Automating Construction of Lexers

Example in javacc

```
TOKEN: {
   <IDENTIFIER: <LETTER> (<LETTER> | <DIGIT> | "_")* >
  | <INTLITERAL: <DIGIT> (<DIGIT>)* >
  | <LETTER: ["a"-"z"] | ["A"-"Z"]>
  | <DIGIT: ["0"-"9"]>
}
SKIP: {
  " "   |  "\n"   |    "\t"
}
```

--> get automatically generated code for lexer!

But how does javacc do it?

A Recap: Simple RE to Programs

Regular Expression

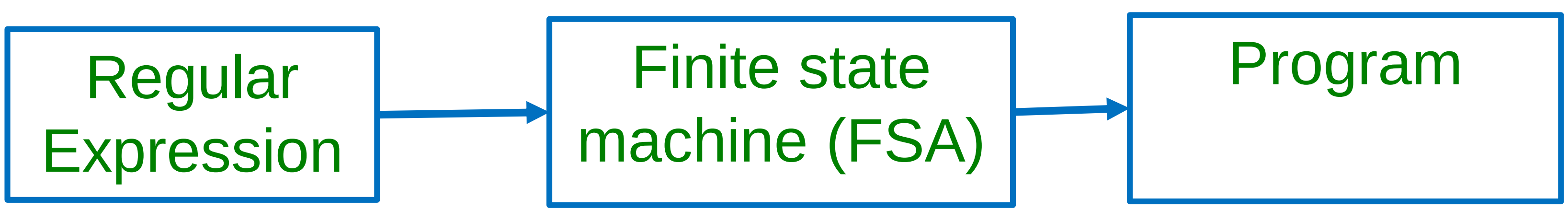
- a
- r1 r2
- (r1 **|** r2)
- r*****

Code

- **if** (current=a) next **else** error
- (code for r1) **;**
  (code for r2)
- **if** (current in first(r1))
    code for r1
  else
    code for r2
- **while**(current in first(r))
    code for r

Regular Expression to Programs

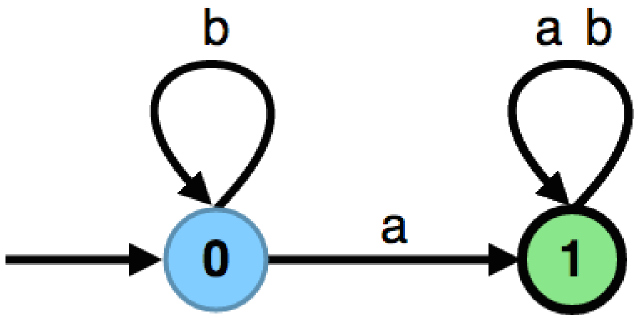
- How can we write a lexer for (a*b | a) ?
- aaaab Vs aaaaa



Finite Automaton (Finite State Machine)

- A = (Σ, Q, q_0, δ, F)

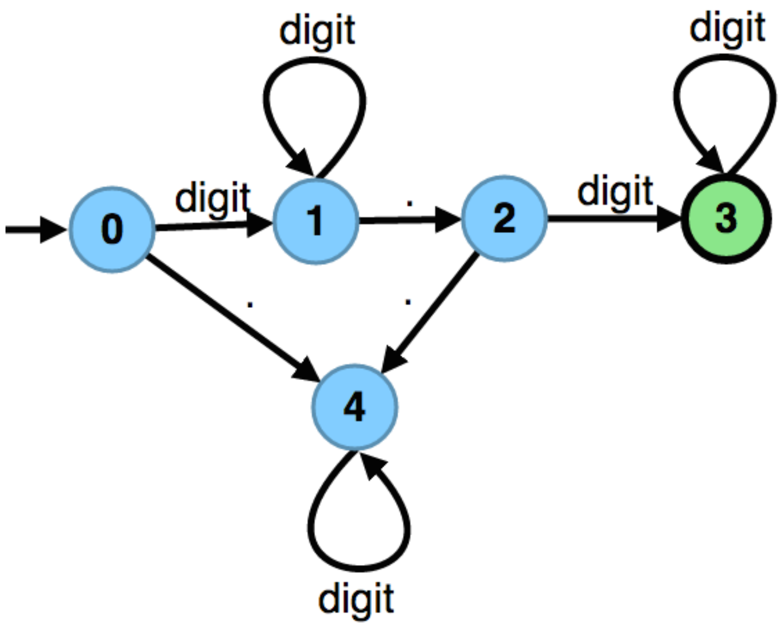
$$\delta \subseteq Q \times \Sigma \times Q,$$
$$q_0 \in Q,$$
$$F \subseteq Q$$



$$q_0 \in Q,$$
$$q_1 \subseteq Q$$
$$\delta = \{ (q_0, a, q_1), (q_0, a, q_0)$$
$$(q_1, a, q_1), (q_1, b, q_1)\}$$

- Σ - alphabet
- Q - states (nodes in the graph)
- q_0 - initial state (with ‘->' sign in drawing)
- δ - transitions (labeled edges in the graph)
- F - final states (double circles)

Numbers with Decimal Point



digit digit* . digit digit*

What if the decimal part is optional?

