

Seminar on Automated Reasoning

Homework 06

Second-Order Logic

1 Linear Order

Show that in an MSO formula the predicate $<$ can be replaced by the successor relation S .

$$\begin{aligned}t &:= 0|x_i \\ f &:= (t < t) | X_i(t) | \neg f | f \wedge f | \exists x_i. f | \exists X_i. f\end{aligned}$$

2 MSO on words

Give FO- or MSO- formulas for the languages that are defined by the following regular expressions.

(a) $a + b^*$

(b) aab^*aa

3 Equivalence of WS1S and Regular Languages

The MSO_0 logic is defined in the slide 8 of the lecture slides.

(a) For the following MSO formula give an equivalent MSO_0 formula.

$$\exists x. \forall y. (x < y \rightarrow P_a(y))$$

(b) Let $\Sigma = \{a, b\}$. The following MSO_0 formula defines a language over $\Sigma \times \{0, 1\}^3$. Construct the equivalent NFA.

$$(X_1 \subseteq X_2) \wedge (X_1 < X_3) \wedge (X_3 \subseteq P_a)$$