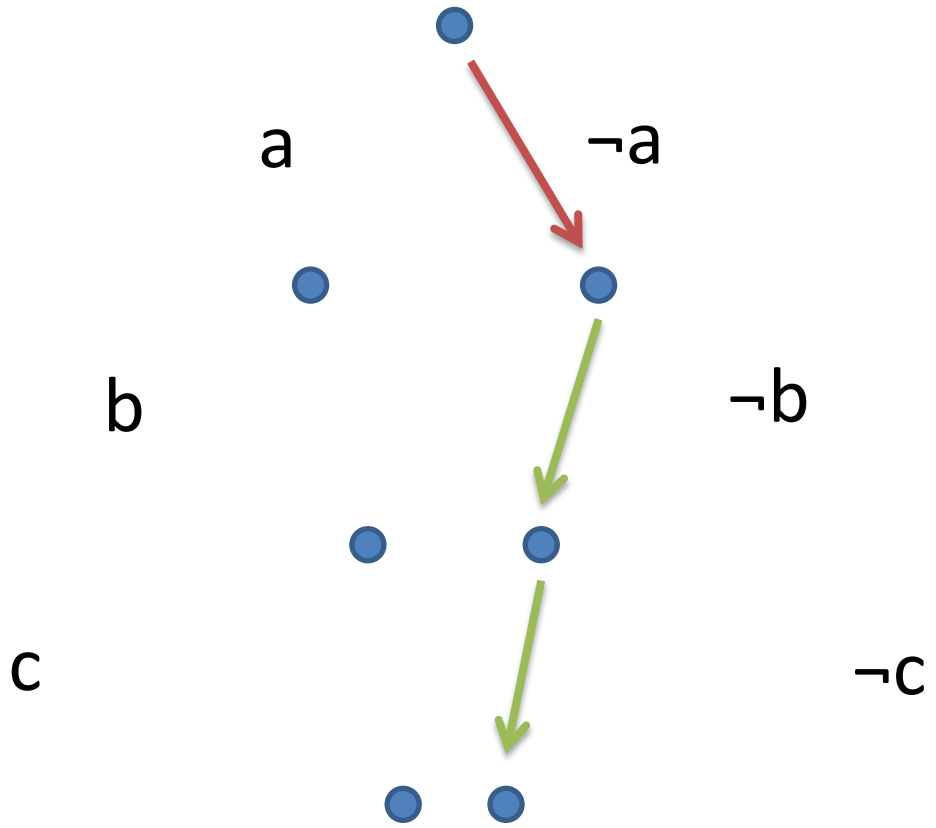
$$
\begin{aligned}
&(\quad\quad b \vee \neg c) \\
\wedge\ &(\neg a \vee b \vee c) \\
\wedge\ &(\neg a \vee \neg b\quad\quad)
\end{aligned}
$$



Two-watched-literal Scheme for BCP

- BCP can cut the search tree dramatically…
- …but checking each clause for potential implications is expensive.

- Observation: as long as at least two literals in a clause are “not false”, that clause does not imply any new literal.
- Idea: for each clause, try to maintain that invariant.

Cutting Deeper: Learning

- Idea: compute new clauses that are logically implied, and that may trigger more BCP.

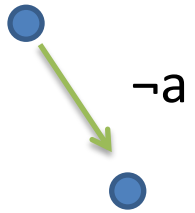
- Use an *implication graph*. When a conflict is derived, look for a *small explanation*.

Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

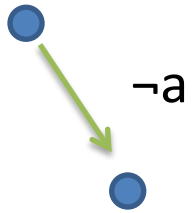
Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)



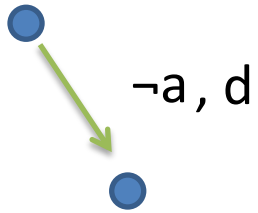
Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)



Learning

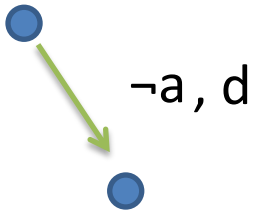
(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

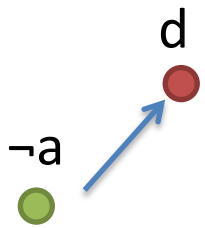


¬a

Learning

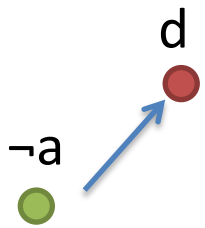
(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

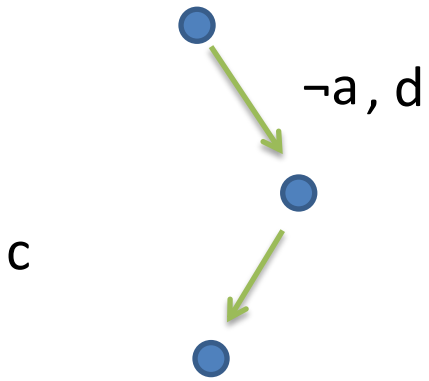




Learning

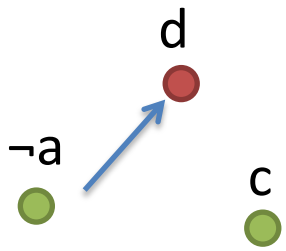
(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

