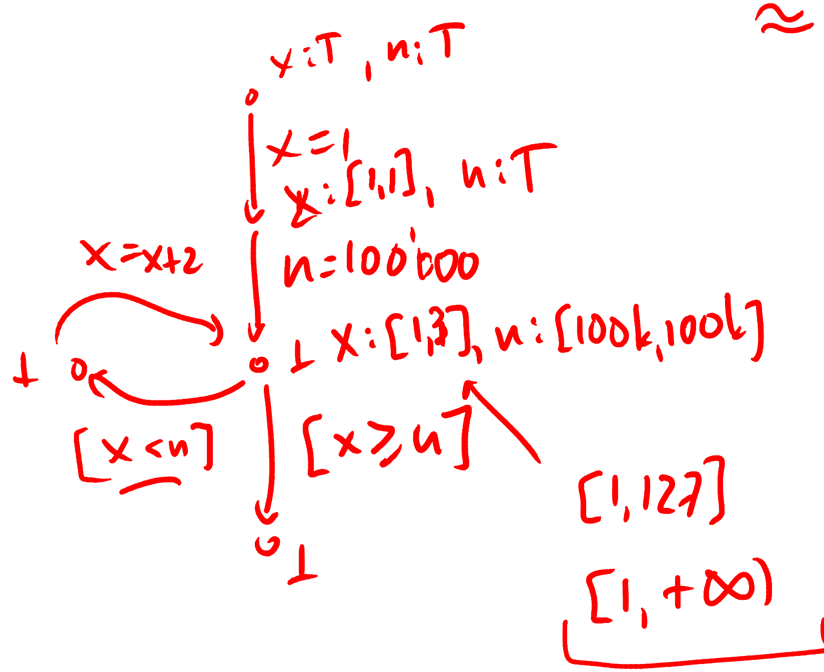
# How Long Does Analysis Take?

# Handling Loops: Iterate **Until Stabilizes**

How many steps until it stabilizes? Compute.

≈50'000 steps

```
x = 1
n = 100000
while (x < n) {
  x = x + 2
}
```



# Handling Loops: Iterate **Until Stabilizes**

How many steps until it stabilizes?

x: [0, +∞)   x ≥ 0

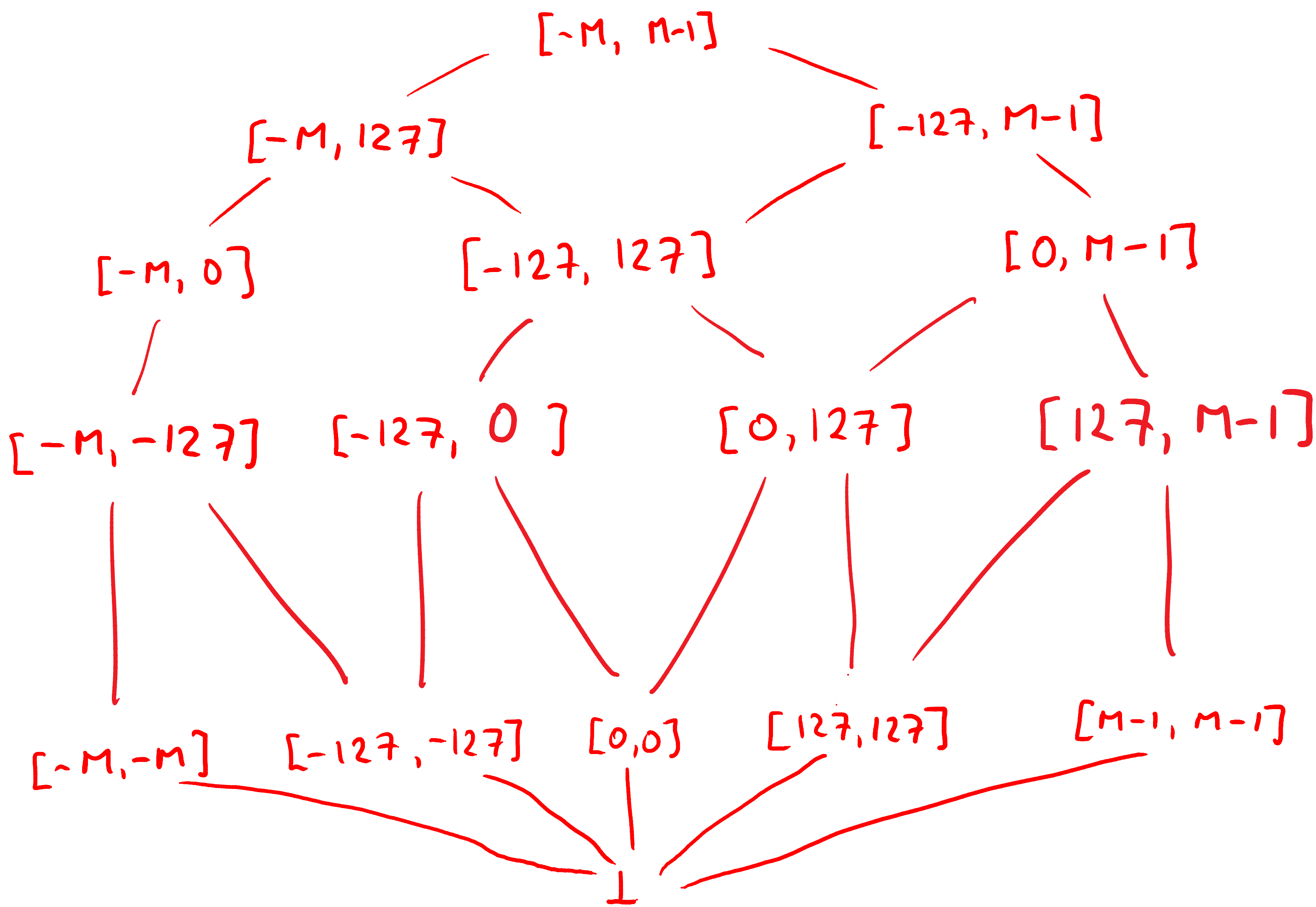
```
var x : BigInt = 1
var n : BigInt = readInput()
while (x < n) {
  x = x + 2
}
```

For unknown program inputs and unbounded domains it may be practically impossible to know how long it takes.

