Automating Construction of Lexers by converting Regular Expressions to Automata

Regular Expression to Programs

- How can we write a lexer that has these two classes of tokens:
  - a*b
  - aaa
- Consider run of lexer on: **aaaab** and on: **aaaaaa**

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  - a*b
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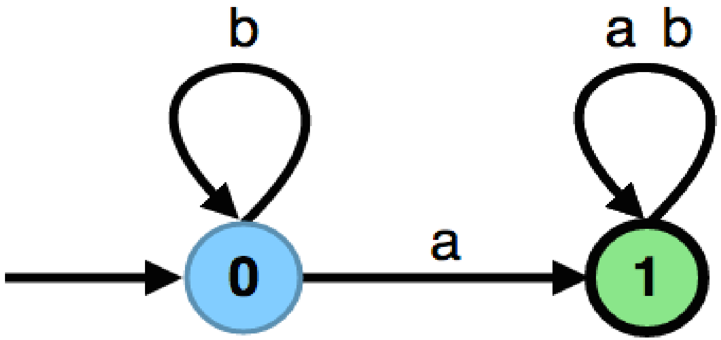
- Consider run of lexer on: **aaaab** and on: **aaaaaa**
- A general approach:

Regular Expression → Finite Automaton → Program

Finite Automaton (Finite State Machine)

$A = (\Sigma, Q, q_0, \delta, F)$

$$\delta \subseteq Q \times \Sigma \times Q,$$
$$q_0 \in Q,$$
$$F \subseteq Q$$

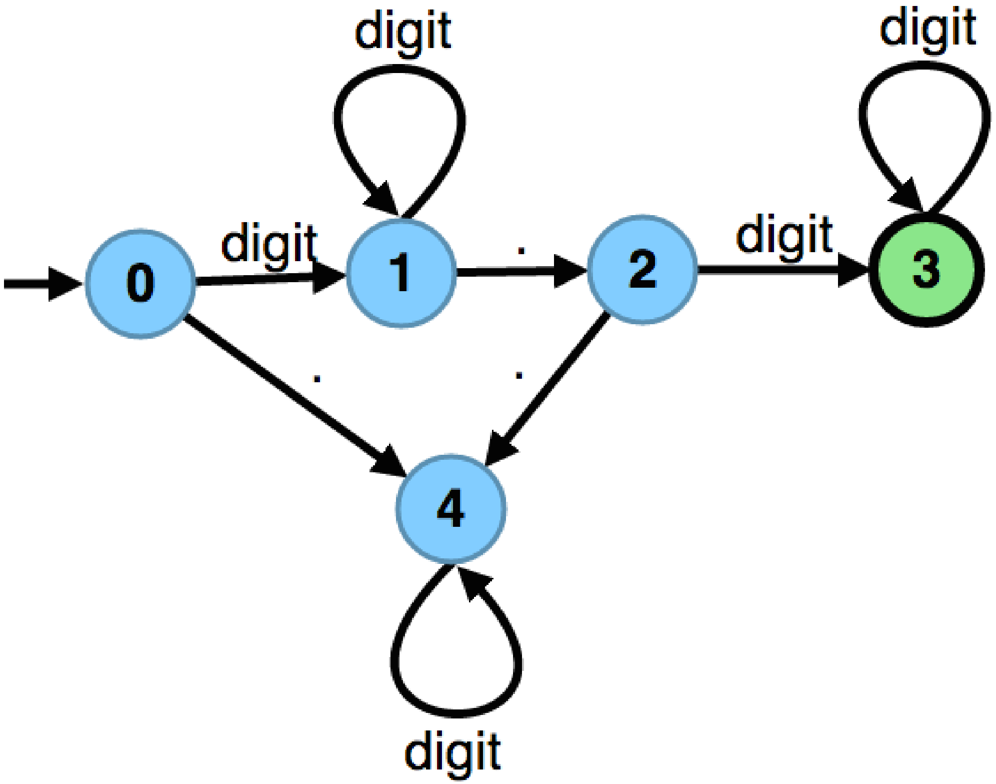


$$q_0 \in Q$$
$$q_1 \subseteq Q$$
$$\delta = \{ (q_0, a, q_1), (q_0, b, q_0), (q_1, a, q_1), (q_1, b, q_1), \}$$

