

$$S_0 \subseteq \text{expand}(S)$$