Solutions

- smaller domain, e.g. only certain intervals [a,b] where a,b in {-**MI**,-127,-1,0,1,127,**MI**-1}
- *widening* techniques (make it less precise on demand)

# Smaller domain: intervals [a,b] where a,b$\in${-**MI**,-127,0,127,**MI**-1} (**MI** denoted M)



# Size of analysis domain

**Interval analysis:**

$D_1 = \{ [a,b] \mid a \le b,\ a,b \in \{-M,-127,-1,0,1,127,M-1\}\} \cup \{\perp\}$

**Constant propagation:**

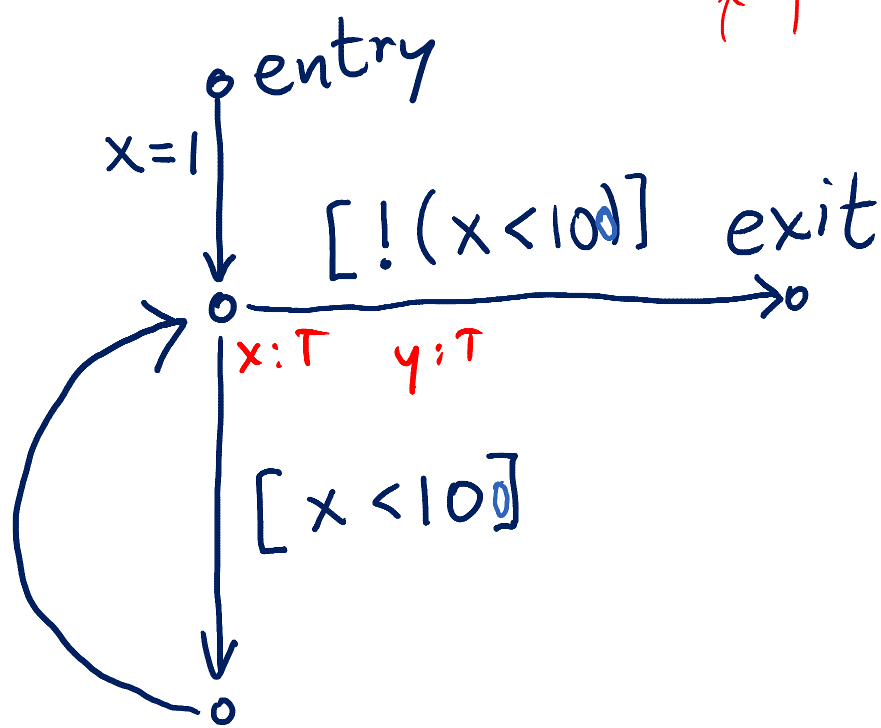
$D_2 = \{ [a,a] \mid a \in \{-M,-(M-1),\ldots,-2,-1,0,1,2,3,\ldots,M-1\}\} \cup \{\perp, T\}$

suppose M is $2^{63}$

$|D_1| = 1 + (7+6+5+\ldots+1)$

$|D_2| = 1 + 2^{64} + 1$

How many steps until it stabilizes, for any program with one variable?



# How many steps does the analysis take to finish (converge)?

**Interval analysis:**

$D_1 = \{ [a,b] \mid a \leq b,\ a,b \in \{-M,-127,-1,0,1,127,M-1\}\} \cup \{\perp\}$

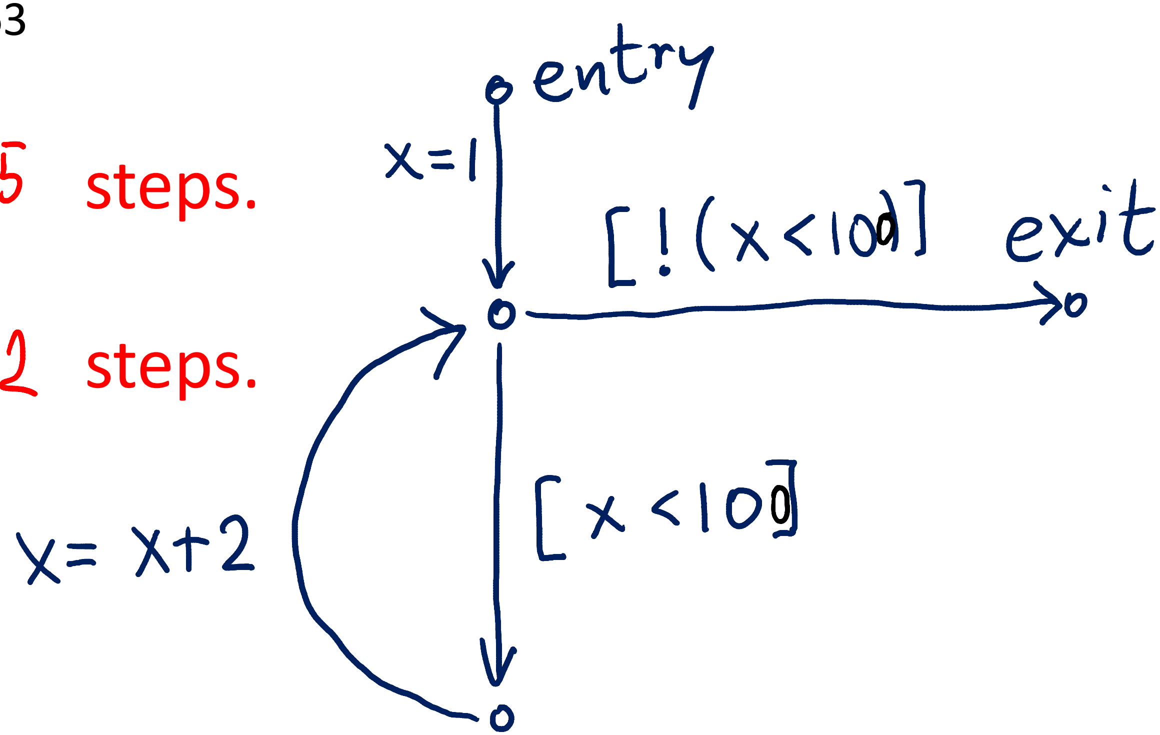
**Constant propagation:**

$D_2 = \{ [a,a] \mid a \in \{-M,-(M-1),\ldots,-2,-1,0,1,2,3,\ldots,M-1\}\} \cup \{\perp,T\}$

