Exercise: Balanced Parentheses

Show that the following balanced parentheses grammar is ambiguous (by finding two parse trees for some input sequence) and find unambiguous grammar for the same language.

$B ::= \varepsilon \mid (B) \mid B\ B$

Dangling Else

The dangling-else problem happens when the conditional statements are parsed using the following grammar.

S ::= S **;** S

S ::= id **:=** E

S ::= **if** E **then** S

S ::= **if** E **then** S else S

Find an unambiguous grammar that accepts the same conditional statements and matches the else statement with the nearest unmatched if.

Left Recursive and Right Recursive

We call a production rule “left recursive” if it is of the form

A ::= A p

for some sequence of symbols p. Similarly, a "right-recursive" rule is of a form

A ::= q A

Is every context free grammar that contains both left and right recursive rule, for a some nonterminal A, ambiguous?

Transforming Grammars into Chomsky Normal Form

1) To parse them using CYK Algorithm
2) To simplify them

Why Parse General Grammars

- Can be difficult or impossible to make grammar unambiguous

- Some inputs are more complex than simple programming languages
  - mathematical formulas:
    x = y /\ z   ?   (x=y) /\ z   x = (y /\ z)
  - future programming languages
  - natural language:

    I saw the man with the telescope.

Ambiguity

1)

2)

I saw the man with the telescope.

CYK Parsing Algorithm

C:

John **Cocke** and Jacob T. Schwartz (1970). Programming languages and their compilers: Preliminary notes. Technical report, Courant Institute of Mathematical Sciences, New York University.

Y:

Daniel H. **Younger** (1967). Recognition and parsing of context-free languages in time n^3. *Information and Control* 10(2): 189–208.

K:

T. **Kasami** (1965). An efficient recognition and syntax-analysis algorithm for context-free languages. Scientific report AFCRL-65-758, Air Force Cambridge Research Lab, Bedford, MA.

Two Steps in the Algorithm

1) Transform grammar to normal form

   called Chomsky Normal Form

   (Noam Chomsky, mathematical linguist)

2) Parse input using transformed grammar

   dynamic programming algorithm

"a method for solving complex problems by breaking them down into simpler steps.
It is applicable to problems exhibiting the properties of overlapping subproblems" (>WP)

Balanced Parentheses Grammar

Original grammar G

$S \rightarrow$ “” | (S) | S S

Modified grammar in Chomsky Normal Form:

$S \rightarrow$ “” | S’ $\leftarrow$ if "" $\in L(G)$

$S' \rightarrow N_(\, N_{S)} \mid N_(\, N_) \mid S' \, S'$
$N_{S)} \rightarrow S' \, N_)$ } Rules $\underbrace{N}_{\text{nonterminals}} \rightarrow \underbrace{N_1 N_2}_{\text{nonterminals}}$

$N_(\rightarrow$ (
$N_) \rightarrow$) } Rules $N \rightarrow t$ nonterminal terminal

- Terminals: ( ) Nonterminals: S S’ $N_{S)}$ $N_)$ $N_($

nonterminal with funny name

Idea How We Obtained the Grammar

$S \rightarrow \underbrace{(}\ \underbrace{S\ )}$

because S can be empty
but S' cannot

$S' \rightarrow N_(\quad N_{S)} \quad | \ \overbrace{N_(\ N_)}$

$N_(\rightarrow ($

$N_{S)} \rightarrow S' \ N_)$

$N_) \rightarrow)$

Chomsky Normal Form transformation
can be done fully mechanically

Transforming to Chomsky Form

Steps:

1. remove unproductive symbols
2. remove unreachable symbols
3. remove epsilons (no non-start nullable symbols)
4. remove single non-terminal productions X::=Y
5. transform productions w/ more than 3 on RHS
6. make terminals occur alone on right-hand side

1) Unproductive non-terminals

How to compute them?

