

$$S_{x,f} \cup S_{y,f} \subseteq S_{y_1}$$