Automata Tutor
www.automatatutor.com

- A website for learning automata
- We have posted some exercises for you to try.
- Create an account for yourself
- Register to the course
  - Course Id: **23EPFL-CL**
  - Password: **GHL2AQ3I**

Exercise

- Design a DFA which accepts all strings in {a, b}* that has an even length

Exercise

- Construct an automaton that recognizes all strings over {a, b} that contain "aba" as a substring

Exercise

- Construct an automaton that recognizes all strings over { a,b} that contain "aba" as a substring and is of even length

Exercise

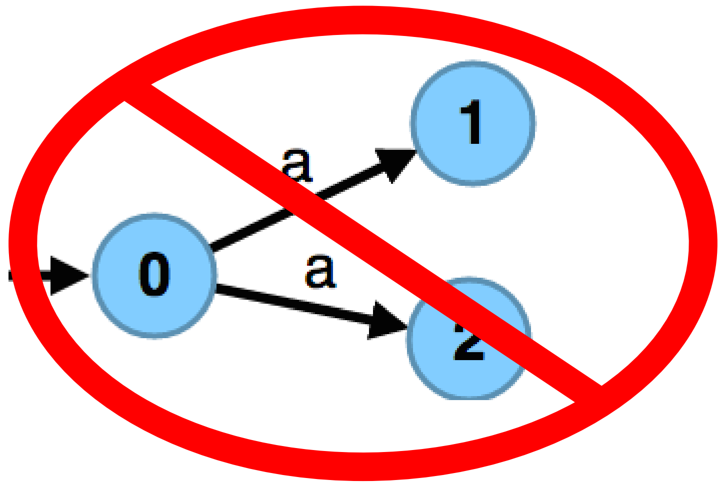
- Design a DFA which accepts all the numbers written in binary and divisible by 2. For example, your automaton should accept the words 0, 10, 100, 110…

Exercise

- Design a DFA which accepts all the numbers written in binary and divisible by 3. For example your automaton should accept the words 0, 11, 110, 1001, 1100 …

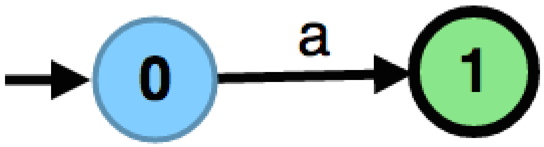
- Can you generalize this to any divisor ‘n’ ?
- Can you generalize this to any base ‘b’ ?

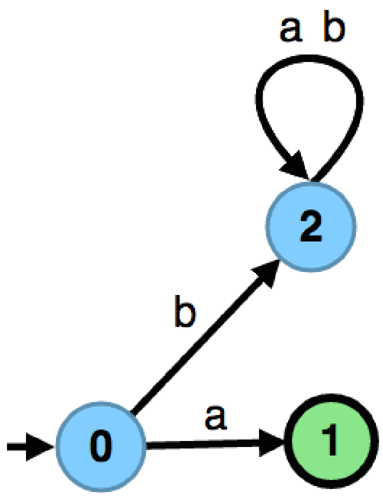
Kinds of Finite State Automata



- Deterministic FA (DFA): δ is a function : $(Q, \Sigma) \mapsto Q$
- Non-deterministic FA (NFA): δ could be a relation
- In NFA there is no unique next state. We have a set of possible next states.

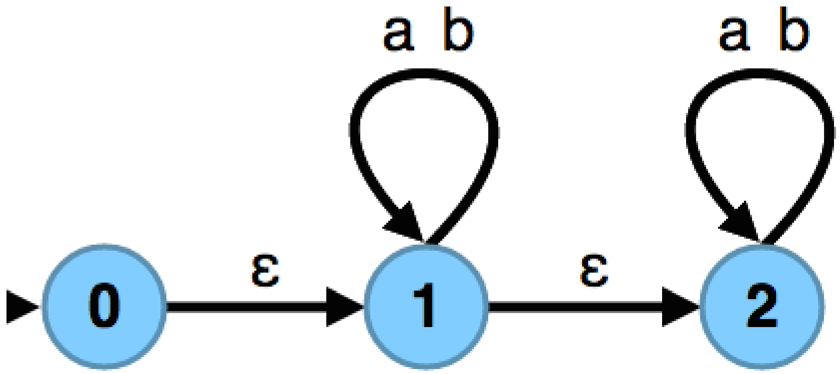
Undefined Transitions





- Undefined transitions lead to a sink state from where no input can be accepted

Epsilon Transitions



- Epsilon transitions: traversing them does not consume anything (empty word)
- More generally, transitions labeled by a word: traversing such transition consumes that entire word at a time

Interpretation of Non-Determinism

- For a given word (string), a path in automaton lead to accepting, another to a rejecting state
- Does the automaton accept in such case?
  - yes, if there **exists** an accepting path in the automaton graph whose symbols give that word

Exercise

- Construct a NFA that recognizes all strings over {a,b} that contain "aba" as a substring

NFA Vs DFA

- For every NFA there exists an equivalent DFA that accepts the same set of strings

- But, NFAs could be exponentially smaller.