suppose M is $2^{63}$

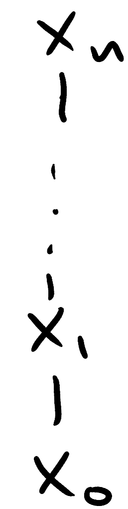
With $D_1$ takes at most 5 steps.

With $D_2$ takes at most 2 steps.



# Chain of length n

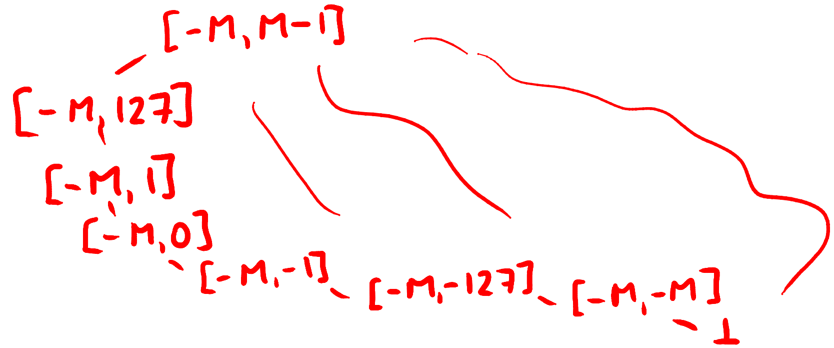
- A set of elements $x_0, x_1, \ldots, x_n$ in D that are linearly ordered, that is $x_0 \leq x_1 \leq \ldots \leq x_n$
- A lattice can have many chains. Its **height** is the maximum n for all the chains, if finite
- If there is no upper bound on lengths of chains, we say lattice has infinite height
- A monotonic sequence of distinct elements has length at most equal to lattice height



# Termination Given by Length of Chains

**Interval analysis:**

$D_1 = \{ [a,b] \mid a \leq b,\ a,b \in \{-M,-127,-1,0,1,127,M-1\}\} \cup \{\perp\}$

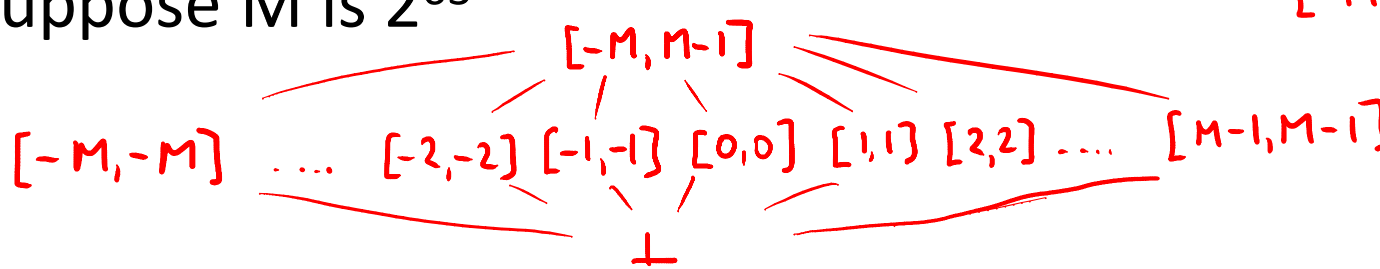



**Constant propagation:**

$D_2 = \{ [a,a] \mid a \in \{-M,\ldots,-2,-1,0,1,2,3,\ldots,M-1\}\} \cup \{\perp,T\}$

suppose M is $2^{63}$

[-M, M-1]



# Product Lattice for All Variables

- If we have N variables, we keep one element for each of them
- This is like N-tuple of variables
- Resulting lattice is product of N lattices for individual variables
- Size is $\underbrace{|D|^{N}}$ $\quad (2+2^{64})^{20}$
- The height is only N times the height of D

$x: \bot \quad y: \bot$

$x: [0,0] \quad y: \bot$

$x: \top \quad y: \bot$

$x: \top, \quad y: [1,1]$

$x: \top, \quad y: \top$

Max height : $h(D) \cdot |V| \cdot |nodes|$ $\quad 2 \cdot \underbrace{20}$

$(\bot, \bot) \in D^2$

$D^N$

# Summary and More Examples of Abstract Interpretation

# Unbounded Range Analysis

**Also called interval analysis** **Z** - integers

$D = \{\perp, T\} \cup \{ [a,b] \mid a,b \in \mathbf{Z}\} \cup$
$\{(-\infty,b] \mid b \in \mathbf{Z}\} \cup \{[a,\infty) \mid a \in \mathbf{Z}\}$

So domain values are:

- bounded intervals of integers [a,b]
- intervals unbounded from one side (-∞,b] , [a,∞)
- empty set ⊥
- the set of all integers T

Convergence not ensured automatically – can increase intervals forever

- even for e.g. 32-bit integers, convergence can take many steps

# Range Analysis of N-bit integers

$\mathbf{B}_{32}$ – 32-bit integers

$D = \{\perp, T\} \cup \{ [a,b] \mid a,b \in \mathbf{B}_{32}\} \cup$
$\{(-\infty,b] \mid b \in \mathbf{B}_{32}\} \cup \{[a,\infty) \mid a \in \mathbf{B}_{32}\}$

What is the height of the lattice for 3 variables and 7 program points?