- Σ - alphabet
- Q - states (nodes in the graph)
- q_0 - initial state (with ‘->' sign in drawing)
- δ - transitions (labeled edges in the graph)
- F - final states (double circles)

Numbers with Decimal Point

digit digit* . digit digit*

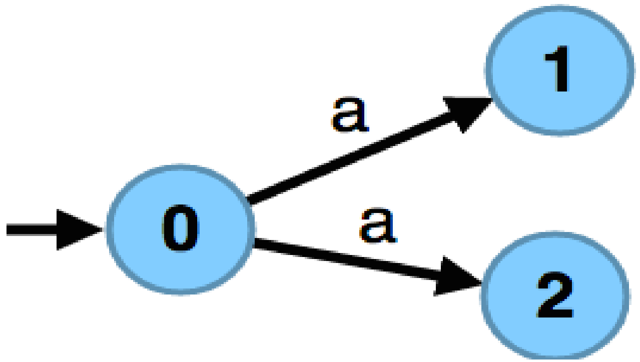


What if the decimal part is optional?

Kinds of Finite State Automata

- DFA: δ is a function : $(Q, \Sigma) \mapsto Q$
- NFA: δ could be a relation

- In NFA there is no unique next state. We have a set of possible next states.



Remark: Relations and Functions

- Relation $r \subseteq B \times C$

  $r = \{ \ldots, (b,c1) , (b,c2) , \ldots \}$

- Corresponding function: $f : B \rightarrow 2^C$

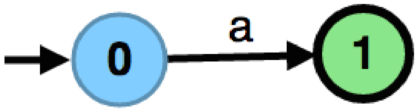
  $f = \{ \ldots (b,\{c1,c2\}) \ldots \}$

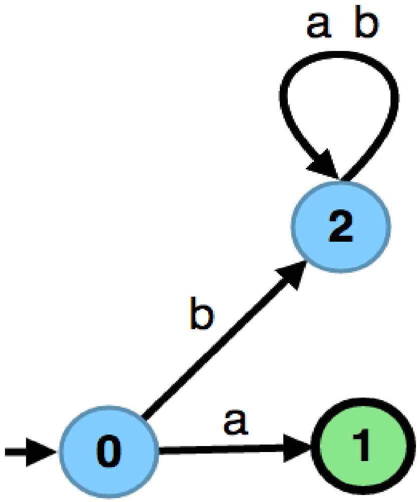
  $f(b) = \{ c \mid (b,c) \in r \}$

- Given a state, next-state function returns a **set** of new states

  for *deterministic* automaton, set has *exactly 1* element

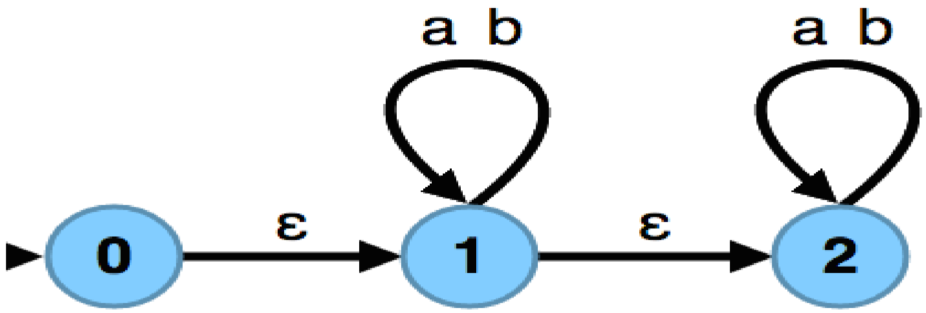
Allowing Undefined Transitions





- Undefined transitions are equivalent to transition into a sink state (from which one cannot recover)

