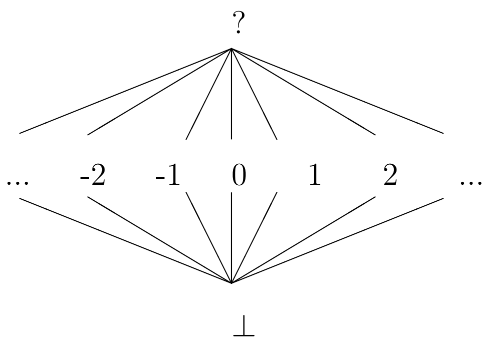
1 Forward Analysis

Each variable maps to a store. Each store maps a field to a constant.

1.1 Standard Constant Propagation lattice

For constants, we use the following lattice:



where

$$x \sqcup y = \begin{cases} x & \text{if } x = y \text{ or } y = \bot \\ y & \text{if} x = \bot \\ ? & \text{otherwise} \end{cases}$$

$$x \sqcap y = \begin{cases} x & \text{if } x = y \text{ or } y = \top \\ y & \text{if } x = \top \\ \bot & \text{otherwise} \end{cases}$$

$$\begin{aligned} \bot &\sqsubseteq x \\ x &\sqsubseteq \top \\ x &\sqsubseteq y \Leftarrow x = y \end{aligned}$$

1.1.1 Interpretation

For the entry point and initial store S_i, we have:

$$[\![entry]\!]_{\mathcal{S}} = \lambda f.c_f$$

Where c_f is the value of field f in the current state. For an S_i not an initial store, we have

$$[\![entry]\!]_{\mathcal{S}} = \lambda f.\bot$$

For a variable assignment, with S_{prev} being the value of store S in the previous state:

$$[\![x := y]\!]_{\mathcal{S}} = S_{prev}$$

For a new instance creation, if S is the store associated with instances at this label:

$$[\![x := \text{new}]\!]_{\mathcal{S}} = \lambda f.\bot$$

For every other store, nohing changes.

For a field assignment where xRS, x is related to S:

$$[\![x.f := a]\!]_{\mathcal{S}} = S_{prev}[f \rightarrow [\![a]\!]_{\mathcal{A}}]$$

For an unrelated field assignment:

$$[\![x.f := a]\!]_{\mathcal{S}} = S_{prev}$$

For an assume statement:

$$[\![\text{assume } cond]\!]_{\mathcal{S}} = \begin{array}{cc} S_{prev} & \text{if } [\![cond]\!]_{\mathcal{B}} \in \{1, \frac{1}{2}\} \\ \bot & \text{otherwise} \end{array}$$

For an arithmetic expression:

$$[\![a_1 * a_2]\!]_{\mathcal{A}} = \left\{ \begin{array}{cc} c_1 * c_2 & \text{if } [\![a_1]\!]_{\mathcal{A}} U_{prev} = c_1 \text{ and } [\![a_2]\!]_{\mathcal{A}} U_{prev} = c_2 \\ ? & \text{otherwise} \end{array} \right.$$

For $* \in \{+, -, \cdot, /\}$

$$[\![x.f]\!]_{\mathcal{A}} = \bigsqcup_{(x,S) \in R_{prev}} S(f)$$

And finally, for relations:

$$[\![a_1 \mathcal{R} a_2]\!]_{\mathcal{B}} = \left\{ \begin{array}{cc} \frac{1}{2} & \text{if } [\![a_1]\!]_{\mathcal{A}} =? \text{ or } [\![a_2]\!]_{\mathcal{A}} =? \\ [\![a_1]\!]_{\mathcal{A}} \mathcal{R} [\![a_2]\!]_{\mathcal{A}} & \text{otherwise} \end{array} \right.$$

For $\mathcal{R} \in \{=, \neq, \leq, <, >, \geq\}$

1.2 Store lattice

A store S is a function from a field name to a constant. $S_i : Fields \rightarrow Constants$. Stores are identified by their definition point i. We define $\sqcup$ as follows:

$$S_1 \sqcup S_2 = \lambda f. S_1(f) \sqcup S_2(f)$$

Also, define $S[f \rightarrow \mathbf{c}]$ to be the store where field f points to value $\mathbf{c}$.

$$S_1 \sqsubseteq S_2 \Leftrightarrow \forall f.\ S_1(f) \sqsubseteq S_2(f)$$

We will have multiple stores, one per program point and one per store (heap object) at the entry vertex, mapping to the current values.

$$\bot = \lambda f. \bot$$

$$\top = \lambda f. ?$$

1.3 Relation between variables and stores

$$R \subseteq X \times S^{|H|+|L|}$$

Where X are the variable names, H are the stores at the entry point and L are the edges where an instance can be created.

$$\bot = \emptyset$$

$$\top = X \times S^{|H|+|L|}$$

Where $\sqsubseteq$ is $\subseteq$, $\sqcup$ is $\cup$ and $\sqcap$ is $\cap$.

1.3.1 Interpretation

For a variable assignment:

$$[\![x := y]\!]_{\mathcal{R}} = (R_{prev} \cup \{(x, S) | (y, S) \in R_{prev}\}) \setminus \{(x, S) | (y, S) \notin R_{prev}\}$$

For a new instance at label l:

$$[\![x := \text{new}]\!]_{\mathcal{R}} = (R_{prev} \cup \{(x, S_l)\}) \setminus \{(x, S) | S \neq S_l\}$$

Nothing changes for field assignment or assume.

1.4 Pointwise

Now we are able to define a lattice for a program point. It is just a product lattice between a relation R and $S^{|I|+|L|}$ stores:

$$((R, S^{|I|+|L|}), \sqsubseteq)$$

where

$$(R_1, S_{11}, \ldots, S_{1n}) \sqsubseteq (R_2, S_{21}, \ldots, S_{2n}) = R_1 \sqsubseteq R_2 \wedge \bigwedge_{i \in 1,\ldots,n} S_{1i} \sqsubseteq S_{2i}$$

$\sqcup$, $\sqcap$ are also defined pointwise.