# Constant Propagation

**Special case of interval analysis:**

D = { [a,a] | a $\in$ {…,-2,-1,0,1,2,3,…}} U {$\perp$,T}

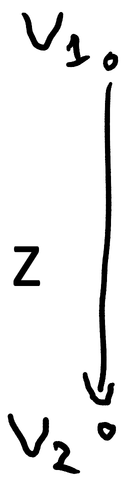
Write [a,a] simply as a. So values are:

- a known constant at this point: a
- "we could not show it is constant": T
- "we did not reach this program point": $\perp$

Convergence fast - lattice has small height

# Transfer Function for Plus

$x = y + z$




For each variable (x,y,z) we store a constant $\perp$, or T

**table for +:**

| z \ y | $\perp$ | $c_y$ | T |
|---|---|---|---|
| $\perp$ | | | |
| $c_z$ | | | |
| T | | | |

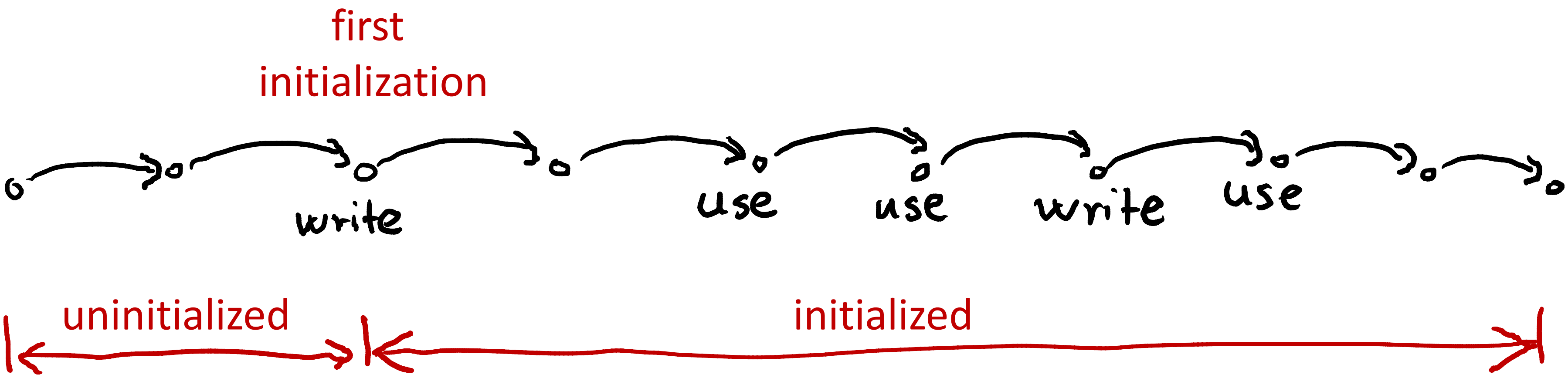
```
abstract class Element
case class Top extends Element
case class Bot extends Element
case class Const(v:Int) extends Element
var facts : Map[Nodes,Map[VarNames,Element]]
```

what executes during analysis:

```
oldY = facts(v1)("y")
oldZ = facts(v1)("z")
newX = tableForPlus(oldY, oldZ)
facts(v2) = facts(v2) join facts(v1).updated("x", newX)
```

```
def tableForPlus(y:Element, z:Element) =
(x,y) match {
case (Const(cy),Const(cz)) => Const(cy+cz)
case (Bot,_) => Bot
case (_,Bot) => Bot
case (Top,Const(cz)) => Top
case (Const(cy),Top) => Top
}
```

# Initialization Analysis



# What does javac say to this:

```
class Test {
    static void test(int p) {
        int n;
        p = p - 1;
        if (p > 0) {
            n = 100;
        }
        while (n != 0) {
            System.out.println(n);
            n = n - p;
        }
    }
}
```

```
Test.java:8: variable n might not have been initialized
        while (n > 0) {
        ^
1 error
```

# Program that compiles in java

```
class Test {
  static void test(int p) {
      int n;
      p = p - 1;
      if (p > 0) {
        n = 100;
      }
      else {
        n = -100;
      }
      while (n != 0) {
        System.out.println(n);
        n = n - p;
      }
  }
} // Try using if (p>0) second time.
```

We would like variables to be initialized on all execution paths.

Otherwise, the program execution could be undesirable affected by the value that was in the variable initially.

We can enforce such check using initialization analysis.

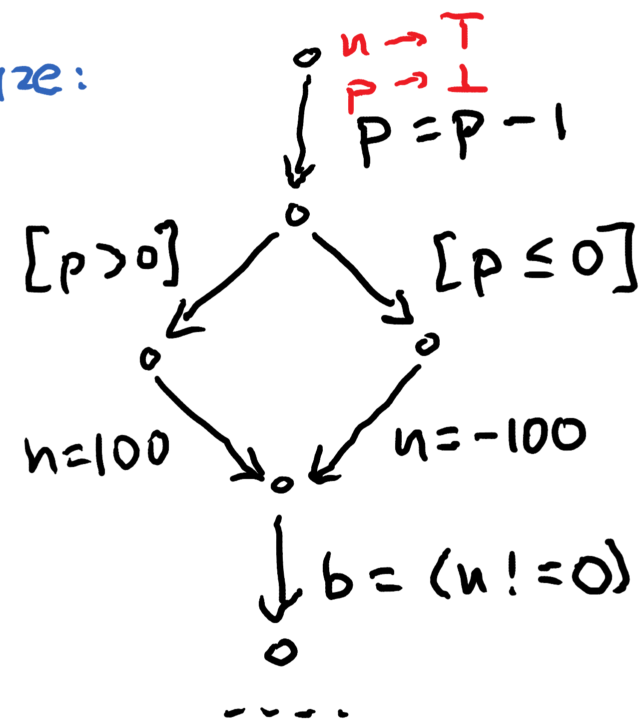
# Initialization Analysis

```
class Test {
  static void test(int p) {
      int n;
      p = p - 1;
      if (p > 0) {
        n = 100;
      }
      else {
        n = -100;
      }
      while (n != 0) {
        System.out.println(n);
        n = n - p;
      }
  }
} // Try using if (p>0) second time.
```