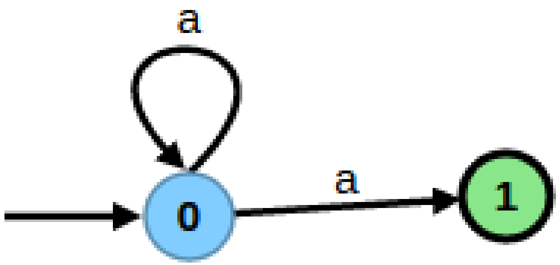
Allowing Epsilon Transitions



- Epsilon transitions:
  - traversing them does not consume anything
- Transitions labeled by a word:
  - traversing them consumes the entire word

When Automaton Accepts a Word

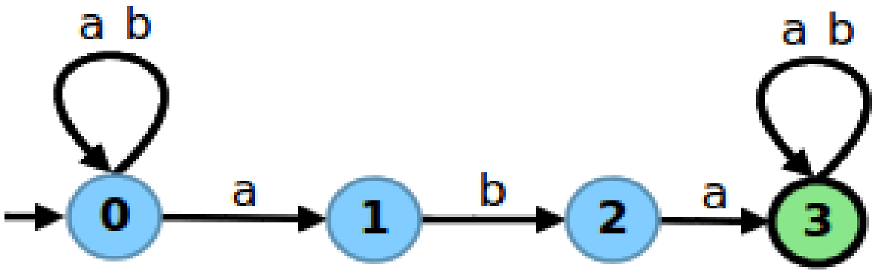
Automaton accepts a word *w* iff there **exists a path** in the automaton from the starting state to some accepting state such that concatenation of words on the path gives *w*.



- Does the automaton accept the word ***a*** ?

Exercise

- Construct a NFA that recognizes all strings over {a,b} that contain "aba" as a substring



Running NFA (without epsilons)

```
def δ(a : Char)(q : State) : Set[States] = { ... }
def δ'(a : Char, S : Set[States]) : Set[States] = {
  for (q1 <- S, q2 <- δ(a)(q1)) yield q2  // S.flatMap(δ(a))
}

def accepts(input : MyStream[Char]) : Boolean = {
 var S : Set[State] = Set(q0) // current set of states
 while (!input.EOF) {
   val a = input.current
   S = δ'(a,S)          // next set of states
 }
 !(S.intersect(finalStates).isEmpty)
}
```

NFA Vs DFA

- Every DFA is also a NFA (they are a special case)
- For every NFA there exists an equivalent DFA that accepts the same set of strings
- But, NFAs could be exponentially smaller (succinct)
- There are NFAs such that **every** DFA equivalent to it has exponentially more number of states

Regular Expressions and Automata

Theorem:

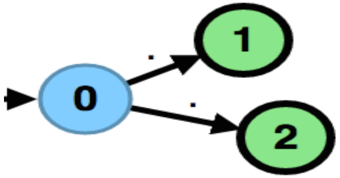
Let L be a language. There exists a regular expression that describes it if and only if there exists a finite automaton that accepts it.

Algorithms:

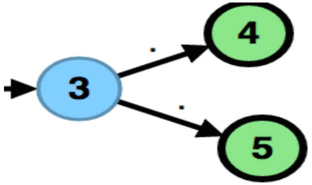
- regular expression → automaton (important!)
- automaton → regular expression (cool)

Recursive Constructions

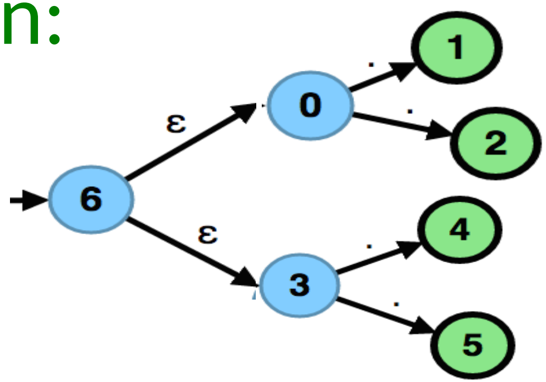
A_1:



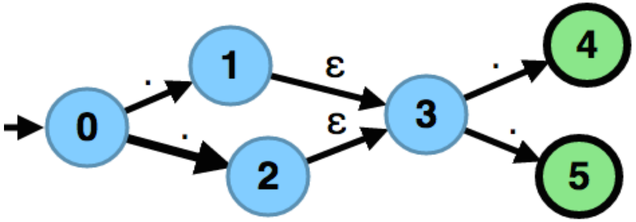
A_2:



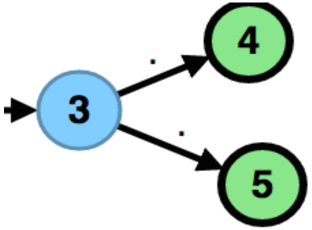
Union:



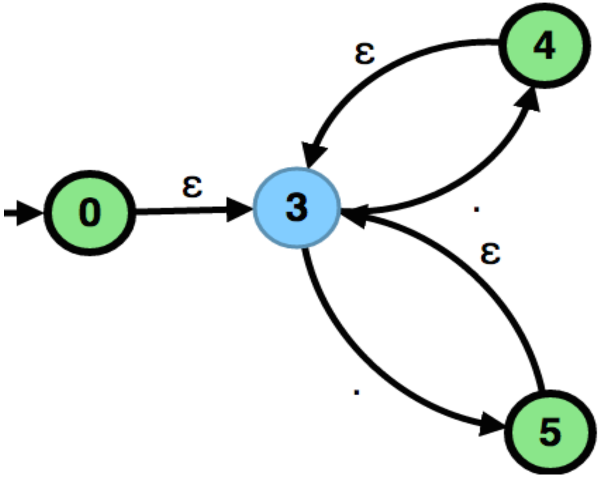
Concatenation:



Recursive Constructions

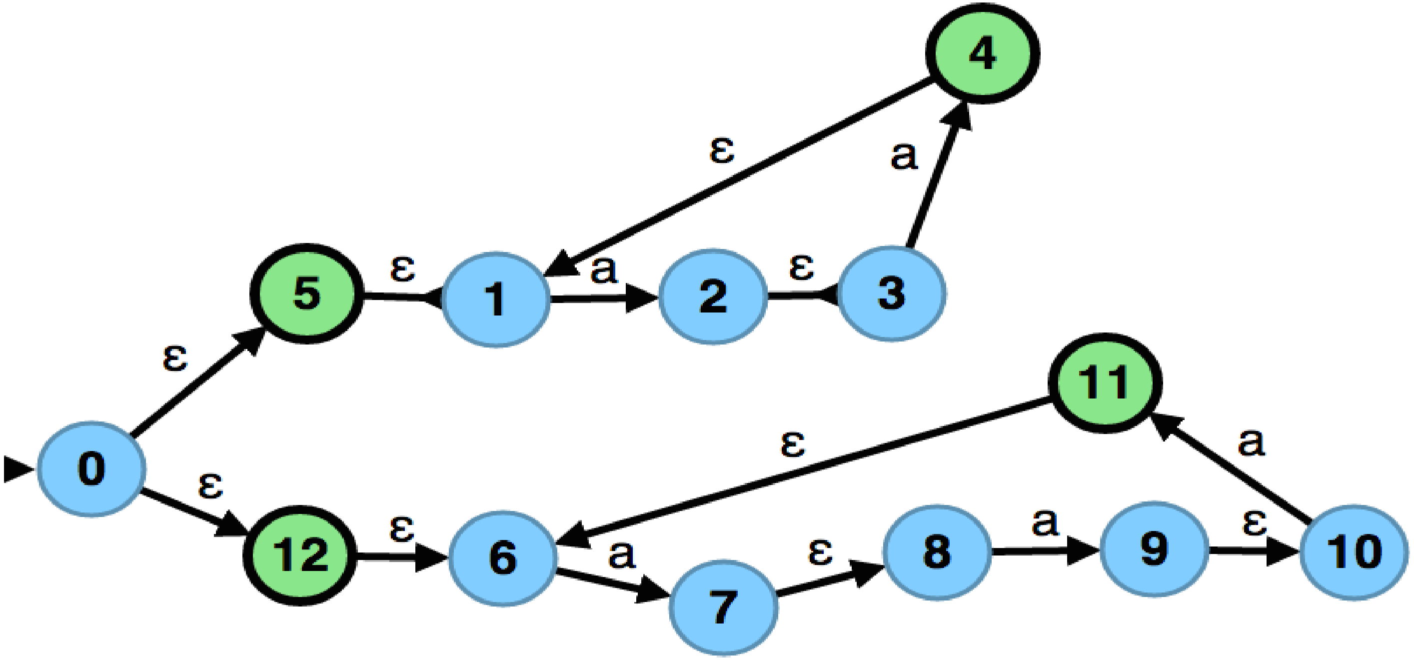


Star:



Exercise: (aa)* | (aaa)*

- Construct an NFA for the regular expression

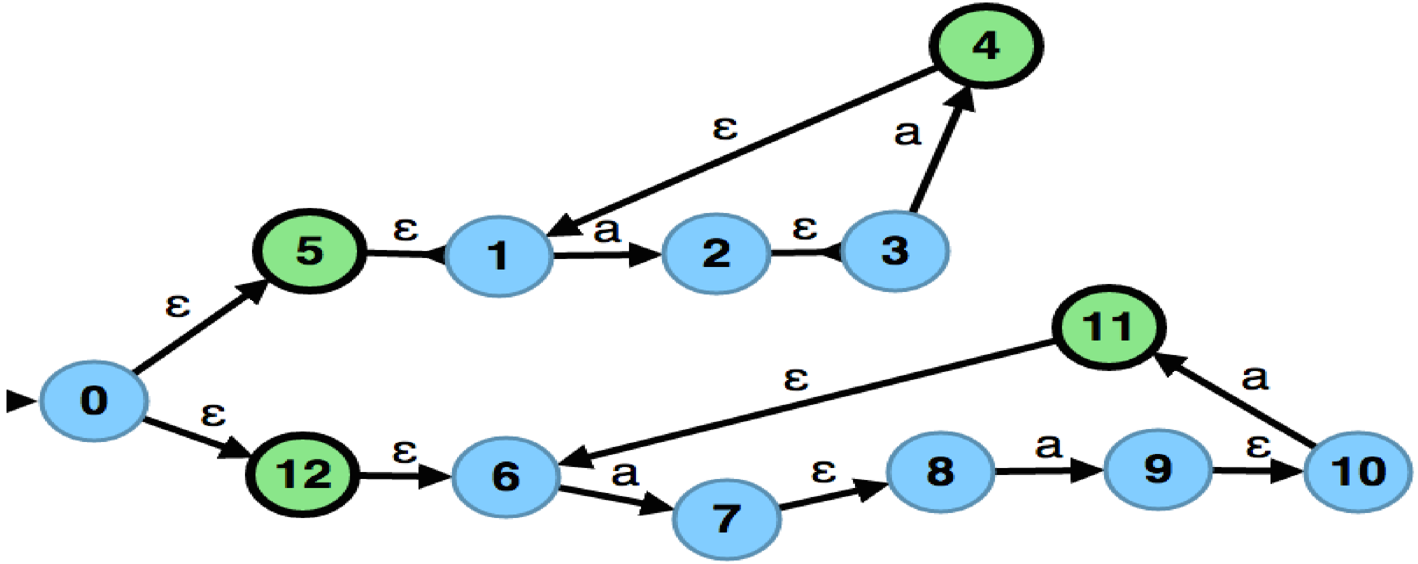


NFAs to DFAs (Determinization)

