

With additional typing rules, the following program type-checks according to the rules given in the course:

```
Class C {
  int f(int x) {
    return 1+f(x);
  }
}
```

The new typing rules are:

$$\frac{\Gamma \vdash x : Int \quad \Gamma \vdash y : Int}{\Gamma \vdash (x + y) : Int}$$

Here is the derivation tree:

$$\frac{\frac{\frac{\frac{\frac{\vdash 1 : Int}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash 1 : Int}}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash x : Int}}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash 1 + f(x) : Int}}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash f(x) : Int}}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash f : Int \rightarrow Int}}{\{(f, Int \rightarrow Int), (x, Int)\} \vdash return\ 1 + f(x) : Int}}{\{(f, Int \rightarrow Int)\} \vdash int\ f(int\ x)\ \{return\ 1 + f(x);\} : Void}}$$