- That is, there are NFAs such that every DFA equivalent to it has exponentially more number of states

Exercise

- Construct a NFA and a DFA that recognizes all strings over {a,b,c} that do not contain all the alphabets a, b and c.

  (let's start with a regular expression)

- Food for thought:
  - Can you prove that every DFA for this language will have exponentially more states than the NFA ?

Regular Expressions and Automata

Theorem:

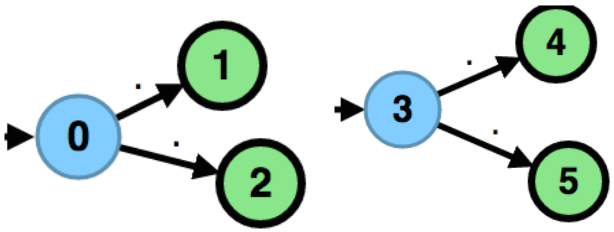
If L is a set of words, it is describable by a regular expression iff (if and only if) it is the set of words accepted by some finite automaton.

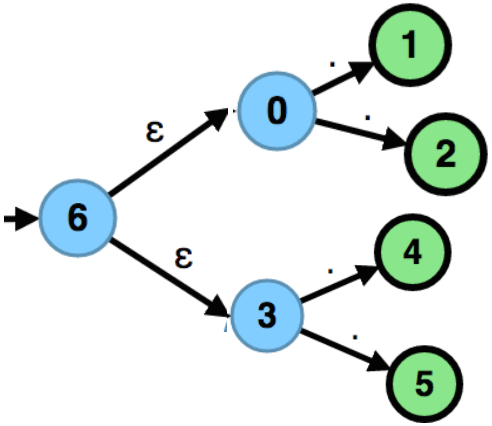
Algorithms:

- regular expression → automaton (important!)
- automaton → regular expression (cool)