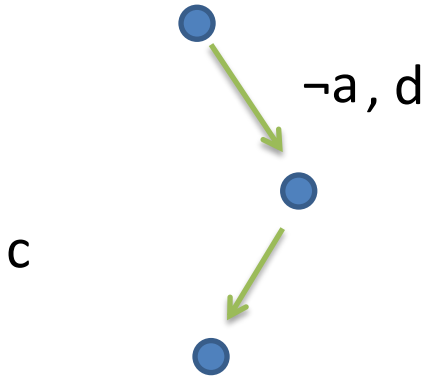




Learning

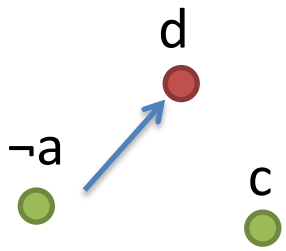
(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

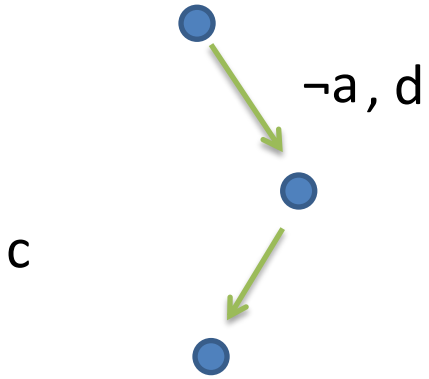




Learning

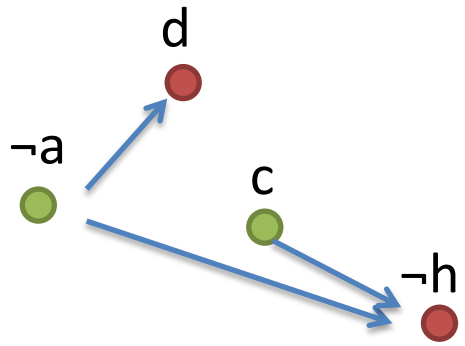
(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

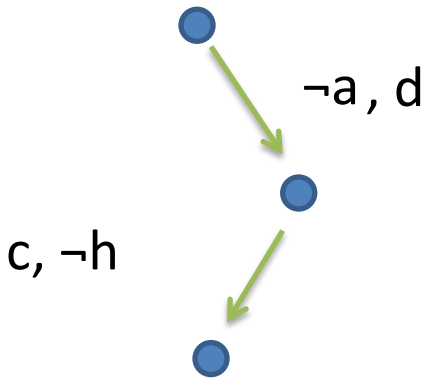




Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)





Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
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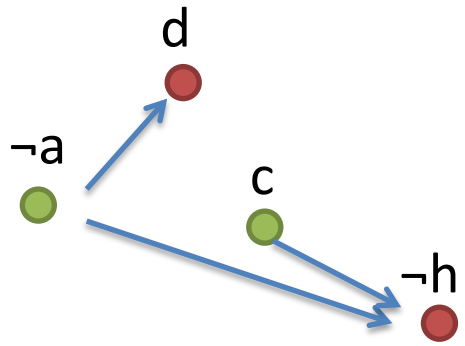