What is funny about this grammar:

stmt ::= identifier := identifier
| while (expr) stmt
| if (expr) stmt else stmt

expr ::= term + term | term – term

term ::= factor * factor

factor ::= (expr)

There is no derivation of a sequence of tokens from expr

Why? In every step will have at least one expr, term, or factor

If it cannot derive sequence of tokens we call it *unproductive*

1) Unproductive non-terminals

- Productive symbols are obtained using these two rules (what remains is unproductive)
  - Terminals (tokens) are productive
  - If $X ::= s_1\ s_2\ \ldots\ s_n$ is rule and each s_i is productive then X is productive

```
stmt ::=  identifier := identifier
        | while (expr) stmt
        | if (expr) stmt else stmt
expr ::= term + term | term – term
term ::= factor * factor
factor ::= ( expr )
program ::= stmt | stmt program
```

Delete unproductive symbols.

Will the meaning of top-level symbol (program) change?

2) Unreachable non-terminals

What is funny about this grammar with starting terminal 'program'

program ::= stmt | stmt program
stmt ::= assignment | whileStmt
assignment ::= expr = expr
ifStmt ::= if (expr) stmt else stmt
whileStmt ::= while (expr) stmt
expr ::= identifier

No way to reach symbol 'ifStmt' from 'program'

2) Computing unreachable non-terminals

What is funny about this grammar with starting terminal 'program'

program ::= stmt | stmt program
stmt ::= assignment | whileStmt
assignment ::= expr = expr
ifStmt ::= if (expr) stmt else stmt
whileStmt ::= while (expr) stmt
expr ::= identifier

What is the general algorithm?

2) Unreachable non-terminals

- Reachable terminals are obtained using the following rules (the rest are unreachable)
  - starting non-terminal is reachable (program)
  - If $X ::= s_1\ s_2 \dots s_n$ is rule and X is reachable then each non-terminal among $s_1\ s_2 \dots s_n$ is reachable

Delete unreachable symbols.

Will the meaning of top-level symbol (program) change?

3) Removing Empty Strings

Ensure only top-level symbol can be nullable

program ::= stmtSeq
stmtSeq ::= stmt | stmt ; stmtSeq
stmt ::= “” | assignment | whileStmt | blockStmt
blockStmt ::= { stmtSeq }
assignment ::= expr = expr
whileStmt ::= while (expr) stmt
expr ::= identifier

How to do it in this example?

3) Removing Empty Strings - Result

program ::= “” | stmtSeq

stmtSeq ::= stmt| stmt ; stmtSeq |

| ; stmtSeq | stmt ; | ;

stmt ::= assignment | whileStmt | blockStmt

blockStmt ::= { stmtSeq } | { }

assignment ::= expr = expr

whileStmt ::= while (expr) stmt

whileStmt ::= while (expr)

expr ::= identifier

3) Removing Empty Strings - Algorithm

- Compute the set of nullable non-terminals
- Add extra rules
  - If $X ::= s_1\ s_2\ \dots\ s_n$ is rule then add new rules of form
    $X ::= r_1\ r_2\ \dots\ r_n$ 2^n

  where r_i is either s_i or, if s_i is nullable then

  r_i can also be the empty string (so it disappears)
- Remove all empty right-hand sides
- If starting symbol S was nullable, then introduce a new start symbol S' instead, and add rule S' ::= S | ""

3) Removing Empty Strings

- Since stmtSeq is nullable, the rule

  blockStmt ::= { stmtSeq }

  gives

  blockStmt ::= { stmtSeq } | { }

- Since stmtSeq and stmt are nullable, the rule

  stmtSeq ::= stmt | stmt ; stmtSeq

  gives

  stmtSeq ::= stmt | stmt ; stmtSeq
  | ; stmtSeq | stmt ; | ;

4) Eliminating single productions

- Single production is of the form

  X ::=Y

where X,Y are non-terminals

```
program ::= stmtSeq
stmtSeq ::= stmt
          | stmt ; stmtSeq
stmt ::= assignment | whileStmt
assignment ::= expr = expr
whileStmt ::= while (expr) stmt
```

4) Eliminate single productions - Result

- Generalizes removal of epsilon transitions from non-deterministic automata

program ::= expr = expr | while (expr) stmt
| stmt ; stmtSeq
stmtSeq ::= expr = expr | while (expr) stmt
| stmt ; stmtSeq
stmt ::= expr = expr | while (expr) stmt
assignment ::= expr = expr
whileStmt ::= while (expr) stmt

now unreachable

4) "Single Production Terminator"

