Verifying pattern matching with guards in Scala

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Outline

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.

[1] The Scala Experiment – Can We Provide Better Language Support for Component Systems? http://lamp.epfl.ch/~odersky/talks/google06.pdf

Scala[1]

- Scala is an object-oriented and functional language which is completely interoperable with Java.
- It removes some of the more arcane constructs of these environments and adds instead:
  1. a uniform object model
  2. pattern matching and higher-order functions
  3. novel ways to abstract and compose programs

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Algebraic Data Types in Scala

- Consider the following ADT definition:

$$\textbf{type}\ \text{Tree} = \text{Node of Tree} * \text{int} * \text{Tree} \\ \qquad\qquad\quad \mid \text{EmptyTree}$$

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| EmptyTree

- In Scala:

abstract class Tree

case class Node (left: Tree, value: Int, right: Tree) **extends** Tree

case object EmptyTree **extends** Tree

Pattern matching in Scala

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```
def search(tree: Tree, value: Int): Boolean = tree match {
    case EmptyTree ⇒ false
    case Node(_,v,_) if(v == value) ⇒ true
    case Node(l,v,_) if(v < value) ⇒ search(l,v)
    case Node(_,v,r) if(v > value) ⇒ search(r,v)
    case _ ⇒ throw new Exception("...")
}
```

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- use wildcards

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Enforcement of these properties varies among languages.

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- unreachable patterns are forbidden

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There is room for improvements using formal verification techniques.

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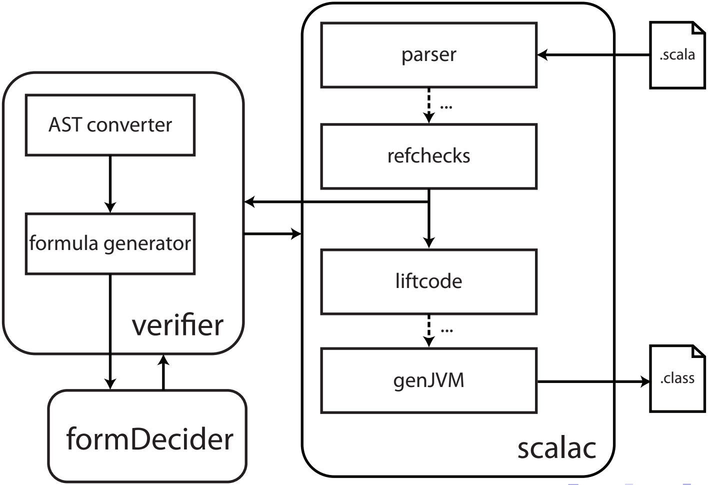
5. Based on the results, warning/error messages are sent back to the compiler.

The big picture



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     - how about primitive types? and strings?
  2. define completeness and disjointness
     - what axioms do we need?
     - how do formulas relate to each other?

Formalizing completeness and disjointness

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- E is complete $\iff \bigvee_i \xi(t, p_i)$
- E is disjoint $\iff \forall i, j,\ i \neq j \implies \neg(\xi(t, p_i) \wedge \xi(t, p_j))$

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Properties of ADT are used to generate axioms

- $\forall t \in Tree, t \in Node(\ldots) \oplus t \in EmptyTree$

Formalizing patterns – cont'd

Objects are represented as singletons

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  $\forall n \in \mathit{Node}\ (\Psi_{\text{Node,left}}(n) \in \mathit{Tree} \wedge \Psi_{\text{Node,right}} \in \mathit{Tree})$

The above transformations, along with the information about the selector's type, define *axioms* about E.

Example – Axioms

abstract class Tree
case class Node(left:Tree,right:Tree) **extends** Tree
case object Leaf **extends** Tree

t: Tree **match** { ... }

$$
\begin{aligned}
& t \in Tree \\
& \quad \wedge\, Node \subseteq Tree \wedge Leaf \subseteq Tree \wedge Leaf = \{leaf_0\} \\
& \quad \wedge\, \forall t_0 \in Tree, t_0 \in Node(...) \oplus t_0 \in Leaf \\
& \quad \wedge\, \forall n \in Node\ (\Psi_{\text{Node,left}}(n) \in Tree \wedge \Psi_{\text{Node,right}} \in Tree)
\end{aligned}
$$

Axioms – cont'd

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- The formula for completeness $\bigvee_i \xi(t, p_i)$ hence becomes $\bigvee_i (A(t) \implies \Pi(p_i))$

Simplified, this becomes: $A(t) \implies \bigvee_i \Pi(p_i)$

Translation of patterns

The "root" type in the pattern is assigned to the selector

- t **match** { **case** Node(...) ⇒ ... } $\longmapsto t \in Node$

[2] the practical implementation slightly differs when proving completeness

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Aliases[2] are bound to fresh names

- **case** Node(left: Node, ...) ⇒ ...
  $\longmapsto left_{\text{fresh}} = \Psi_{\text{Node,left}}(t) \wedge left_{\text{fresh}} \in Node$

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Wildcards generate no constraints

- **case** _ ⇒ ... $\longmapsto true$

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Translation of patterns – cont'd

Guards are, to some extent, translated to formulas:

- equality and arithmetic operators are kept "as it"
- `equals` is always considered side-effect free
- dynamic type tests are converted to set membership
  - `o.isInstanceOf[`*Type*`]` $\longmapsto o \in \mathit{Type}$
- other method calls are ignored

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The result of the transformation is a predicate, whose parameters are the selector and the aliases defined in the pattern.

It is added as a conjunction to the main formula.

Matching on lists

Scala, as a language making an extensive use of lists, has a dedicated syntax for them:

z **match** {
    case Nil ⇒ ...
    case x :: xs ⇒ ...
}

... but this is essentially syntactic sugar for the following hierarchy:

sealed abstract class List
case final class ::(List, List) **extends** List
case object Nil **extends** List

Future work

Some issues we want to address in the future:

- Actually plug it into scalac :)
- Allow matching on string constants.
- Improve support for primitive types.
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- Actually plug it into scalac :)
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- Implement limited support for external variables and functions
- . . . oh, well, you always find something to do

Questions ?

One for the road...