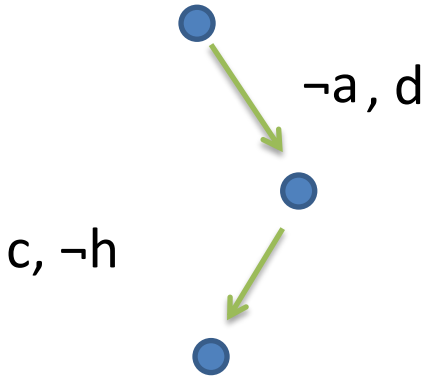

Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
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∧ (¬g ∨ ¬c ∨ i)
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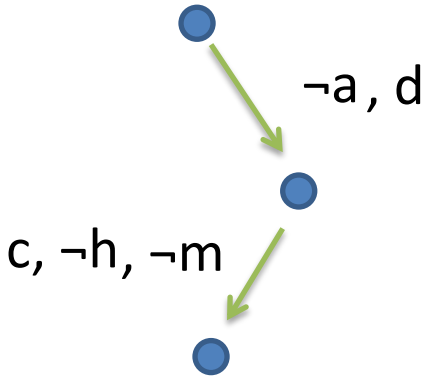
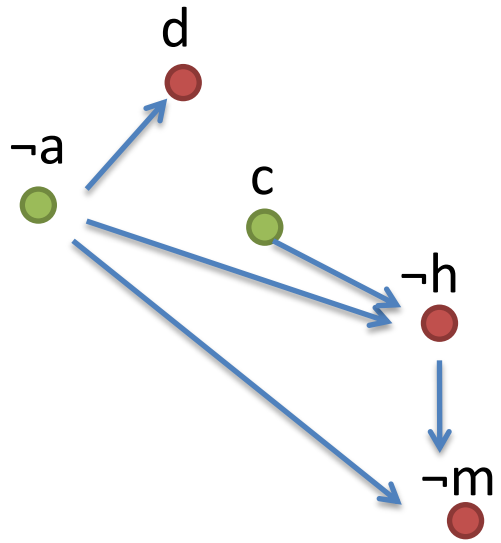
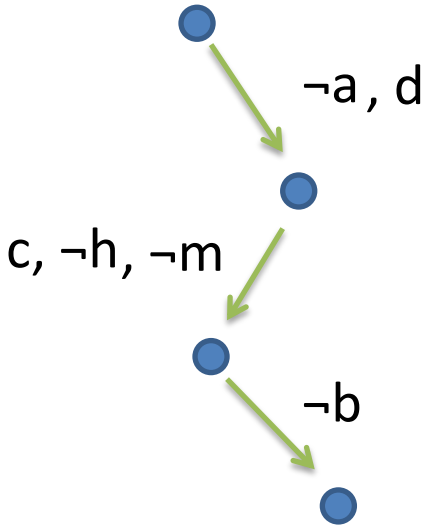
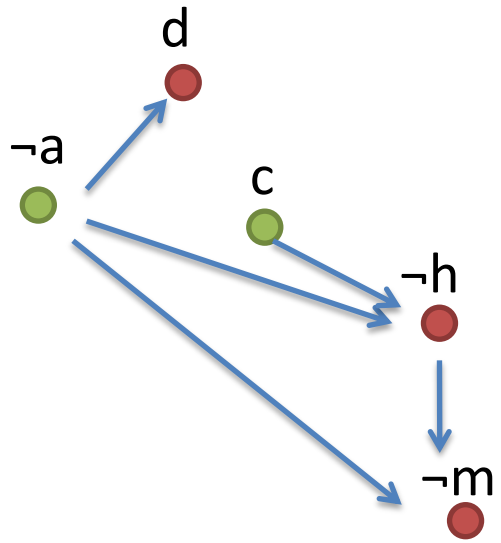




Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)





Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
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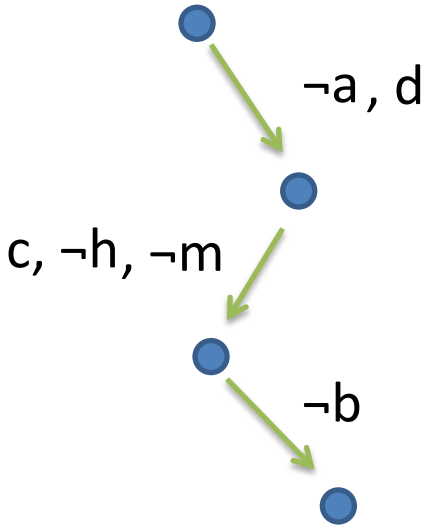
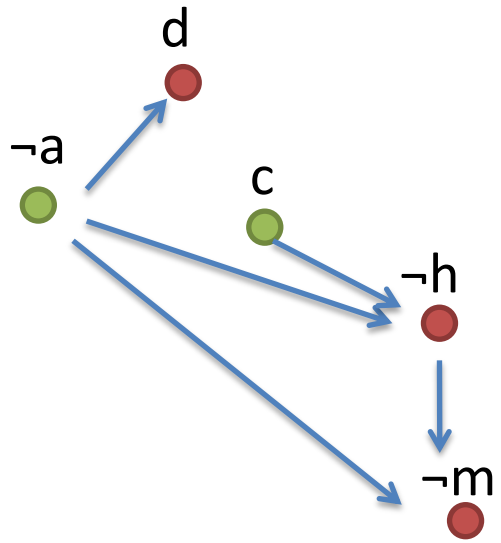
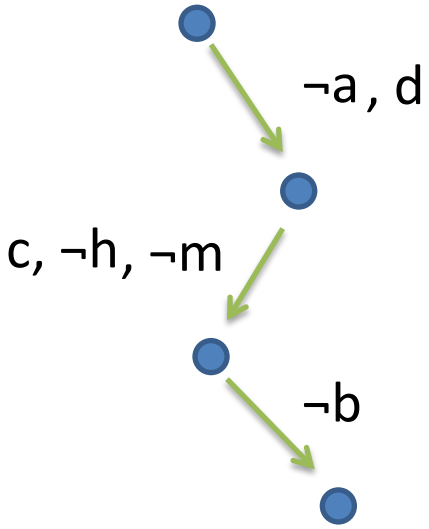