- keep track of a set of all possible states in which the automaton could be

- view this finite set as one state of new automaton

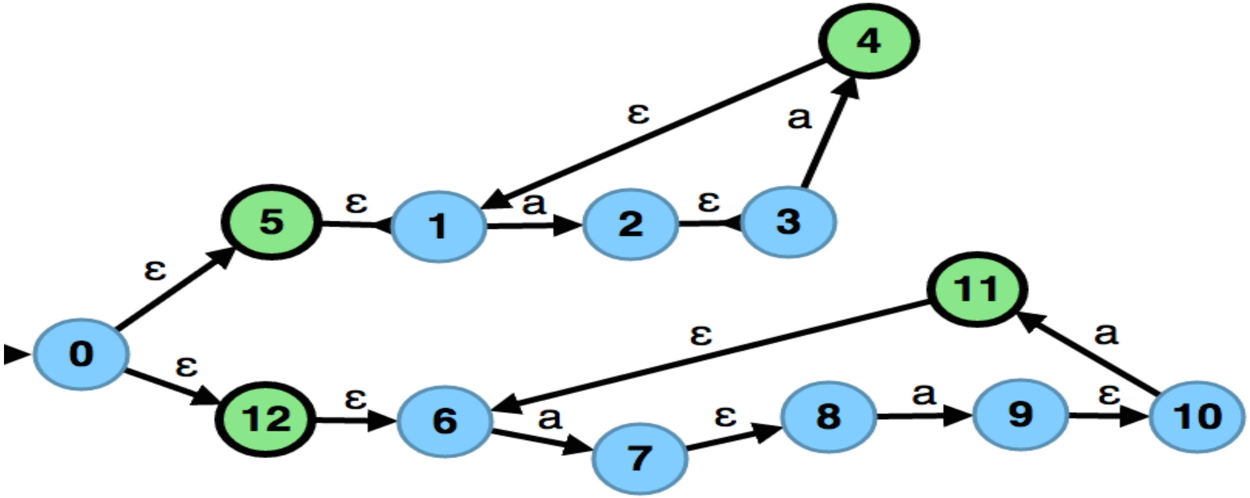
NFA to DFA Conversion



Possible states of the DFA: 2^Q

{ { } , { 0},...{12}, {0,1}, ...,{0,12}, ...{12, 12},
{0,1,2} ..., { 0,1,2...,12 } }

NFA to DFA Conversion



Epsilon Closure

– All states reachable from a state through epsilon

– q $\in E(q)$

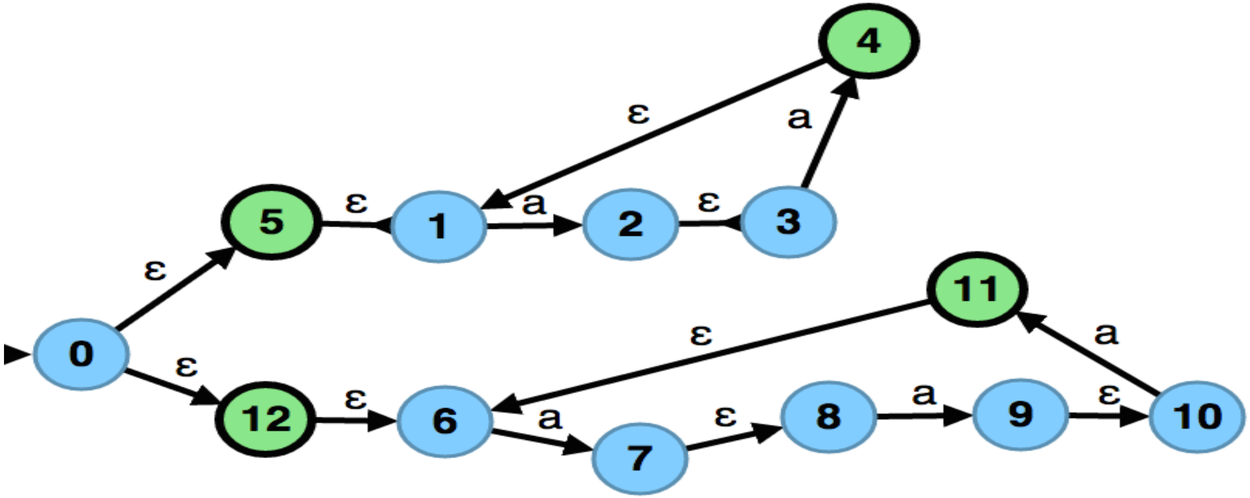
– If $q_1 \in E(q)$ and $\delta(q_1, \epsilon, q_2)$ then $q_2 \in E(q)$

E(0) = { }   E(1) = { }   E(2) = { }

NFA to DFA Conversion

- DFA: $(\Sigma, 2^Q, q_0', \delta', F')$
- $q_0' = E(q_0)$
- $\delta'(q', a) = \bigcup_{\{\exists q_1 \in q',\, \delta(q_1, a, q_2)\}} E(q_2)$
- $F' = \{\, q' | q' \in 2^Q, q' \cap F \neq \emptyset\}$

NFA to DFA Conversion through Examle



Clarifications

- what happens if a transition on an alphabet ‘a’ is not defined for a state ‘q’ ?
- $\delta'(\{q\}, a) = \emptyset$
- $\delta'(\emptyset, a) = \emptyset$

- Empty set represents a state in the NFA
- It is a trap/sink state: a state that has self-loops for all symbols, and is non-accepting.