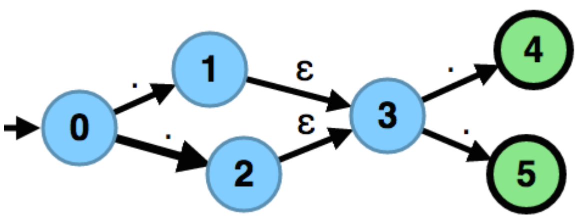
Recursive Constructions

- Union



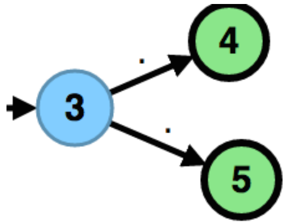


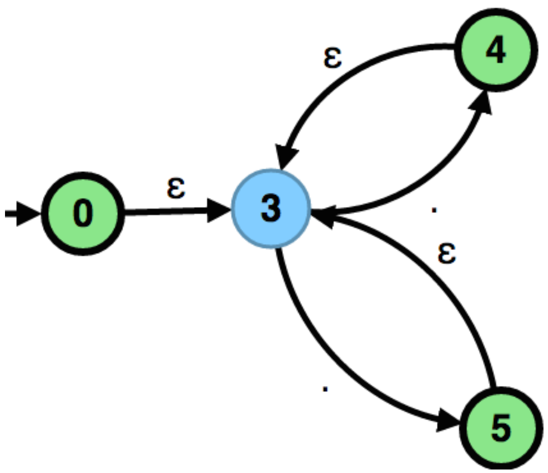
- Concatenation



Recursive Constructions

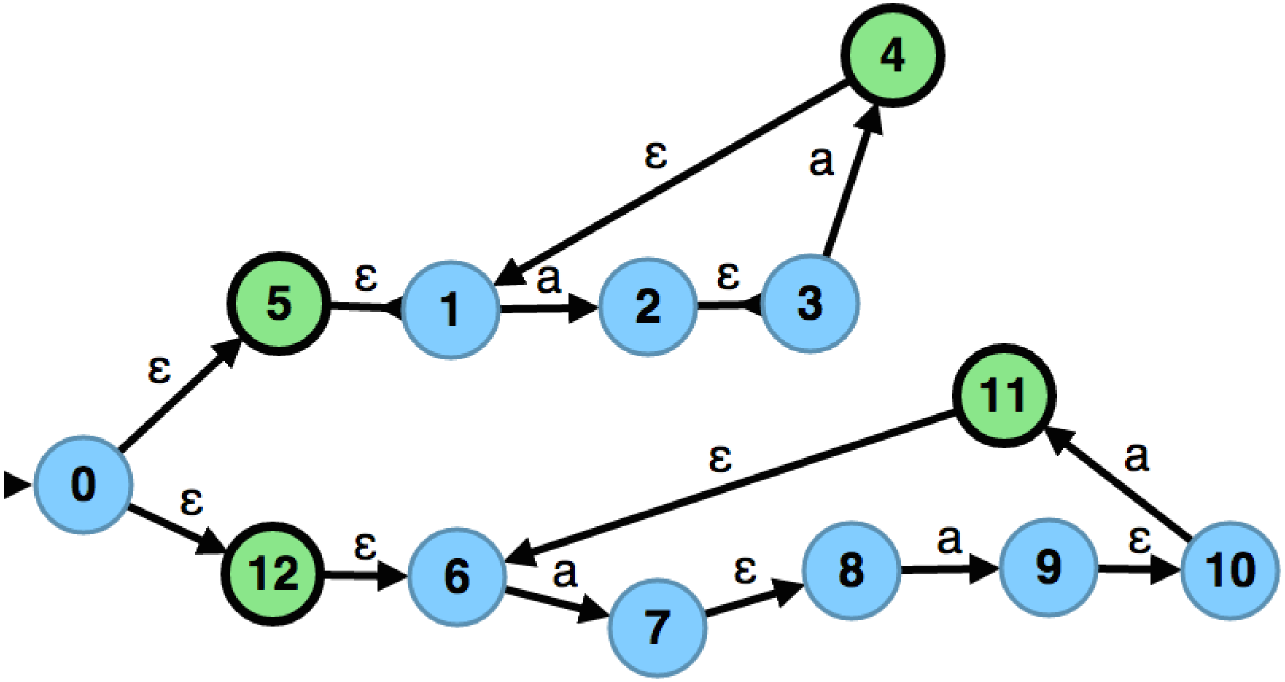
- Star





Exercise: (aa)* | (aaa)*

- Construct an NFA for the regular expression

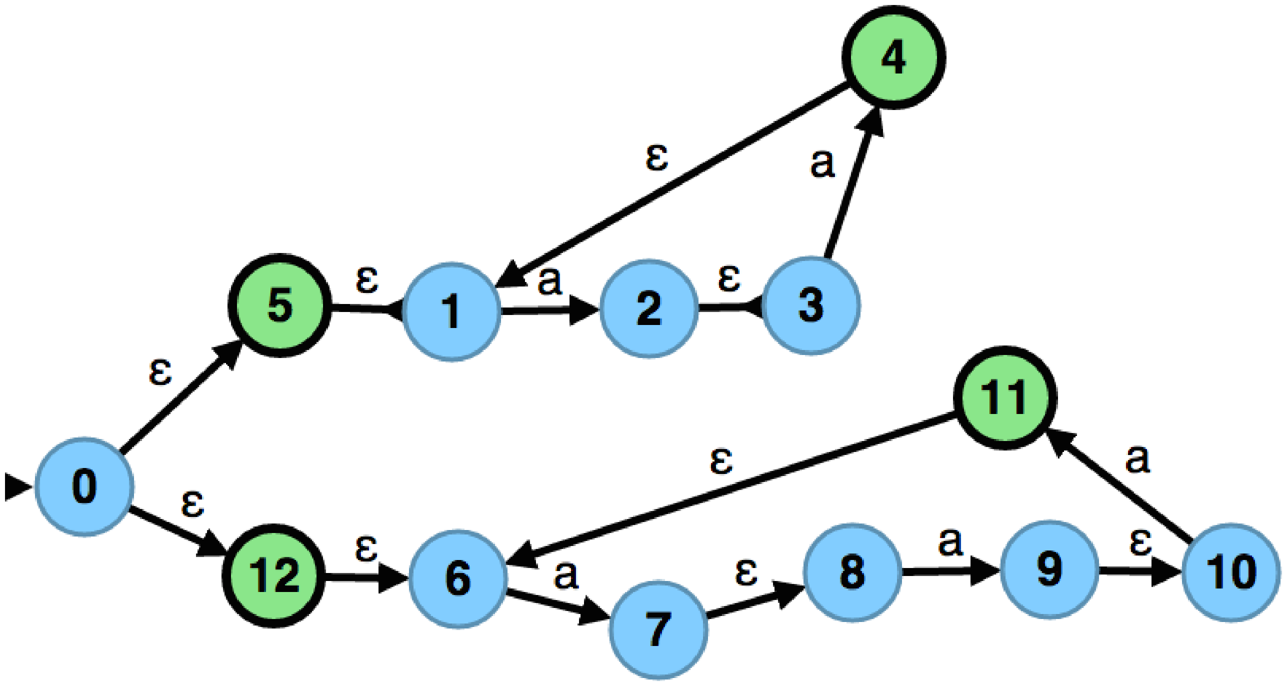


NFAs to DFAs (Determinisation)