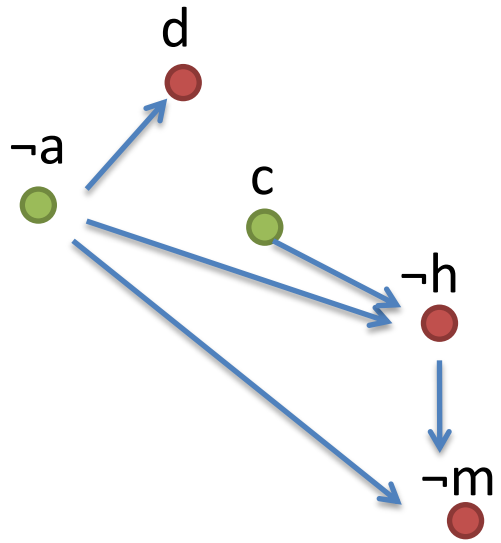


¬b

Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)

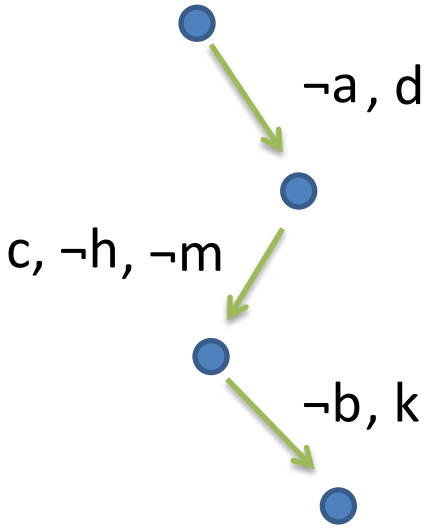


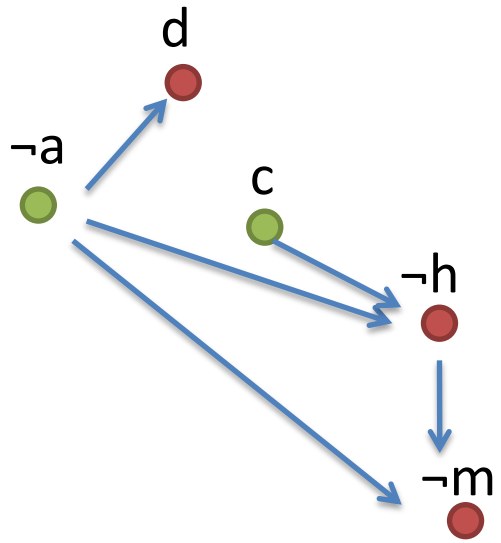


¬b

Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
∧ (¬g ∨ ¬c ∨ i)
∧ (¬g ∨ h ∨ ¬i)
∧ (g ∨ h ∨ ¬j)
∧ (g ∨ j ∨ ¬m)





Learning

(a ∨ d)
∧ (a ∨ ¬c ∨ ¬h)
∧ (a ∨ h ∨ ¬m)
∧ (b ∨ k)
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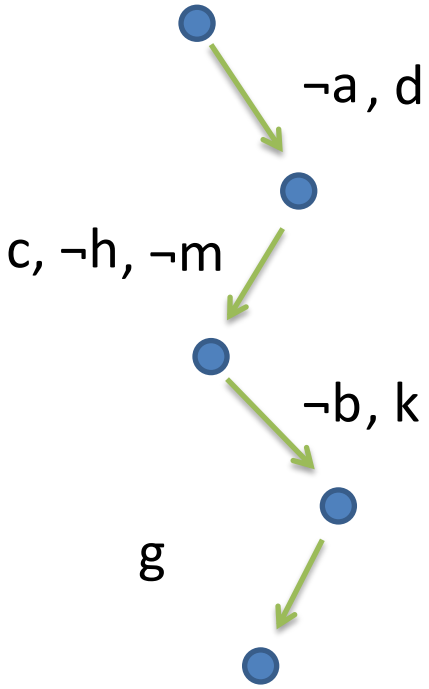

