CS-320

Computer Language Processing

Exercise Session 2

October 8, 2018

Overview

Today we will have a deeper look at lexers.

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How can we automatically generate lexers?

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What does a lexer (*aka.* tokenizer) do?
$\Rightarrow$ It transforms a stream of symbols into a stream of tokens.

How can we automatically generate lexers?
$\Rightarrow$ We will use regular languages and automata.

Lexer

First, let us define a lexer as an ordered set of token names and regular expressions

$$\langle \mathit{Token}_1 := e_1,\ \mathit{Token}_2 := e_2,\ \ldots \rangle$$

where earlier token classes have higher priority than later ones.

E.g.

$$\langle \mathit{ID} := \text{letter } (\text{letter} \mid \text{digit})^*,\ \mathit{LE} := \texttt{<=},\ \mathit{LT} := \texttt{<},\ \mathit{EQ} := \texttt{=} \rangle$$

Ambiguity in tokenization

Tokenization differs from matching using a single regular expression (say $(e_1 \mid e_2 \mid \ldots)^*$).

Rather, the result of tokenizing an input stream of symbols is a stream of tokens. Each token maps to a subsequence of the input stream, and none of the tokens' subsequences overlap.

E.g.

$$\texttt{i0 <= size} \quad \overset{tokenize}{\Rightarrow} \quad \underset{\texttt{i0}}{ID} \;\; \underset{\texttt{<=}}{LE} \;\; \underset{\texttt{size}}{ID}$$

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$$\texttt{i0 <= size} \overset{tokenize}{\Rightarrow} \underset{\texttt{i0}}{ID}\ \underset{\texttt{<=}}{LE}\ \underset{\texttt{size}}{ID}$$

- How do we avoid ambiguities?

$$\texttt{i0 <= size} \overset{???}{\Rightarrow} \underset{\texttt{i0}}{ID}\ \underset{\texttt{<}}{LT}\ \underset{\texttt{=}}{EQ}\ \underset{\texttt{s}}{ID}\ \underset{\texttt{iz}}{ID}\ \underset{\texttt{e}}{ID}$$

Tokenization rules

Given an input string w the lexer will match tokens on a prefix u of $w = uv$, output the matching token and repeat the process on the remaining string v.

To disambiguate between different possible tokenizations we employ two additional rules:

- *Longest match*: If we find matching tokens for prefixes of varying lengths, we pick the longer prefix.
- *Token priority*: If multiple tokens match a prefix of the same length, we pick the token that has higher priority.