- If there is single production

  X ::=Y put an edge (X,Y) into graph

- If there is a path from X to Z in the graph, and there is rule $Z ::= s_1\ s_2\ \ldots\ s_n$ then add rule

$$X ::= s_1\ s_2\ \ldots\ s_n$$

At the end, remove all single productions.

program ::= expr = expr | while (expr) stmt
                | stmt ; stmtSeq
stmtSeq ::= expr = expr | while (expr) stmt
                | stmt ; stmtSeq
stmt ::= expr = expr | while (expr) stmt

5) No more than 2 symbols on RHS

stmt ::= while $\underbrace{(\underbrace{\text{expr}\ \underbrace{)\ \text{stmt}}_{3}}_{2}}_{1}$

becomes

stmt ::= while stmt_1

stmt_1 ::= (stmt_2

stmt_2 ::= expr stmt_3

stmt_3 ::=) stmt

6) A non-terminal for each terminal

$stmt ::= \text{while} \; (expr) \; stmt$

becomes

$stmt ::= N_{while} \; stmt_1$

$stmt_1 ::= N_{(} \; stmt_2$

$stmt_2 ::= expr \; stmt_3$

$stmt_3 ::= N_{)} \; stmt$

$N_{while} ::= \text{while}$

$N_{(} ::= ($

$N_{)} ::=)$

Parsing using CYK Algorithm

- Transform grammar into Chomsky Form:
  1. remove unproductive symbols
  2. remove unreachable symbols
  3. remove epsilons (no non-start nullable symbols)
  4. remove single non-terminal productions X::=Y
  5. transform productions of arity more than two
  6. make terminals occur alone on right-hand side

  Have only rules X ::= Y Z, X ::= t, and possibly S ::= “”
- Apply CYK dynamic programming algorithm

Dynamic Programming to Parse Input

Assume Chomsky Normal Form, 3 types of rules:

$S \rightarrow$ “” | S'	(only for the start non-terminal)
$N_j \rightarrow t$	(names for terminals)
$N_i \rightarrow N_j\ N_k$	(just **2** non-terminals on RHS)

Decomposing long input:

$$\overbrace{\overbrace{(\ (\ (\ )\ (\ )\ )\ (\ )\ )}^{N_j}\ \overbrace{(\ (\ )\ )}^{N_k}}^{N_i}$$

find all ways to parse substrings of length 1,2,3,...