T indicates presence of flow from states where variable was not initialized:

- If variable is possibly uninitialized, we use T
- Otherwise (initialized, or unreachable): ⊥

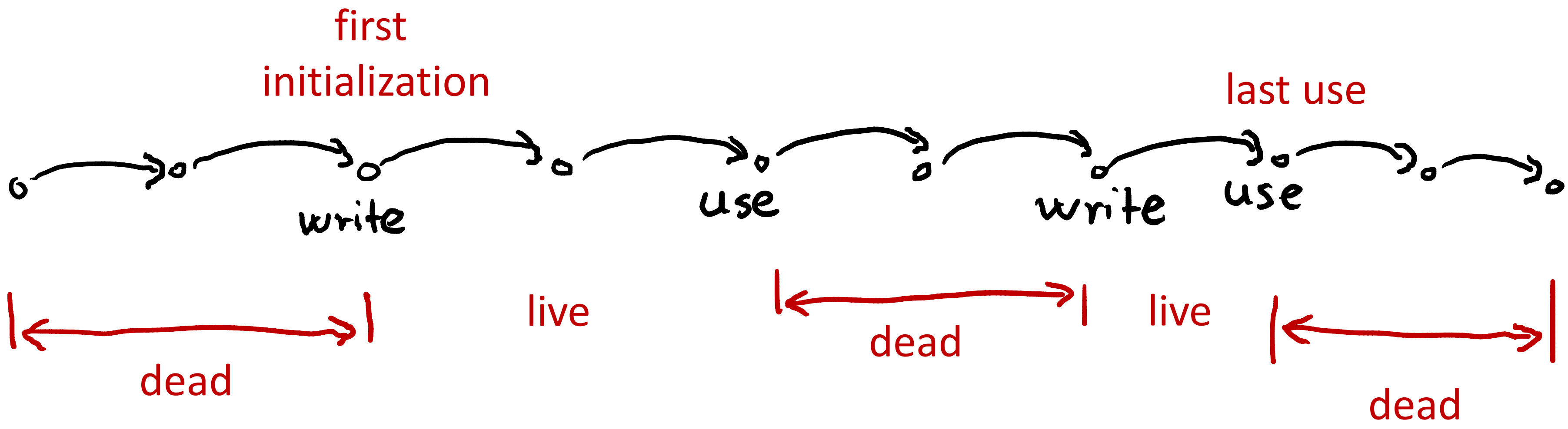
analyze:




If var occurs anywhere but left-hand side of assignment and has value T, report error

# Liveness Analysis

Variable is dead if its current value will not be used in the future. If there are no uses before it is reassigned or the execution ends, then the variable is sure dead at a given point.



# What is Written and What Read

x = y + x

written

read

if (x > y)

## Example:

$\{z\}$

$x = 42$

$\{x, z\}$

$y = x + 3$

$\{x, y, z\}$

$z = y + z$

$\{x\}$

$y = 3 + x$

$\emptyset$

## Purpose:

Register allocation:

find good way to decide which variable should go to which register at what point in time.

# How Transfer Functions Look

$L_i$ - set of live variables

$L_0$

$x = x + y$ (st)

$L_2$

$L_0 = L_1 \cup \{x, y\}$

read(x,y)

$L_1 = L_2 \setminus \{x\}$

write(x)

$L_2$

$$L_0 = (L_2 \setminus \{x\}) \cup \{x, y\}$$

Generally

$$L_0 = (L_2 \setminus \mathrm{def}(st)) \cup \mathrm{use}(st)$$

# Initialization: Forward Analysis

```
while (there was change)
   pick edge (v1,statmt,v2) from CFG
           such that facts(v1) has changed
   facts(v2)=facts(v2) join transferFun(statmt, facts(v1))
}
```

# Liveness: Backward Analysis

```
while (there was change)
   pick edge (v1,statmt,v2) from CFG
           such that facts(v2) has changed
   facts(v1)=facts(v1) join transferFun(statmt, facts(v2))
}
```

# Example

```
x = m[0]
y = m[1]
xy = x * y
z = m[2]
yz = y*z
xz = x*z
res1 = xy + yz
m[3] = res1 + xz
```

# Data Representation Overview

# Original and Target Program have Different Views of Program State

- Original program:
  - local variables given by names (any number of them)
  - each procedure execution has fresh space for its variables (even if it is recursive)
  - fields given by names
- Java Virtual Machine
  - local variables given by slots (0,1,2,…), any number
  - intermediate values stored in operand stack
  - each procedure **call** gets fresh slots and stack
  - fields given by names and object references
- **Machine code:** program state is a large arrays of bytes and a finite number of registers

