

$$2^{S \times S \times \{1, 2, \dots, n\}}$$