Parsing an Input

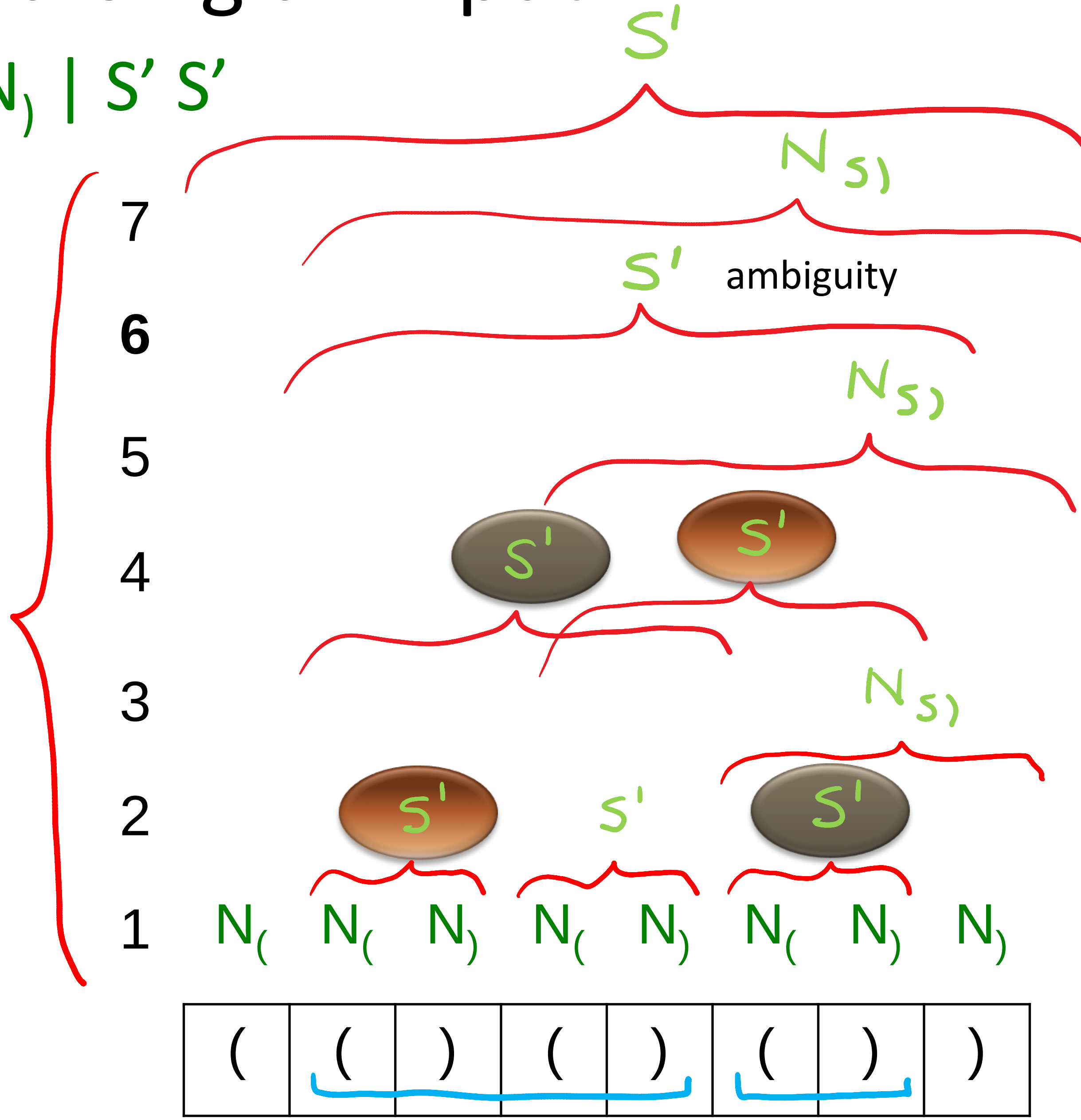
$S' \rightarrow N_(\, N_{S)} \mid N_(\, N_) \mid S' \, S'$

$N_{S)} \rightarrow S' \, N_)$

$N_(\rightarrow ($

$N_) \rightarrow)$

substring length



Algorithm Idea

$S' \rightarrow S'\ S'$

w_{pq} – substring from p to q

d_{pq} – all non-terminals that could expand to w_{pq}

Initially d_{pp} has $N_{w(p,p)}$

key step of the algorithm:

if $X \rightarrow Y\ Z$ is a rule,

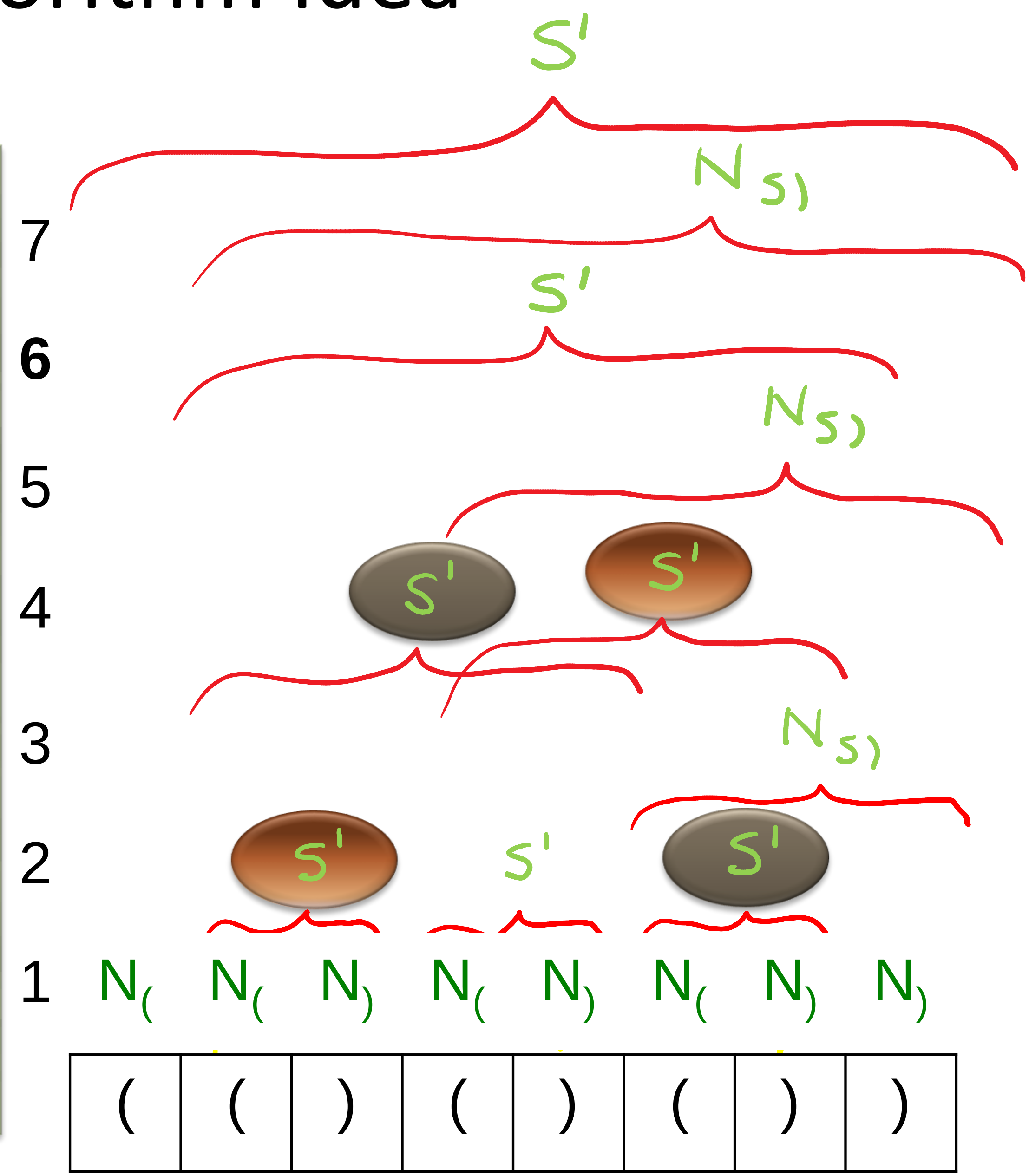
Y is in $d_{p\,r}$, and

Z is in $d_{(r+1)q}$

then put X into d_{pq}

$(p \leq r < q)$,

in increasing value of (q-p)



Algorithm

```
INPUT:  grammar G in Chomsky normal form
        word w to parse using G
OUTPUT: true iff (w in L(G))
N = |w|
var d : Array[N][N]
for p = 1 to N {
  d(p)(p) = {X | G contains X->w(p)}
  for q in {p + 1 .. N} d(p)(q) = {} }
for k = 2 to N // substring length
 for p = 0 to N-k // initial position
  for j = 1 to k-1 // length of first half
    val r = p+j-1; val q = p+k-1;
    for (X::=Y Z) in G
     if Y in d(p)(r) and Z in d(r+1)(q)
       d(p)(q) = d(p)(q) union {X}
return  S in d(0)(N-1)
```

What is the running time as a function of grammar size and the size of input?

O(   )

	p			r		q	
(	(	)	(	)	(	)	)

Parsing another Input

$S' \rightarrow N_{(}\ N_{S)}\ |\ N_{(}\ N_{)}\ |\ S'\ S'$

$N_{S)} \rightarrow S'\ N_{)}$

$N_{(} \rightarrow ($

$N_{)} \rightarrow)$

substring length

7								
6								
5								
4								
3								
2								
1	$N_{(}$	$N_{)}$	$N_{(}$	$N_{)}$	$N_{(}$	$N_{)}$	$N_{(}$	$N_{)}$
	(	)	(	)	(	)	(	)

Number of Parse Trees

- Let w denote word ()()()
  - it has two parse trees
- Give a lower bound on number of parse trees of the word w^n (n is positive integer)

  w^5 is the word

  ()()() ()()() ()()() ()()() ()()()

- CYK represents all parse trees compactly
  - can re-run algorithm to extract first parse tree, or enumerate parse trees one by one