```
sealed abstract class Arith
case class Sum(l: Arith, r: Arith) extends Arith
case class Prod(n: Num, f: Arith) extends Arith
case class Num(n: Int) extends Arith

def eval(a: Arith): Int = (a: @verified) match {
    case Sum(l, r) => eval(l) + eval(r)
    case Prod(Num(n), f) if(n == 0) => 0
    case Prod(Num(n), f) if(n != 0) => n * eval(f)
    case Num(n) => n
}
```

$$
\begin{aligned}
& a \in Arith \wedge Sum \subseteq Arith \wedge Prod \subseteq Arith \wedge Num \subseteq Arith \\
& \quad \wedge \forall a_0 \in Arith, ((a_0 \in Sum \oplus a_0 \in Prod) \wedge (a_0 \in Sum \oplus a_0 \in Num) \\
& \quad \wedge (a_0 \in Prod \oplus a_0 \in Num)) \wedge \forall s_0 \in Sum, (\Psi_{\text{Sum,l}}(s_0) \in Arith \\
& \quad \wedge \Psi_{\text{Sum,r}}(s_0) \in Arith) \wedge \forall p_0 \in Prod, (\Psi_{\text{Prod,n}}(p_0) \in Num \\
& \quad \wedge \Psi_{\text{Prod,f}}(s_0) \in Arith) \wedge \forall n_0 \in Num, \Psi_{\text{Num,n}}(n_0) \in \mathbb{N}
\end{aligned}
$$

$$\Longrightarrow$$

$$
\begin{aligned}
& ((l_{\text{fresh}} = \Psi_{\text{Sum,l}}(a) \wedge r_{\text{fresh}} = \Psi_{\text{Sum,r}}(a)) \implies a \in Sum) \\
& \vee ((f_{\text{fresh}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod \\
& \quad \wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh}} = 0) \\
& \vee ((f_{\text{fresh'}} = \Psi_{\text{Prod,f}}(a) \wedge n_{\text{fresh'}} = \Psi_{\text{Num,n}}(\Psi_{\text{Prod,l}}(a))) \implies a \in Prod \\
& \quad \wedge \Psi_{\text{Prod,l}}(a) \in Num \wedge n_{\text{fresh'}} \neq 0) \\
& \vee (n_{\text{fresh''}} = \Psi_{\text{Num,n}}(a) \implies a \in Num)
\end{aligned}
$$