# Compilation Performs Automated Data Refinement

s

x → 0
y → 5
z → 8

Data Refinement Function **R**

1 → 0
2 → 5
3 → 8

c

P: **x = y + 7 * z**

s'

x → 61
y → 5
z → 8

[[**x = y + 7 * z**]] =

iload 2
iconst 7
iload 3
imul
iadd
istore 1

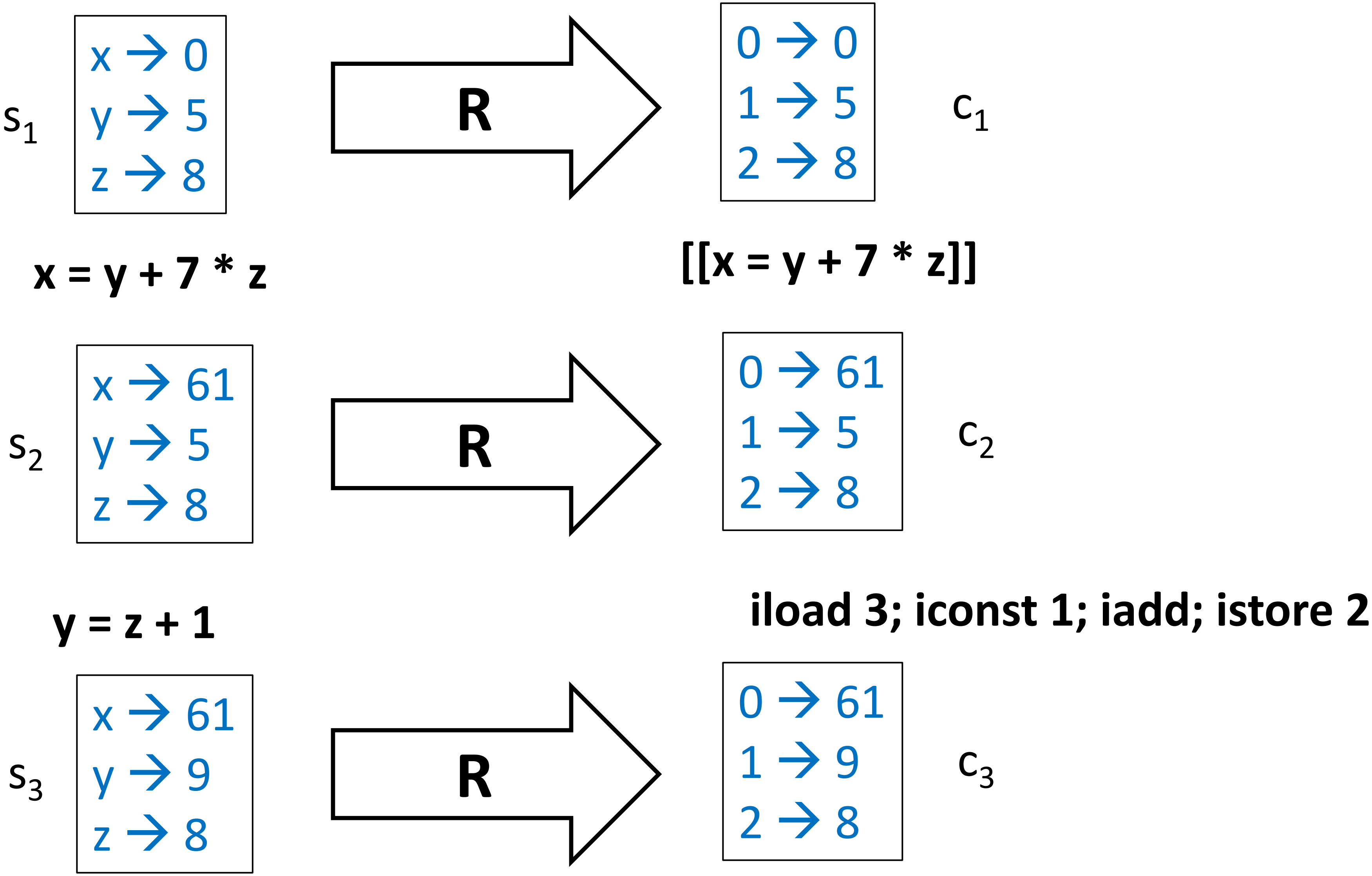
**R** maps s to c and s' to c'

1 → 61
2 → 5
3 → 8

c'

If interpreting program P leads from s to s' then running the compiled code [[P]] leads from R(s) to R(s')

# Inductive Argument for Correctness




(R may need to be a relation, not just function)

# A Simple Theorem

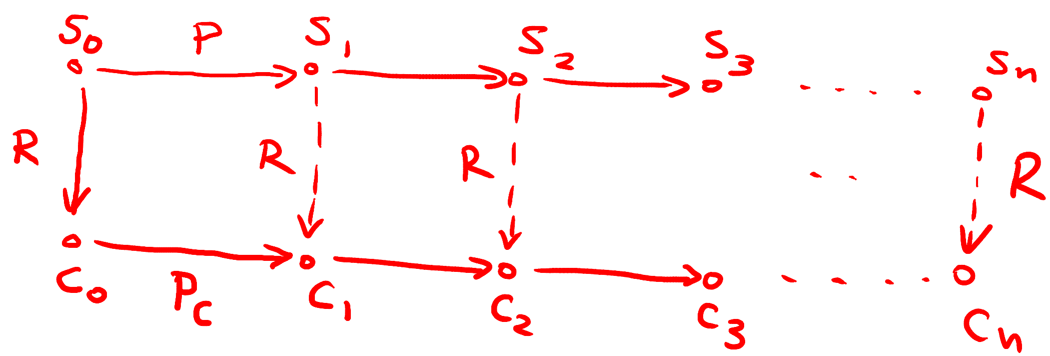
$P : S \rightarrow S$ is a program meaning function

