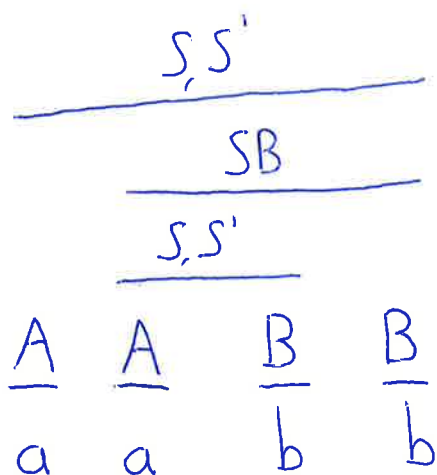## Ex 1

$E \rightarrow E\ PE\ |\ E\ TE\ |\ O\ EC\ |\ id$

$PE \rightarrow P\ E$

$TE \rightarrow T\ E$

$P \rightarrow +$

$T \rightarrow *$

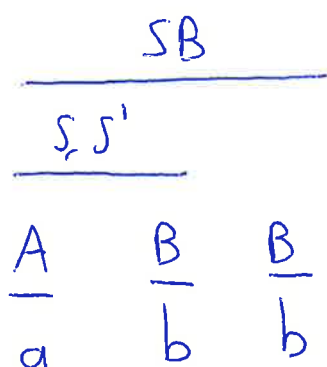$O \rightarrow ($

$EC \rightarrow E\ C$

$C \rightarrow)$

Ex 2

$S' \rightarrow A\ B\ |\ \varepsilon\ |\ A\ SB$

$S \rightarrow A\ B\ |\ A\ SB$

$SB \rightarrow S\ B$

$A \rightarrow a$

$B \rightarrow b$

Parsing a a b b

$$S, S'$$

$$SB$$

$$S, S'$$

A A   B   B

a   a   b   b

Parsing a b b

$$SB$$

$$S, S'$$

A   B   B

a   b   b

No parse...
The non-terminal S' isn't
at the top.

Ex 3

First, we notice that:

- C is not productive.
- W is not reachable.

We can therefore eliminate all rules involving C or W.
Once this is done, we get:

$$S \rightarrow P ;$$
$$P \rightarrow I ; P \mid I$$
$$I \rightarrow \text{if } E \text{ then } P R \mid \text{print } E$$
$$R \rightarrow \text{else } P \mid \varepsilon$$
$$E \rightarrow L \mid E \text{ or } E$$
$$L \rightarrow \text{true} \mid \text{false}$$

Then, we make terminals appear alone on the right-hand side.

$$S \rightarrow P T_;$$
$$P \rightarrow I T_; P \mid I$$
$$I \rightarrow T_{if} E T_{then} P R \mid T_{print} E$$
$$R \rightarrow T_{else} P \mid \varepsilon$$
$$E \rightarrow L \mid E T_{or} E$$
$$L \rightarrow \text{true} \mid \text{false}$$

$$T_; \rightarrow ;$$
$$T_{if} \rightarrow \text{if}$$
$$T_{then} \rightarrow \text{then}$$
$$T_{print} \rightarrow \text{print}$$
$$T_{else} \rightarrow \text{else}$$
$$T_{or} \rightarrow \text{or}$$

Then, we reduce arity of every production to ≤ 2.

$S \rightarrow P \; T_;$

$P \rightarrow I \; P_2 \mid I$

$P_2 \rightarrow T_; \; P$

$I \rightarrow T_{if} \; I_2 \mid T_{print} \; E$

$I_2 \rightarrow E \; I_3$

$I_3 \rightarrow T_{then} \; I_4$

$I_4 \rightarrow P \; R$

$R \rightarrow T_{else} \; P \mid \varepsilon$

$E \rightarrow L \mid E \; E_2$

$E_2 \rightarrow T_{or} \; E$

$L \rightarrow true \mid false$

$T_; \rightarrow ;$

$T_{if} \rightarrow if$

$T_{then} \rightarrow then$

$T_{print} \rightarrow print$

$T_{else} \rightarrow else$

$T_{or} \rightarrow or$

Next step is to remove ε's. I will only show the rule which are affected.

$R \rightarrow T_{else} \; P$

$I_4 \rightarrow P \; R \mid P$

Now, we remove unit productions.

We have the following unit productions:

$$P \to I \ , \quad E \to L \ , \quad I_4 \to P$$

We get therefore: (after removing unproductive and then unreachable symbols)

$S \to P \ T_;$

$P \to I \ P_2 \mid T_{if} \ I_2 \mid T_{print} \ E$

$P_2 \to T_; \ P$

$I \to T_{if} \ I_2 \mid T_{print} \ E$

$I_2 \to E \ I_3$

$I_3 \to T_{then} \ I_4$

$I_4 \to P \ R \mid I \ P_2 \mid T_{if} \ I_2 \mid T_{print} \ E$

$R \to T_{else} \ P$

$E \to true \mid false \mid E \ E_2$

$E_2 \to T_{or} \ E$

$T_; \to \ ;$

$T_{if} \to if$

$T_{then} \to then$

$T_{print} \to print$

$T_{else} \to else$

$T_{or} \to or$

The rules $L \to true \mid false$ are removed since L is no longer reachable.

①

$$\frac{S}{I, I_4, P}$$

$$\overline{T_{if}} \quad \overline{E} \quad \overline{T_{print}} \quad \overline{E} \quad \overline{T_;}$$

if   true   print   true   ;

Zero parse trees.

②

$$\frac{S, S}{I, I_4, P, I, I_4, P}$$

$$\frac{E, E}{}$$

$$\frac{E_2}{}$$

$$\frac{I, I_4, P}{}$$

$$\frac{E}{} \qquad \frac{E}{}$$

$$\frac{I, I_4, P}{} \qquad \frac{E_2}{} \qquad \frac{E_2}{}$$

$$\overline{T_{print}} \quad \overline{E} \quad \overline{T_{or}} \quad \overline{E} \quad \overline{T_{or}} \quad \overline{E} \quad \overline{T_;}$$

print   true   or   false   or   true   ;

2 parse trees.

③

S, S

I, I_4, P, I, I_4, P

I_2

S

I_3

I, I_4, P

I, I_4, P

I_2

S

S

I_3

P_2

I, I_4, P

I, I_4, P

T_{if} E T_{then} T_{print} E $T_;$ T_{print} E $T_;$

if   true   then   print   true ;   print   false ;

We get 2 different parse trees.

First example has 4 parse trees.

Second example has 2 parse trees.

Last example has 3 parse trees.