- keep track of a set of all possible states in which the automaton could be

- view this finite set as one state of new automaton

NFA to DFA Conversion



Possible states of the DFA: 2^{Q}

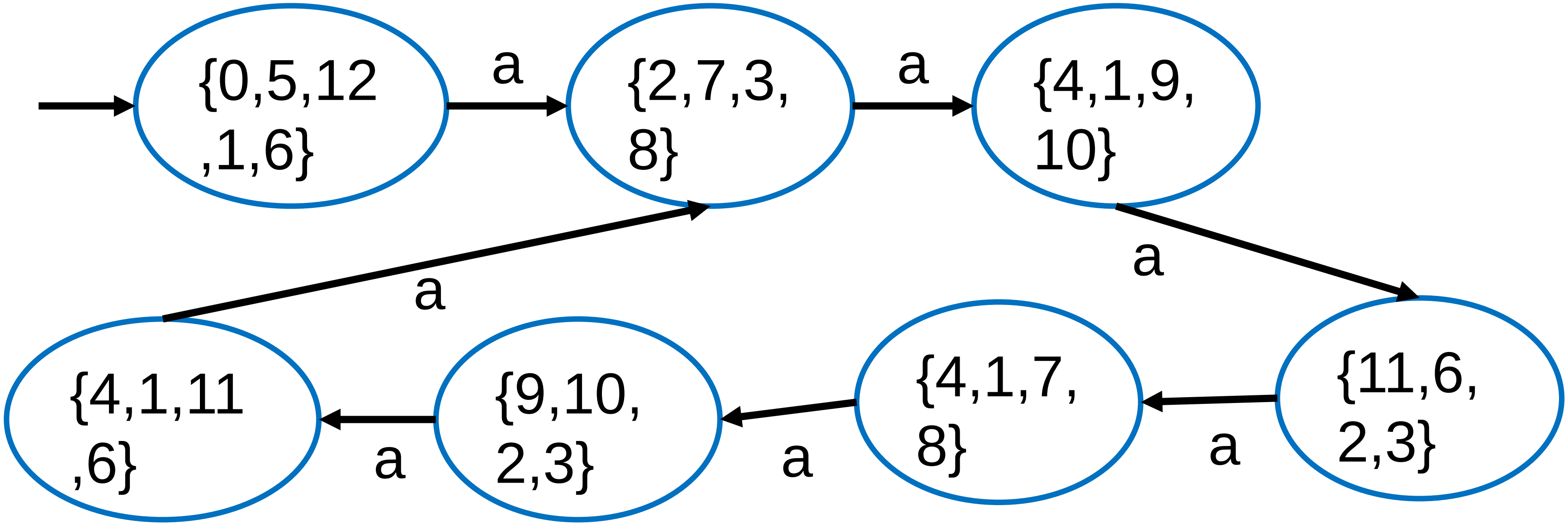
{ { } , { 0},…{12}, {0,1}, …,{0,12}, …{12, 12},
{0,1,2} …, { 0,1,2…,12 } }

NFA to DFA Conversion

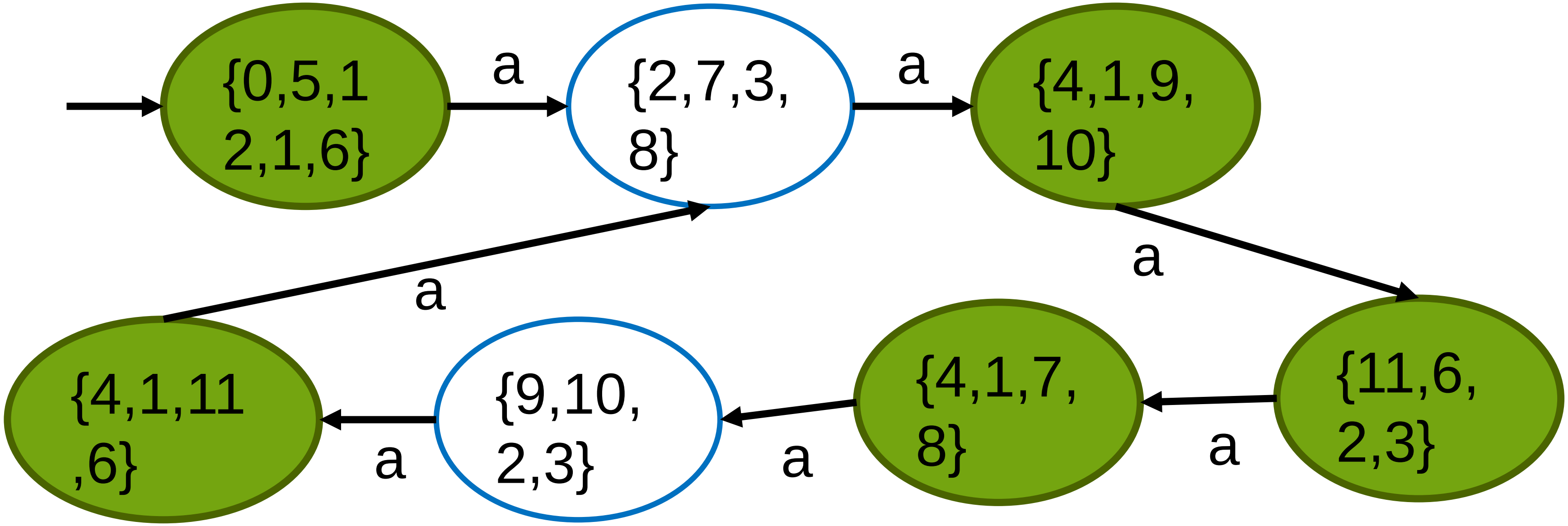
- Epsilon Closure
- E(0) = { 0,5,1,2,6}, E(1) = { 1}, E(2) = {
- $E(q) = \{\, q_1 \mid \delta(q, \epsilon, q_1) \,\}$