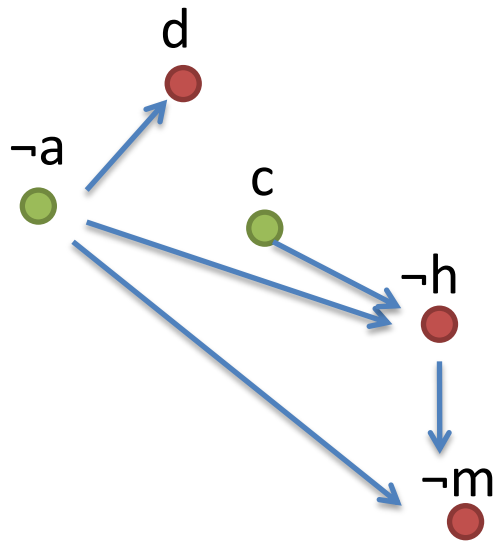

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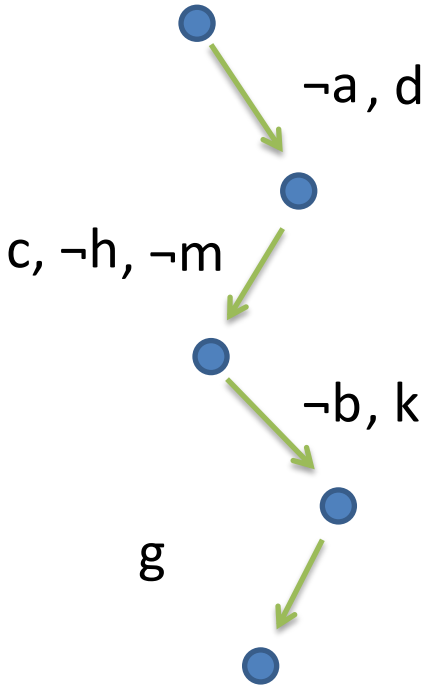

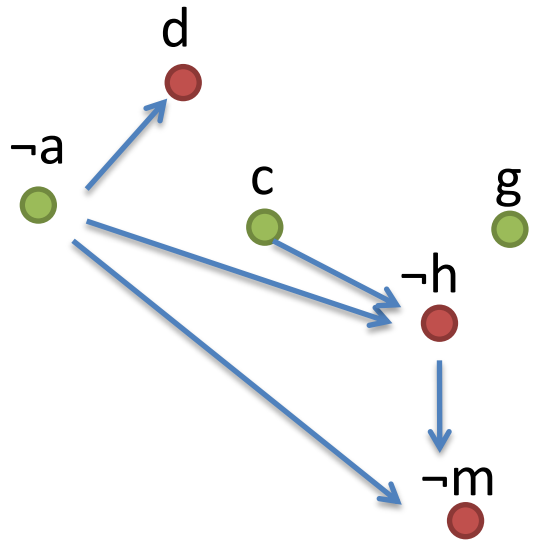

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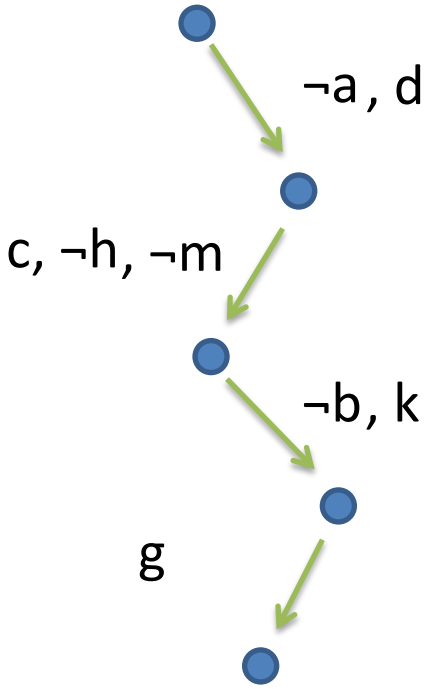

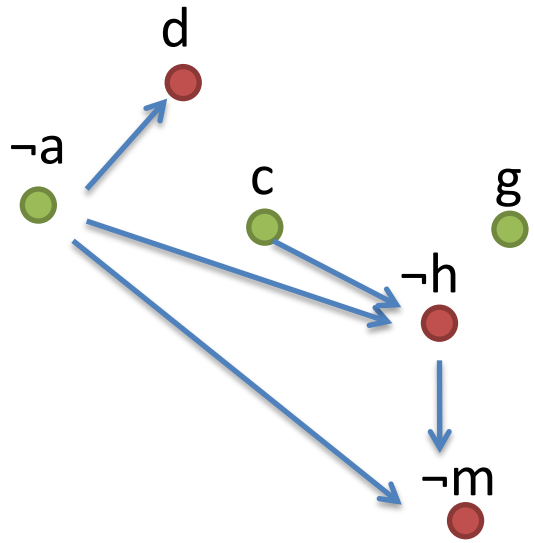

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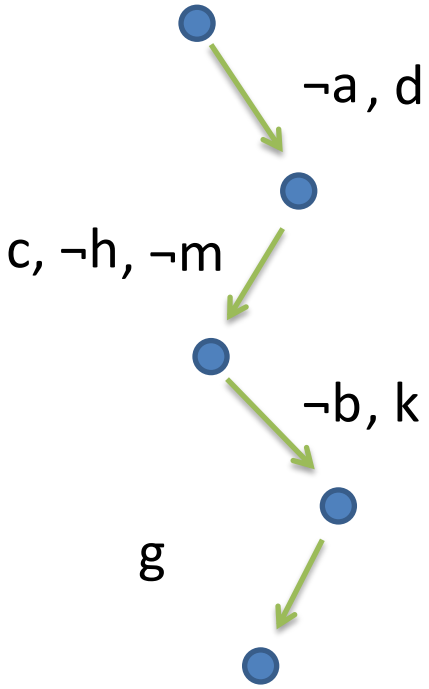