$P_c : C \rightarrow C$ is meaning function for the compiled program

$R : S \rightarrow C$ is data representation function

Let $s_{n+1} = P(s_n)$, $n = 0,1,\ldots$ be interpreted execution

Let $c_{n+1} = P(c_n)$, $n = 0,1,\ldots$ be compiled execution




**Theorem:** If

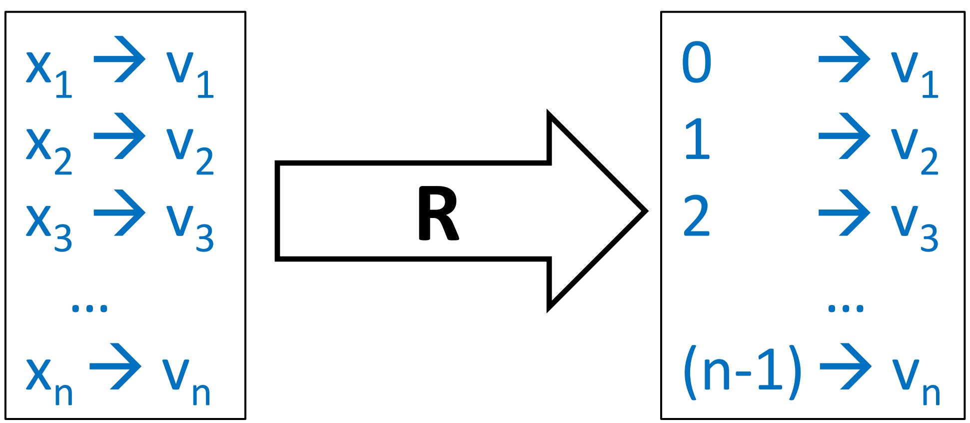
- $c_0 = R(s_0)$
- for all s, $P_c(R(s)) = R(P(s))$

then $c_n = R(c_n)$ for all n.

**Proof:** immediate, by induction. R is often called **simulation relation.**

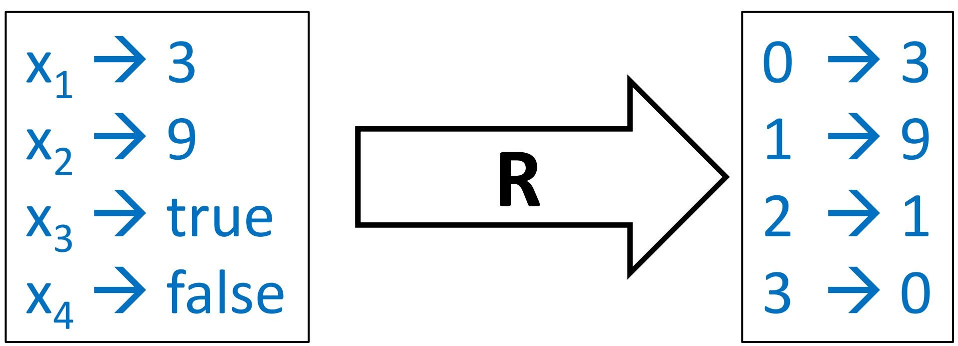
# Example of a Simple R

- Let the received, the parameters, and local variables, in their order of declaration, be $x_{1,}\ x_2 \ldots x_n$
- Then R maps program state with only integers like this:



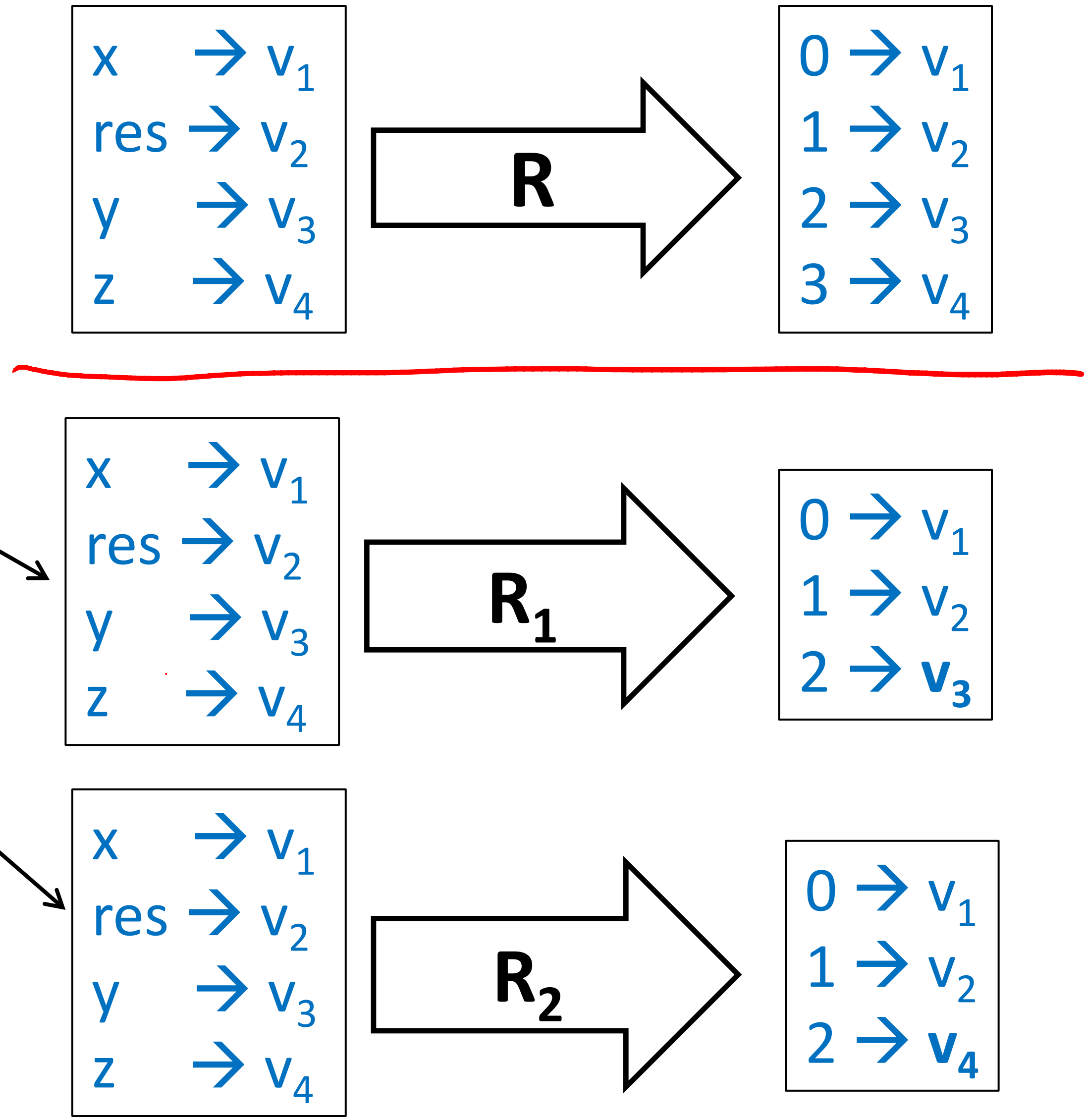
# R for Booleans

- Let the received, the parameters, and local variables, in their order of declaration, be $x_1, x_2 \ldots x_n$
- Then R maps program state like this, where $x_1$ and $x_2$ are integers but $x_3$ and $x_4$ are Booleans:



# R that depends on Program Point

```
def main(x:Int) {
  var res, y, z: Int
  if (x>0) {
    y = x + 1
    res = y
  } else {
    z = -x - 10
    res = z
  }
  ...
}
```




Map y,z to same slot.

Consume fewer slots!

# Packing Variables into Memory

- If values are not used at the same time, we can store them in the same place
- This technique arises in
  - **Register allocation**: store frequently used values in a bounded number of fast registers
  - ‘malloc’ and ‘free’ manual memory management: *free* releases memory to be used for later objects
  - Garbage collection, e.g. for JVM, and .NET as well as languages that run on top of them (e.g. Scala)

# Register Machines

Better for most purposes than stack machines

- closer to modern CPUs (RISC architecture)
- closer to control-flow graphs
- simpler than stack machine

Example: [ARM architecture]

Directly Addressable RAM (large - GB, slow)

A few fast registers

R0,R1,…,R31

# Basic Instructions of Register Machines

$R_i \leftarrow Mem[R_j]$ load

$Mem[R_j] \leftarrow R_i$ store

$R_i \leftarrow R_j * R_k$ compute: for an operation *

Efficient register machine code uses as few loads and stores as possible.

# State Mapped to Register Machine

Both dynamically allocated heap and stack expand

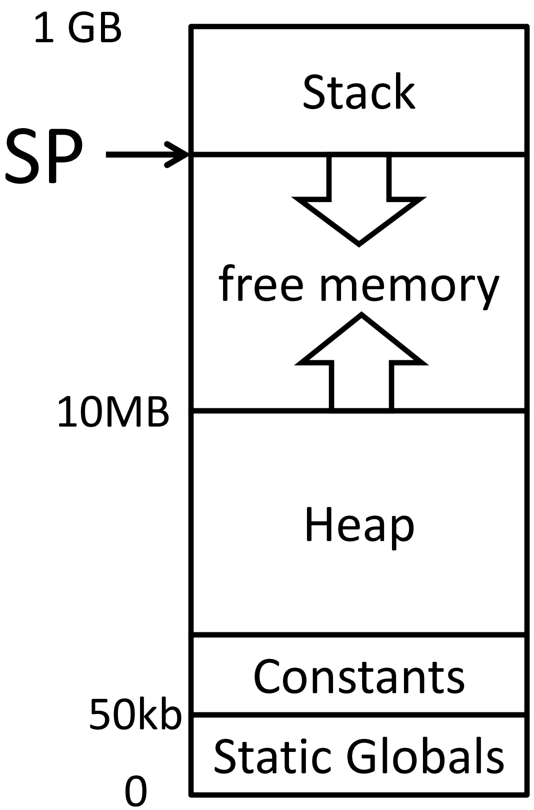
- heap need not be contiguous can request more memory from the OS if needed
- stack grows downwards

Heap is more general:

- Can allocate, read/write, and deallocate, in any order
- Garbage Collector does deallocation automatically
  - Must be able to find free space among used one, group free blocks into larger ones (compaction),…

Stack is more efficient:

- allocation is simple: increment, decrement
- top of stack pointer (SP) is often a register
- if stack grows towards smaller addresses:
  - to allocate N bytes on stack (**push**): **SP := SP - N**
  - to deallocate N bytes on stack (**pop**): **SP := SP + N**




Exact picture may depend on hardware and OS

# JVM vs General Register Machine Code

JVM:

imul

Register Machine:

R1 ← Mem[SP]

SP = SP + 4

R2 ← Mem[SP]

R2 ← R1 * R2

Mem[SP] ← R2

# Register Allocation

# How many variables?

x,y,z,xy,xz,res1

Do we need 6 distinct registers if we wish to avoid load and stores?

```
x = m[0]
y = m[1]
xy = x * y
z = m[2]
yz = y*z
xz = x*z
res1 = xy + yz
m[3] = res1 + xz
```

```
x = m[0]
y = m[1]
xy = x * y
z = m[2]
yz = y*z
y = x*z          // reuse y
x = xy + yz      // reuse x
m[3] = x + y
```

# Idea of Graph Coloring

- Register Interference Graph (RIG):
  - indicates whether there exists a point of time where both variables are live
  - if so, we draw an edge
  - we will then assign different registers to these variables
  - finding assignment of variables to K registers corresponds to coloring graph using K colors!

# Graph Coloring Algorithm

**Simplify**
If there is a node with less than K neighbors, we will always be able to color it!
so we can remove it from the graph
This reduces graph size (it is incomplete)
Every planar can be colored by at most 4 colors (yet can have nodes with 100 neighbors)

**Spill**
If every node has K or more neighbors, we remove one of them
we mark it as node for potential spilling
then remove it and continue

**Select**
Assign colors backwards, adding nodes that were removed
If we find a node that was spilled, we check if we are lucky that we can color it
if yes, continue
if no, insert instructions to save and load values from memory
restart with new graph (now we have graph that is easier to color, we killed a variable)

# Examples