- NFA: $(\Sigma, 2^Q, q_0', \delta', F')$
- $q_0' = E(q_0)$
- $\delta'(q', a) = \bigcup_{\{\exists q_1 \in q', \delta(q_1, a, q_2)\}} E(q_2)$
- $F' = \{\, q' | q' \in 2^Q, q' \cap F \neq \emptyset\}$

NFA to DFA Conversion



NFA to DFA Example



Remark: Relations and Functions

- Relation $r \subseteq B \times C$

  $r = \{ \ldots, (b,c1) , (b,c2) ,\ldots \}$

- Corresponding function: $f : B \text{ -> } 2^C$

  $f = \{ \ldots (b,\{c1,c2\}) \ldots \}$

$f(b) = \{ c \mid (b,c) \in r \}$

- Given a state, next-state function returns the set of new states
  - for deterministic automaton, the set has exactly 1 element

Clarifications

- what happens if a transition on an alphabet 'a' is not defined for a state 'q' ?
- $\delta'(\{q\}, a) = \emptyset$
- $\delta'(\emptyset, a) = \emptyset$

- Empty set represents a state in the NFA
- It is a trap/sink state: a state that has self-loops for all symbols, and is non-accepting.

Running NFA (without epsilons) in Scala

```
def δ(q : State, a : Char) : Set[States] = { ... }
def δ'(S : Set[States], a : Char) : Set[States] = {
   for (q1 <- S, q2 <- δ(q1,a)) yield q2
}

def accepts(input : MyStream[Char]) : Boolean = {
  var S : Set[State] = Set(q0) // current set of states
  while (!input.EOF) {
    val a = input.current
    S = δ'(S,a)             // next set of states
  }
  !(S.intersect(finalStates).isEmpty)
}
```

Running NFA in Scala

- Modify this to handle epsilons transitions.

```
def δ(q : State, a : Char) : Set[States] = { ... }
def δ'(S : Set[States], a : Char) : Set[States] = {
  for (q1 <- S, q2 <- δ(q1,a))
        for(q <- δ(q2, ϵ)) yield q
}
```

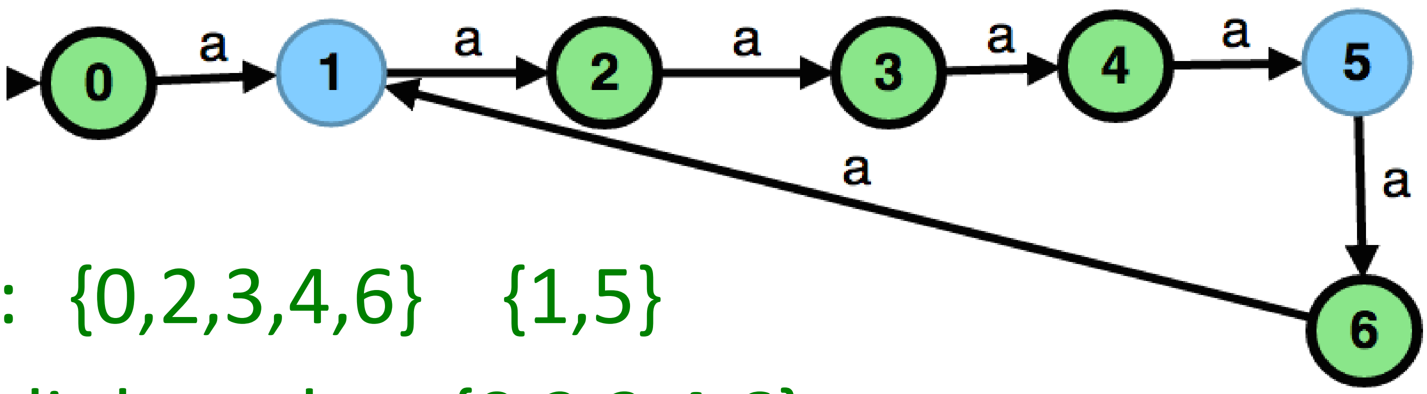
Minimizing DFAs

- Merge equivalent states.
  - q_0 and q_1 are equivalent iff there is no distinguishing string
  - $\hat{\delta}(q_0, z) \in F \Leftrightarrow \hat{\delta}(q_1, z) \in F$
  - Corollary of *Myhill-Nerode Theorem*
- Final and non-final states are not equivalent as ϵ distinguishes them