A simple tokenization example

Exercise 1

- Given the lexer

$$\langle T_1 := a(ab)^*,\ T_2 := b^*(ac)^*,\ T_3 := cba,\ T_4 := c^+ \rangle$$

tokenize the following input strings:

```
c a c c a b a c a c c b a b c
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tokenize the following input strings:

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$$\texttt{c c c a a b a b a c c b a b c c b a b a c}$$

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$$\underbrace{\texttt{c c c}}_{T_4}\ \underbrace{\texttt{a a b a b}}_{T_1}\ \underbrace{\texttt{a c}}_{T_2}\ \underbrace{\texttt{c b a}}_{T_3}\ \underbrace{\texttt{b}}_{T_2}\ \underbrace{\texttt{c c}}_{T_4}\ \underbrace{\texttt{b}}_{T_2}\ \underbrace{\texttt{a}}_{T_1}\ \underbrace{\texttt{b a c}}_{T_2}$$

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▷ Are there alternative tokenizations if we disregard the longest match rule?

Constructing a lexer

To automatically construct a lexer from token class definitions we go through a series of transformations:

$$\textbf{Token def.s} \overset{\textit{translate}}{\Rightarrow} \textbf{NFA} \overset{\textit{determinize}}{\Rightarrow} \textbf{DFA} \overset{\textit{minimize}}{\Rightarrow} \textbf{DFA}$$

The resulting DFA is then repeatedly used to produce tokens for an input string.

(The minimization step is optional.)

Constructing a lexer (2)

(Token def.s $\Rightarrow$ NFA)

Let $e_1, \ldots, e_n$ be the regular expressions for each token class and consider the regular expression $(e_1 \mid \cdots \mid e_n)$.

E.g., for the token classes

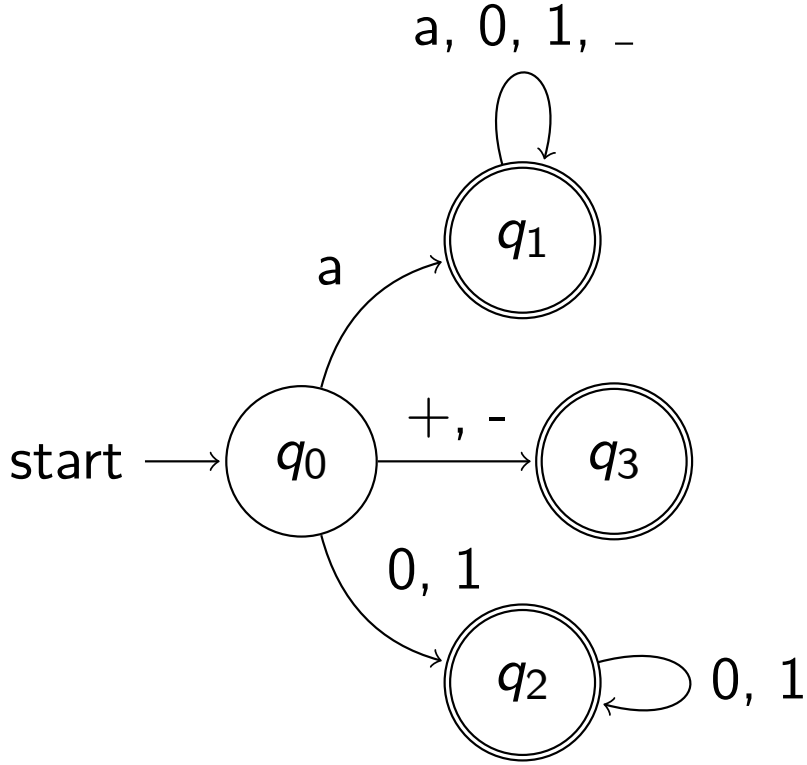
$$\langle ID := a\,(a \mid 0 \mid 1 \mid \_)^*,\ INT := (0 \mid 1)\,(0 \mid 1)^*,\ OP := + \mid - \rangle$$

we have

$$a\,(a \mid 0 \mid 1 \mid \_)^* \mid (0 \mid 1)\,(0 \mid 1)^* \mid (+ \mid -).$$

Constructing a lexer (3)

Convert the regular expression to an automaton and specify the token class being recognized by each accepting state:



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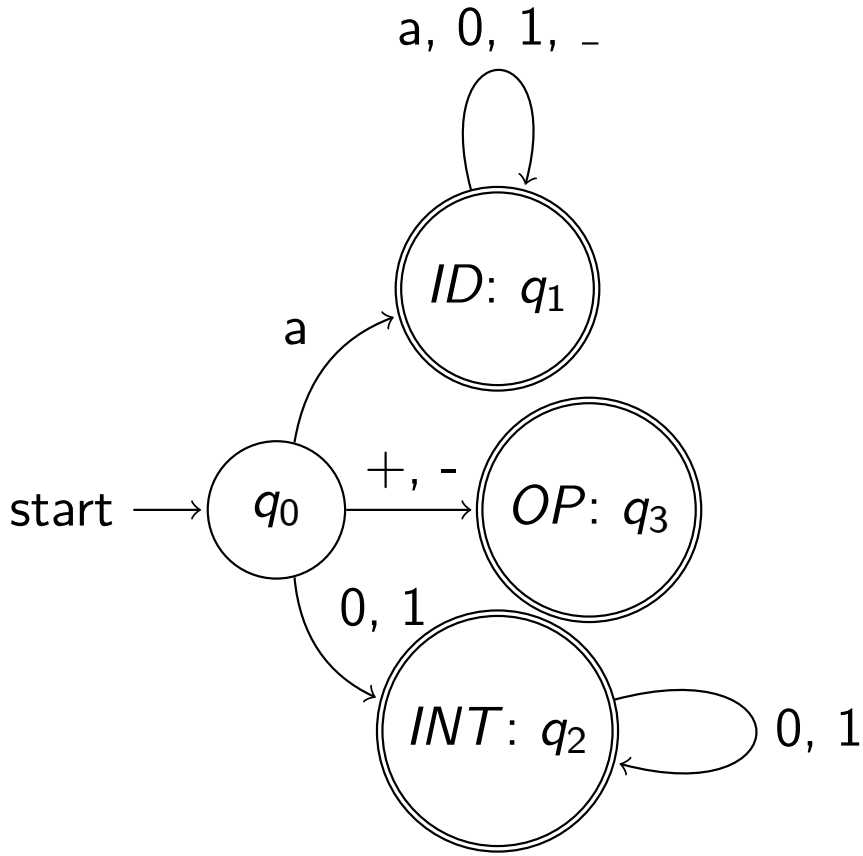

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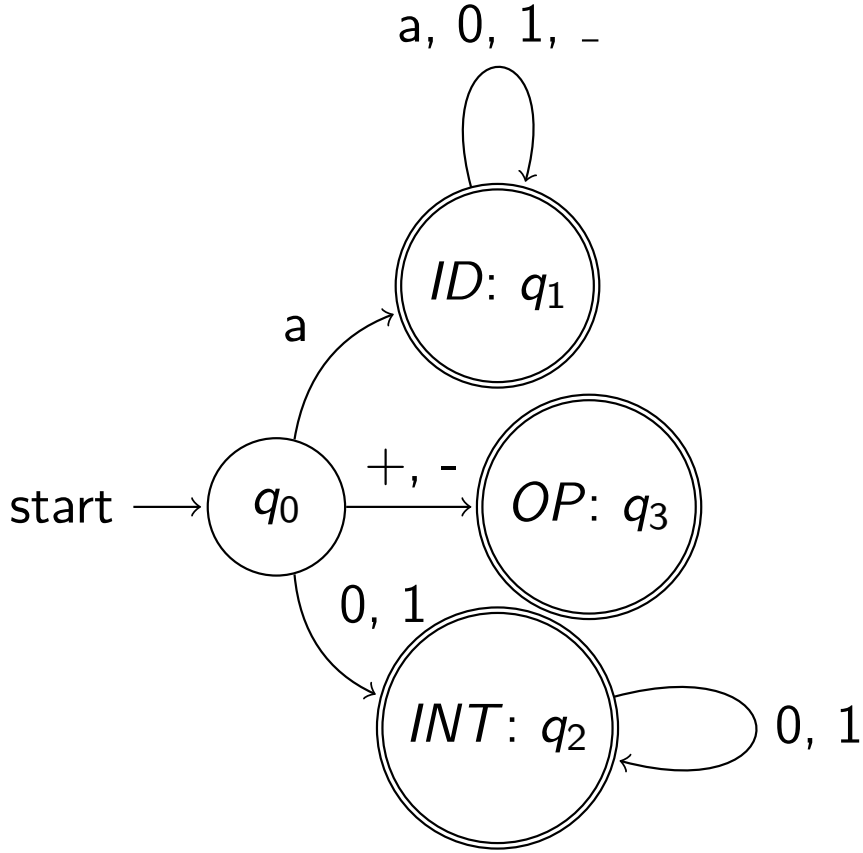
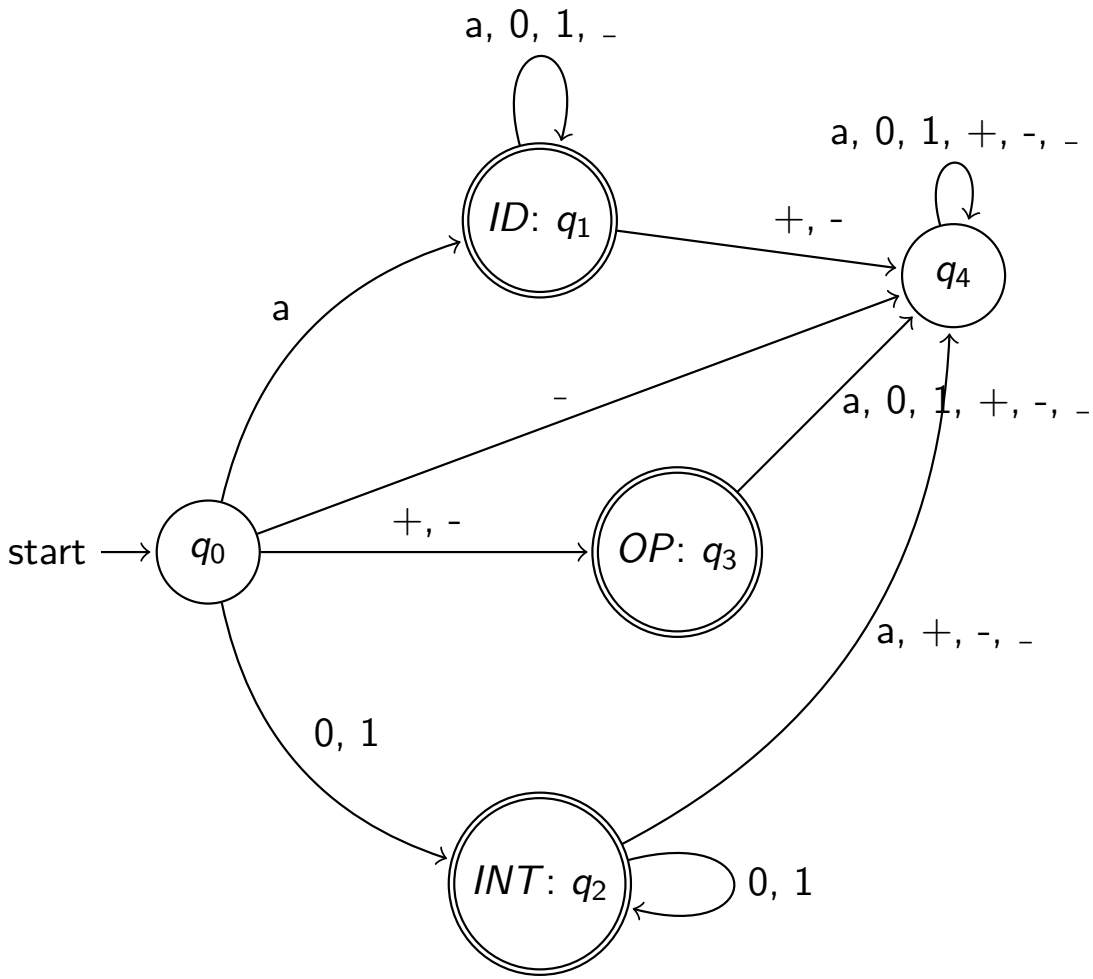


In case of ambiguities we pick the token class of higher priority.

Constructing a lexer (4)

Finally, we determinize and minimize the automaton:



Lexing

We can then produce tokens for an input string as follows:

1. Initialize variables `lastToken` and `lastTokenPos` to the initial value $\bot$, resp. -1, and set the automaton's state to q_0.
2. Consume the next input character and make the corresponding transition in the automaton.
   - If we have arrived in an accepting state, update `lastToken` with the corresponding token and set `lastTokenPos` to the current position in the input string.
   - If we have arrived in a state which cannot lead to acceptance (a trap state, effectively):
     - If `lastToken` $= \bot$, report an error.
     - Otherwise, output `lastToken`.
   - Reset `lastToken`, and restart the automaton with the input string starting at `lastTokenPos` $+ 1$ (Continue with step 2).
3. If there is no more input to consume, output the `lastToken`, or report an error, if `lastToken` $= \bot$.

Alternate way to build the lexer

Instead of building a single regular expression and converting it to a NFA and then a DFA, we can convert each regular expression to a DFA independently and then build the *parallel composition* of the DFAs as our lexer.
The parallel composition of multiple DFAs $\langle \Sigma, Q_i, s_i, \delta_i, F_i \rangle$ on the same alphabet Σ is the DFA $\langle \Sigma, Q, s, \delta, F \rangle$ where:

- Q is the cartesian product of all Q_i,
- $s \in Q$ is the tuple of all initial states s_i,
- $\delta : (Q \times \Sigma) \to Q$ applies the different δ_i to the respective points, and
- $F \subseteq Q$ is the set of accepting states. A tuple $q \in Q$ is in F iff there exists a q_i such that $q_i \in F_i$.