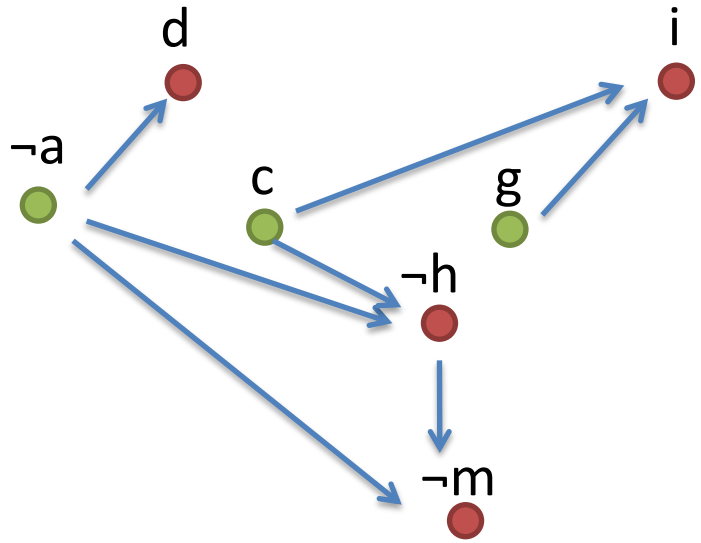

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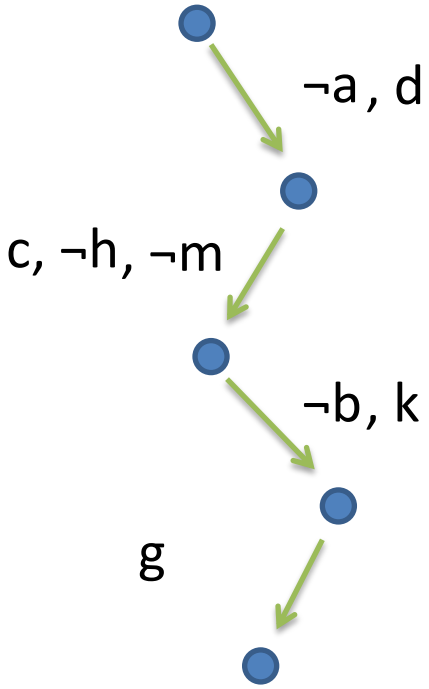

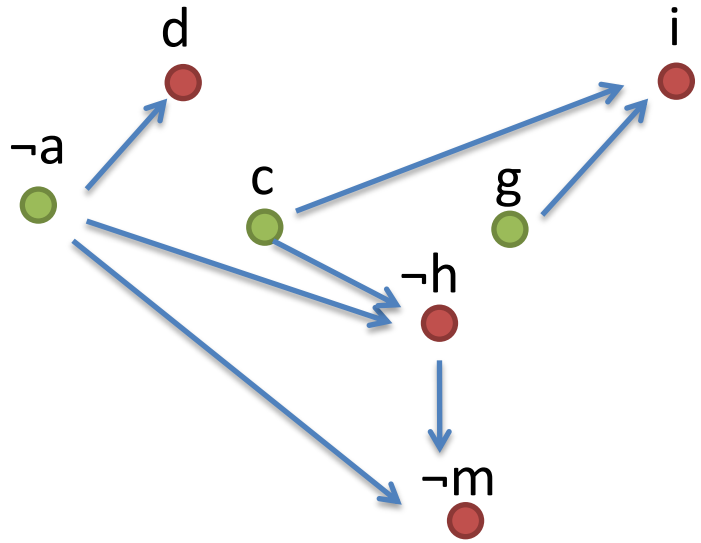

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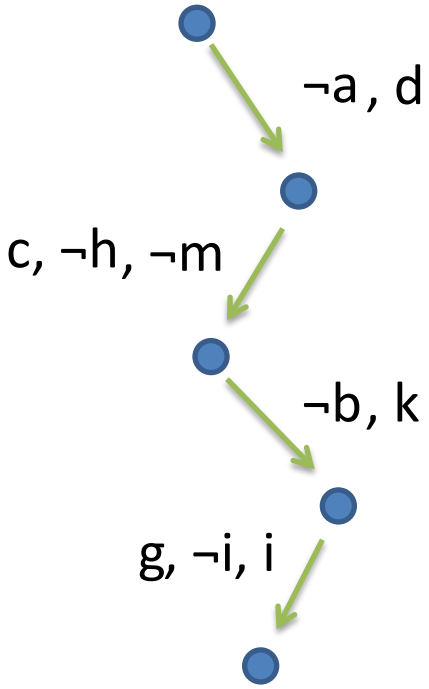