Minimizing DFAs: Procedure

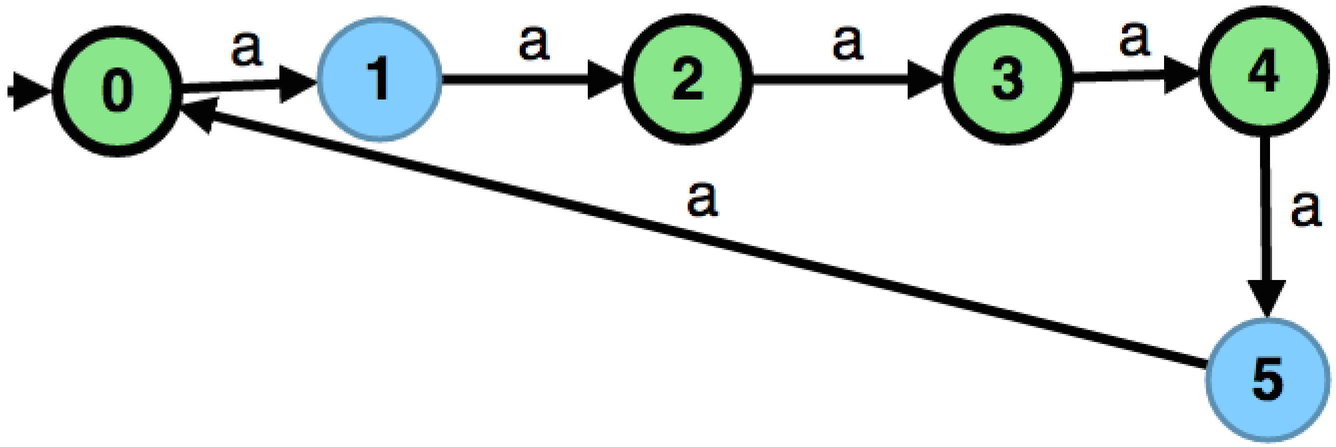
- Maintain a partition A of states
- Every set in the partition has a different behavior i.e, they have a distinguishing string
- States within a partition may or may not be equivalent
- Initially, we have (F, Q - F)

Minimizing DFAs: Procedure



- A: {0,2,3,4,6} {1,5}
- split based on {0,2,3,4,6}
  - A: {0,4,6} {2,3} {1} {5}
- split based on {1}
  - A: {0,6} {4} {2,3} {1} {5} {6}
- split based on {4}
  - A: {0,6} {4} {2} {3} {1} {5} {6}

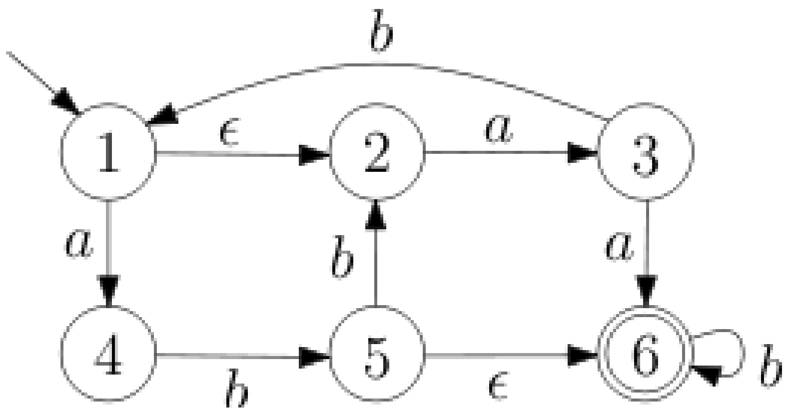
Minimizing DFAs: Procedure

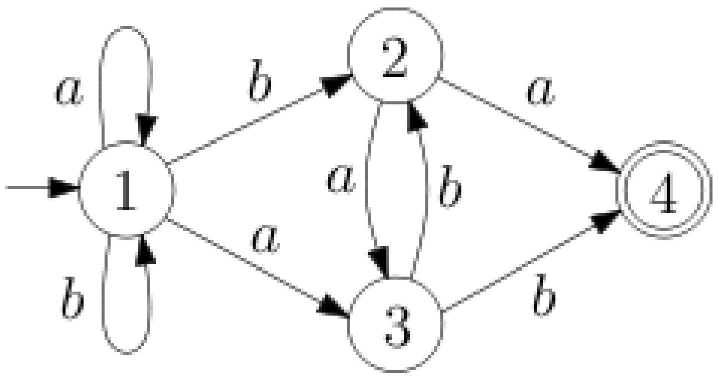


- The minimal DFA is unique (up to isomorphism)
- Implication of *Myhill-Nerode theorem*
- *Food For Thought: Can we minimize NFA ?*