Exercise 2

Build a DFA that accepts binary numbers that are multiples of 2 or 3. There should be different (accepting) states for multiples of 2 only, multiples of 3 only, and multiples of 2 and 3.

Hint: Build 2 DFAs, and then use parallel composition.

A lexer for XML

Exercise 3 (Quiz 2015)

Your goal is to construct a lexer (i.e., a DFA) that tokenizes an XML input stream into the tokens listed below. Note that *WS* denotes a whitespace character.

Token name	**Regular expression**
OP	$<$
CL	$>$
OPSL	$< /$
CLSL	$/ >$
EQ	$=$
NAME	$\mathit{letter}(\mathit{letter} \mid \mathit{digit})^*$
NONNAME	$(\mathit{digit} \mid \mathit{special})(\mathit{letter} \mid \mathit{digit} \mid \mathit{special})^*$
STRING	$''(\mathit{letter} \mid \mathit{digit} \mid \mathit{special})^*''$
COMMENT	$<! -- (\mathit{letter} \mid \mathit{digit} \mid \mathit{special})^* -- >$
SKIP	*WS*

A lexer for XML (2)

Exercise 3 (Quiz 2015)

▷ Construct the labelled DFA described in the lectures for the tokens defined above. Note that every final state should be labelled by the token class(es) it accepts.

A lexer for XML (2)

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Consider the following XML string.

```
<jsonmessage>
    <!--CommunicationOfJSonObjects-->
    <from  ip="">EPFLserver</from>
    <message>{"field":1}</message>
</jsonmessage>
```

▷ Show the list of tokens that should be generated by the lexer for the above XML string. You **need not** show SKIP tokens, which correspond to whitespaces.

Supporting tokens in FSMs

Once we try to put things together as outlined before we note that the usual notion of finite-state automatons does not support tokens.

What we really want is a notion of outputs rather than accepting states.

▷ How can we adapt the formal definitions of finite-state automatons to support such outputs?
▷ Identify where and how the transformations we have seen (regular expressions to NFAs, determinization and minimization) need to be adapted.