Minimizing DFAs: Procedure

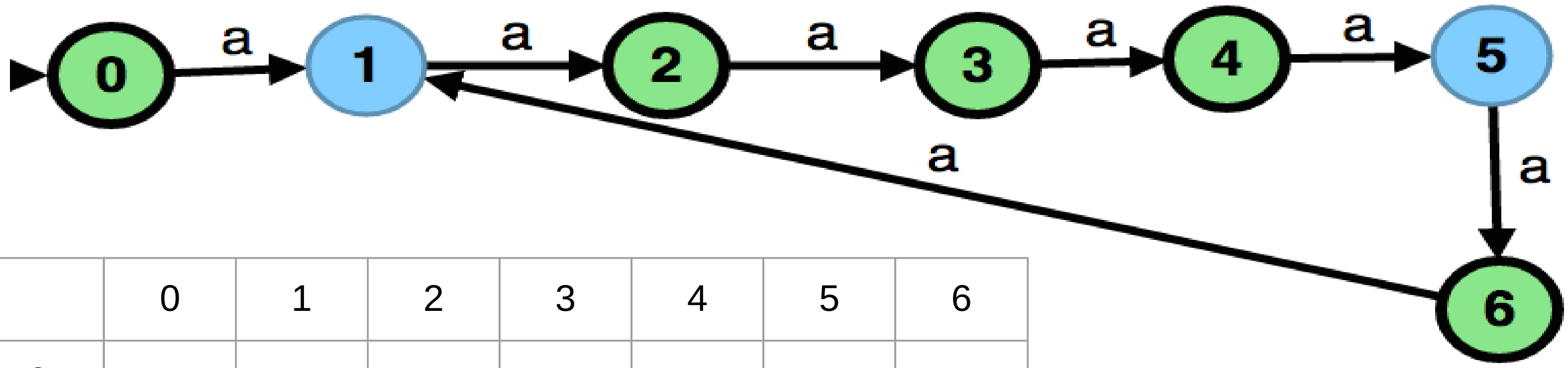
- Write down all pairs of state as a table
- Every cell in the table denotes whether the corresponding states are equivalent

	q1	q2	q3	q4	q5
q1	x	?	?	?	?
q2		x	?	?	?
q3			x	?	?
q4				x	?
q5					x

Minimizing DFAs: Procedure

- Inititalize cells (q1, q2) to false if one of them is final and other is non-final
- Make the cell (q1, q2) false, if q1 → q1’ on some alphabet symbol and q2 → q2’ on ‘a’ and q1’ and q2’ are not equivalent
- Iterate the above process until all non-equivalent states are found

Minimizing DFAs: Illustration



	0	1	2	3	4	5	6
0	x						
1		x					
2			x				
3				x			
4					x		
5						x	
6							x

Properties of Automata

Complement:

- Given a DFA A, switch accepting and non-accepting states in A gives the complement automaton A^c
- $L(A^c) = (\Sigma^* \setminus L(A))$

Note this does not work for NFA

Intersection: $L(A') = L(A_1) \cap L(A_2)$

- $A' = (\Sigma, Q_1 \times Q_2, (q_0^1, q_0^2), \delta', F_1 \times F_2)$
- $\delta'((q_1, q_2), a) = \delta(q_1, a) \times \delta(q_2, a)$

Emptiness of language, inclusion of one language into another, equivalence – they are all decidable

Exercise 0.1: on Equivalence

Prove that (a*b*)* is equivalent to (a|b)*

Sequential Hardware Circuits are Automata

$A = (\Sigma, Q, q_0, \delta, F)$

Q – states of flip-flops, registers, etc.

Each state q_i is given by values $v : Vars \to \{0,1\}$

δ – combinational circuit that determines next state: given v compute v' according to a given logical circuit

Circuit can be exponentially smaller than graph