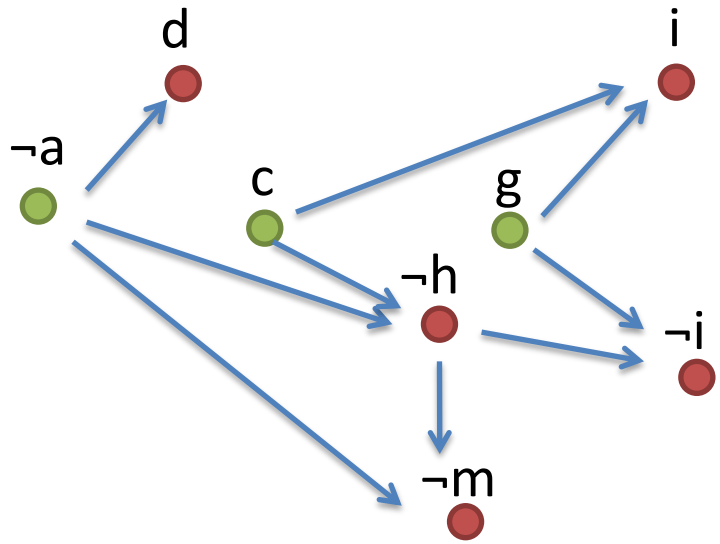

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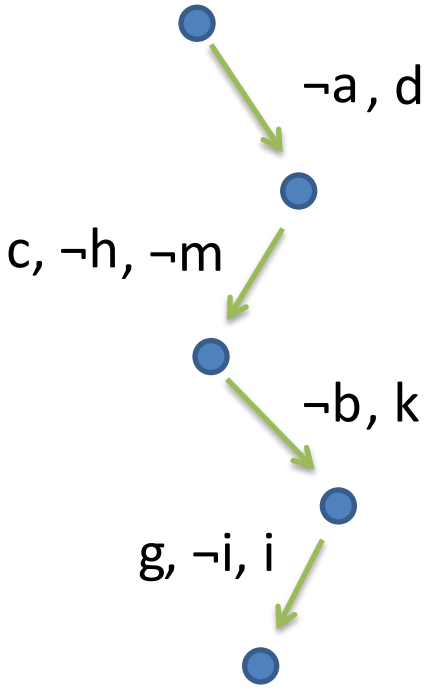

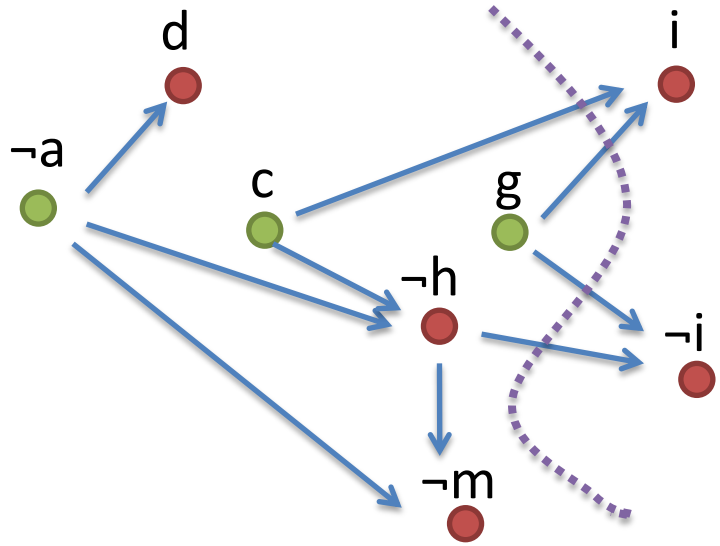

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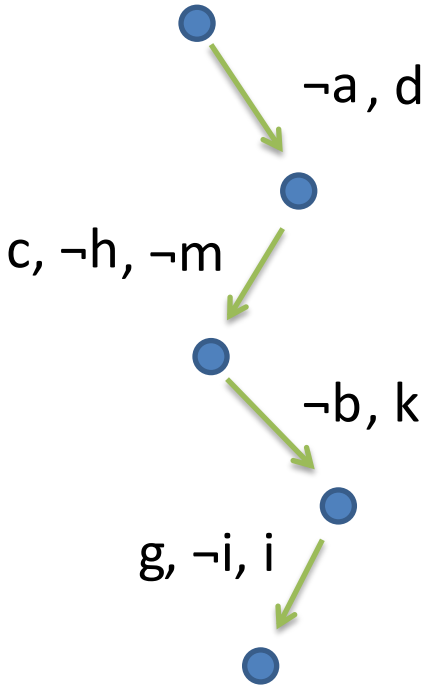

