Example to Type Check

```
object World {
  var z : Boolean
  var u : Int
  def f(y : Boolean) : Int {
    z = y
    if (u > 0) {
      u = u – 1
      var z : Int
      z = f(!y) + 3
      z+z
    } else { 0 }
  }
}
```

Γ_0 = {
(z, Boolean),
(u, Int),
(f, Boolean → Int) }

$\Gamma_1 = \Gamma_0 \oplus \{(y, \text{Boolean})\}$

$$\frac{\Gamma_1 \vdash z\colon \text{Boolean} \qquad \Gamma_1 \vdash y\colon \text{Boolean}}{\Gamma_1 \vdash (z{=}y)\colon \text{void}}$$

Exercise:

$$\frac{???}{\Gamma_1 \vdash \text{if } (n > 0) \{ \text{ body } \} \text{ else } \{ 0 \} : \text{Int}}$$

Overloading of Operators

Int $\times$ Int $\rightarrow$ Int

$$\frac{\Gamma \vdash e_1 \colon \text{Int} \qquad \Gamma \vdash e_2 \colon \text{Int}}{\Gamma \vdash (e_1 + e_2) \colon \text{Int}}$$

Not a problem for type checking from leaves to root

String $\times$ String $\rightarrow$ String

$$\frac{\Gamma \vdash e_1 \colon \text{String} \qquad \Gamma \vdash e_2 \colon \text{String}}{\Gamma \vdash (e_1 + e_2) \colon \text{String}}$$

Arrays

Using array as an expression, on the right-hand side

$$\frac{\Gamma \vdash \mathrm{a}:\ \mathrm{Array(T)} \qquad \Gamma \vdash \mathrm{i}:\ \mathrm{Int}}{\Gamma \vdash \mathrm{a[i]}:\mathrm{T}}$$

Assigning to an array

$$\frac{\Gamma \vdash \mathrm{a}:\ \mathrm{Array(T)} \qquad \Gamma \vdash \mathrm{i}:\ \mathrm{Int} \qquad \Gamma \vdash \mathrm{e}:\ \mathrm{T}}{\Gamma \vdash (\mathrm{a[i]}) = \mathrm{e}):\ \mathrm{void}}$$

Example with Arrays

```
def next(a : Array[Int], k : Int) : Int = {
  a[k] = a[a[k]]
}
```

Given Γ = {(a, Array(Int)), (k, Int)}, check $\Gamma \vdash$ a[k] = a[a[k]]: Int

$$\dfrac{\dfrac{\Gamma \vdash \text{a: Array(Int)} \qquad \dfrac{\Gamma \vdash \text{a: Array(Int)} \qquad \Gamma \vdash \text{k: Int}}{\Gamma \vdash \text{a[k]: Int}}}{\Gamma \vdash \text{a[a[k]]:Int}} \qquad \dfrac{\Gamma \vdash \text{a: Array(Int)} \qquad \Gamma \vdash \text{k:Int}}{\Gamma \vdash \text{a[k]: Int}}}{\Gamma \vdash \text{a[k]} = \text{a[a[k]]: Int}}$$

Type Rules (1)

$$\frac{(\mathrm{x}:\ \mathrm{T}) \in \Gamma}{\Gamma \vdash \mathrm{x}:\ \mathrm{T}}$$ variable

$$\frac{}{\mathrm{IntConst(k)}:\ \mathrm{Int}}$$ constant

$$\frac{\Gamma \vdash e_1 : T_1 \ \ldots\ \Gamma \vdash e_n : T_n \qquad \Gamma \vdash f : (T_1 \times \cdots \times T_n \to T)}{\Gamma \vdash f(e_1, \ldots, e_n) : T}$$ function application

$$\frac{\Gamma \vdash e_1 :\ \mathrm{Int} \qquad \Gamma \vdash e_2 :\ \mathrm{Int}}{\Gamma \vdash (e_1 + e_2) :\ \mathrm{Int}}$$ plus $$\frac{\Gamma \vdash e_1 :\ \mathrm{String} \qquad \Gamma \vdash e_2 :\ \mathrm{String}}{\Gamma \vdash (e_1 + e_2) :\ \mathrm{String}}$$

$$\frac{\Gamma \vdash \mathrm{b}:\ \mathrm{Boolean} \qquad \Gamma \vdash e_1 :\ \mathrm{T} \qquad \Gamma \vdash e_2 :\ \mathrm{T}}{\Gamma \vdash (\mathrm{if(b)}\ e_1\ \mathrm{else}\ e_2) :\ \mathrm{T}}$$ if

$$\frac{\Gamma \vdash \mathrm{b}:\ \mathrm{Boolean} \qquad \Gamma \vdash \mathrm{s}:\ \mathrm{void}}{\Gamma \vdash (\mathrm{while(b)\ s}):\ \mathrm{void}}$$ while

$$\frac{(\mathrm{x},\ \mathrm{T}) \in \Gamma \qquad \Gamma \vdash \mathrm{e}:\ \mathrm{T}}{\Gamma \vdash (\mathrm{x{=}e}):\ \mathrm{void}}$$ assignment

Type Rules (2)

$$\frac{\Gamma \vdash \text{e}: \text{T}}{\Gamma \vdash \{\text{e}\}: \text{T}} \qquad \frac{}{\Gamma \vdash \{\}: \text{void}}$$

$$\frac{\Gamma \oplus \{(x, T_1)\} \vdash \{t_2; \ \ldots \ ; t_n\}: \text{T}}{\Gamma \vdash \{\text{var } x : T_1; t_2; \ \ldots \ ; t_n\}: \text{T}}$$

$$\frac{\Gamma \vdash s_1: \text{void} \qquad \Gamma \vdash \{t_2; \ \ldots \ ; t_n\}: \text{T}}{\Gamma \vdash \{s_1; t_2; \ \ldots \ ; t_n\}: \text{T}}$$

block

$$\frac{\Gamma \vdash \text{a}: \text{Array(T)} \qquad \Gamma \vdash \text{i}: \text{Int}}{\Gamma \vdash \text{a[i]}: \text{T}}$$

array use

$$\frac{\Gamma \vdash \text{a}: \text{Array(T)} \qquad \Gamma \vdash \text{i}: \text{Int} \qquad \Gamma \vdash \text{e}: \text{T}}{\Gamma \vdash \text{a[i]} = \text{e}}$$

array assignment

Type Rules (3)

Γ^c - top-level environment of class C

```
class C {
  var x: Int;
  def m(p: Int): Boolean = {…}
}
```

Γ^c = { (x, Int), (m, C $\times$ Int $\rightarrow$ Boolean)}

$$\frac{\Gamma \vdash e : C \quad \Gamma^C \vdash m : T \times T_1 \times \ \ldots \ \times T_n \rightarrow T_{n+1} \qquad \Gamma \vdash e_i : T_i \quad 1 \leq i \leq n}{\Gamma \vdash e.m(e_1, \ \ldots \ , e_n) : T_{n+1}}$$

method invocation

$$\frac{\Gamma \vdash \text{e: C} \qquad \Gamma^C \vdash \text{f: T}}{\Gamma \vdash \text{e.f: T}}$$

field use

$$\frac{\Gamma \vdash \text{e: C} \qquad \Gamma^C \vdash \text{f: T} \qquad \Gamma \vdash \text{x: T}}{\Gamma \vdash (\text{e.f} = \text{x}): \text{ void}}$$

field assignment