Exercise

Convert the following NFAs to deterministic finite automata.

a) 

b) 

Properties of Automatons

- **Complement:** $(\Sigma^* \setminus L(A))$
  - switch accepting and non-accepting states in **deterministic automaton**
  - Does not work for non-deterministic automatons
- **Intersection:** $L(A_1) \cap L(A_2)$
  - $(\Sigma, Q_1 \times Q_2, (q_0^1, q_0^2), \delta', \ F_1 \times F_2)$
  - $\delta'\big((q_1, q_2), a\big) = \delta(q_1, a) \times \delta(q_2, a)$

Properties of Automatons

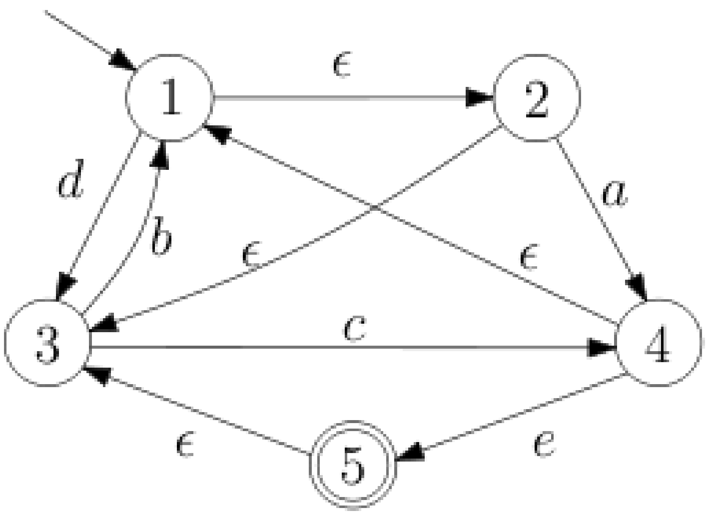
- **Intersection:**
  - complement union of complements
- **Set difference**: intersection with complement
- **Inclusion**: emptiness of set difference
- **Equivalence:** two inclusions

Exercise

- Design a DFA which accepts all the numbers written in binary and divisible by 6. For example your automaton should accept the words 0, 110 (6 decimal) and 10010 (18 decimal).

Exercise: first, nullable

- For each of the following languages find the *first* set. Determine if the language is *nullable*.
  - (a|b)* (b|d) ((c|a|d)* | a*)
  - language given by automaton:



Automated Construction of Lexers

- let $r_1, r_2, \ldots, r_n$ be regular expressions for token classes
- consider combined regular expression: $(r_1 \mid r_2 \mid \ldots \mid r_n)$
- recursively map a regular expression to a non-deterministic automaton
- eliminate epsilon transitions and determinize
- optionally minimize A_3 to reduce its size $\rightarrow A_4$
- **the result only checks that input can be split into tokens, does not say how to split it**

From $(r_1 | r_2 | \ldots | r_n)$ to a Lexer

- For each accepting state of r_i specify the token class *i* being recognized
- **Longest match rule:** remember last token and input position for a last accepted state
- When no accepting state can be reached (effectively: when we are in a trap state)
  - revert position to last accepted state
  - return last accepted token
- *Why can't we simply use $(r_1|r_2|\ldots|r_n)^*$?*

Exercise

Build lexical analyzer for the following two tokens using longest match. The first token class has a higher priority:

binaryDigit ::= $(\mathbf{z}|1)^*$

ternaryDigit ::= $(0|1|2)^*$

1111z1021z1 →

Realistic Exercise: Integer Literals of Scala

- Integer literals are in three forms in Scala: decimal, hexadecimal and octal. The compiler discriminates different classes from their beginning.
  - Decimal integers are started with a non-zero digit.
  - Hexadecimal numbers begin with 0x or 0X and may contain the digits from 0 through 9 as well as upper or lowercase digits from A to F.
  - If the integer number starts with zero, it is in octal representation so it can contain only digits 0 through 7.
  - l or L at the end of the literal shows the number is Long.
- Draw a single DFA that accepts all the allowable integer literals.
- Write the corresponding regular expression.

Exercise

- Let L be the language of strings over {<, =} defined by regexp (<|=| <====*). That is, L contains <,=, and words <$=^n$ for n >= 3.
- Construct a DFA that accepts L
- Describe how the lexical analyzer will tokenize the following inputs.

  1) <=====

  2) ==<==<==<==<==

  3) <=====<

More Questions

- For which of the following languages can you find an automaton or regular expression:
  - Sequence of open or closed parentheses of even length? E.g. (), ((,)),)()))(, ...
  - as many digits before as after decimal point?
  - Sequence of balanced parentheses

    ((()) ()) - balanced

    ( )) (( ) - not balanced
  - Comments from // until LF
  - Nested comments like /* ... /* */ ... */