Problem 4. A grammar G is given in Chomsky Normal Form with exactly n non-terminals. Find a counter-example or prove correct the following statement:

$$L(G) = L_{n+1}(G)$$

$L_{n+1}(G)$ is the language obtained by considering only derivations of G of index up to $n + 1$.

Counter Example. Consider the following grammar.

$$\begin{aligned} S &\to AA \\ A &\to SA \\ S &\to b \\ A &\to a \end{aligned}$$

We claim $aabaaaba \in L(G)$ but $aabaaaba \notin L_3(G)$. This can be easily seen from the derivation tree of the word. This grammar is unambiguous for this input so there is only one parse tree.