

Exercises on Grammars

1. Consider the following grammar:

$S \rightarrow (L) \mid a$

$L \rightarrow L , S \mid S$

- Is this grammar ambiguous ?
- Is this grammar LL(1) ?
- Compute the First and Follow sets for the new grammar.
- Construct the parsing table for the LL(1) parser

Finding an LL(1) grammar

- No procedural way ! Practice ...
- But there are some recommended practices that generally help in finding one.
- Eg. try to eliminate left recursion.
 - There is a procedure for this but you don't have to faithfully follow the entire approach.
 - Just think of what left recursion brings and what can be done to eliminate them

Removing Left Recursion

$S \rightarrow (L) \mid a$

$L \rightarrow L, S \mid S$

- How does a derivation starting from 'L' look ?

- $L \Rightarrow L, S$

$\Rightarrow L, S, S$

$\Rightarrow^* L, S, \dots, S$

$\Rightarrow S, \dots, S$

- $L \rightarrow L, S \mid S$ is equivalent to $L \rightarrow S, L \mid S$

$S \rightarrow (L) \mid a$

$L \rightarrow S, L \mid S$

Removing Left Recursion

- In general, $L \rightarrow L \alpha \mid \beta_1 \mid \dots \mid \beta_n$
 - $L \rightarrow \beta_1 Z \mid \dots \mid \beta_n Z$
 - $Z \rightarrow \alpha Z \mid \epsilon$
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- This will remove immediate recursion
 - But, what if
 - $S \rightarrow L a$
 - $L \rightarrow S a \mid b$

Removing Left Recursion

- Order nonterminals Eg. (1) S , (2) L
- Enforce that if $A \rightarrow B$ then A should precede B in the ordering
- $S \rightarrow L a$ and $L \rightarrow b$ satisfy the constraint but $L \rightarrow S a$ doesn't
- Inline the production of S in $L \rightarrow S a$
- $L \rightarrow L a a$
 - Remove left recursion. Result ??
 - If this wasn't left recursive or doesn't satisfy the constraints, inline again.

Example 1 [Cont.]

$S \rightarrow (L) \mid a$

$L \rightarrow L , S \mid S$

- After eliminating left recursion

$S \rightarrow (L) \mid a$

$L \rightarrow S , L \mid S$

- Is this LL(1) now ?

Example 1 [Cont.]

$S \rightarrow (L) \mid a$

$L \rightarrow L , S \mid S$

- After eliminating left recursion

$S \rightarrow (L) \mid a$

$L \rightarrow S , L \mid S$

- Is this LL(1) now ?

Left factorization

$S \rightarrow (L) \mid a$

$L \rightarrow S , L \mid S$

- Identify a common prefix and push the suffixes to a new nonterminal.

$S \rightarrow (L) \mid a$

$L \rightarrow S Z$

$Z \rightarrow , L \mid \epsilon$

- Is this LL(1) now ?

Exercise 2

Consider a grammar for expressions where the multiplication sign is optional.

$ex ::= ex + ex \mid ex * ex \mid ex \ ex \mid ID$

- Find a LL(1) grammar recognizing the same language
- Using your grammar to derive a string
- Create the LL(1) parsing table.

Exercise 3

Balanced Parentheses over $\{ (, [\}$

$S ::= (S) \mid [S] \mid S S \mid \epsilon$

- Find a LL(1) grammar recognizing the language

Exercise 4

Prove that every LL(1) grammar is unambiguous.

Exercise 5

Say that a grammar has a cycle if there is a *reachable, productive* non-terminal A such that $A \Rightarrow^* A$, i.e. it is possible to derive the nonterminal A from A by a nonempty sequence of production rules.

Show that if a grammar has a cycle, then it is not LL(1).

Exercise 6

Show that the regular languages can be recognized with LL(1) parsers. Describe a process that, given a regular expression, constructs an LL(1) parser for it.

- a
- Concatenation: $r_1 r_2$
- Union: $r_1 \mid r_2$
- Closure: r_1^*