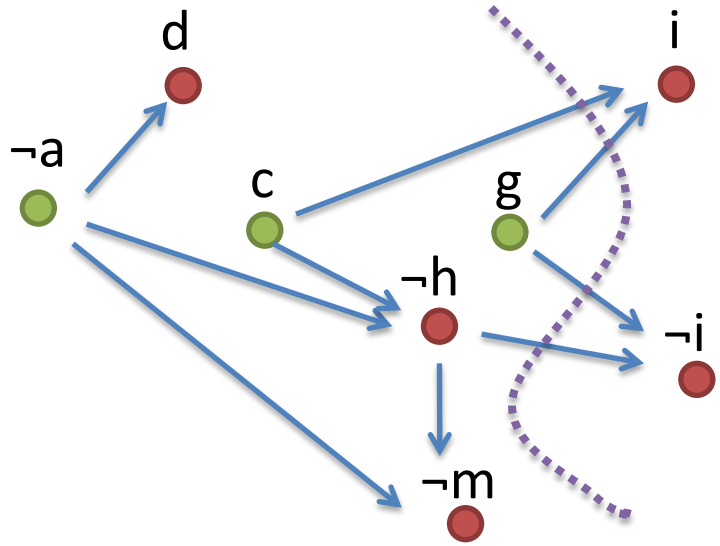

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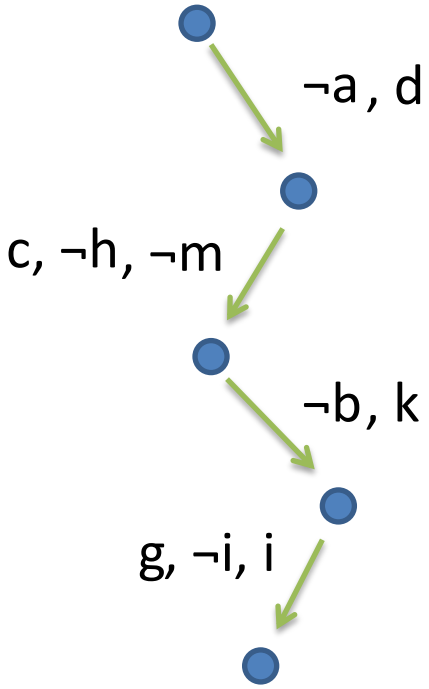

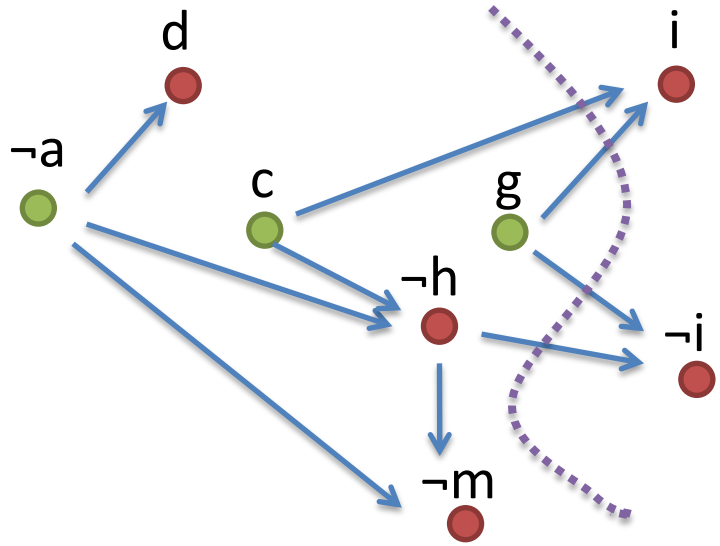

¬(c ∧ g ∧ ¬h)

…and backtrack to c, then assert ¬g !

Learning

- Learning has a dramatically positive impact.

- Learning also makes *restarts* possible:
  - Idea: after some number of literal assignments, drop the assignment stack and restart from zero.
  - Goal: avoid locally difficult subtrees.
  - Clauses encode previous knowledge and make new search faster.

Picking Variable Assignments

- Potential strategies:
  - Fixed ordering,
  - Frequency based,
  - “Maximal impact”.

Picking Variable Assignments

- Potential strategies:
  - Fixed ordering,
  - Frequency based,
  - “Maximal impact”.

- Overall favorite are activity-based heuristics:
  - Pick variables that you have seen a lot in conflicts.
  - Decay weights to favor recent conflicts.
  - Cheap to compute/update.

More Engineering…

- SAT dirty little secret: the enormous impact of preprocessing.
  - Problems are generated automatically ("compiled"); many redundancies, symmetry, etc.
  - Preprocessors look for subsumed clauses, equivalent clauses, etc.
  - Typically, run with timeout, then DPLL search.

More Engineering…

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- Parallel SAT
  - State-of-the-art is to run instances with different parameters in parallel.

Beyond SAT

- SMT solvers
  - Idea: use a SAT solver for the propositional structure, and theory solvers for conjunction of literals.
- QBF
  - SAT with quantifiers. PSPACE complete.