

Recall the example from the course:

```

class World {
  boolean z;
  int u;
  int f(boolean y) {
    z = y;
    if (u > 0) {
      int z;
      z = f(u) + 3;
      return z+z;
    } else {
      return 0;
    }
  }
}

```

Let Γ_1 be the environment obtained from the *Class* declaration (with methods and global variables):

$\Gamma_0 = (z : Bool), (u : Int), (f : Bool \rightarrow Int)$

Let us type check the code inside the method **f**: we add a binding for **f**'s parameter in the environment:

$\Gamma_1 = \Gamma_0 \oplus (y : Bool)$ We obtain the tree $Tree_0$:

$$\begin{array}{c}
 \text{Tree}_0 : \\
 \frac{\frac{\frac{(z, Bool) \in \Gamma_1}{\Gamma_1 \vdash z : Bool} \quad \frac{(y, Bool) \in \Gamma_1}{\Gamma_1 \vdash y : Bool}}{\Gamma_1 \vdash z = y : void} \quad \frac{\frac{\frac{(u, Int) \in \Gamma_1}{\Gamma_1 \vdash u : Int} \quad \frac{\vdash 0 : Int}{\Gamma_1 \vdash 0 : Int}}{\Gamma_1 \vdash u > 0 : Bool} \quad \frac{\text{Tree}_1}{\Gamma_1 \vdash \{int\ z; z = f(u) + 3; return\ z + z;\} : Int} \quad \frac{\frac{\vdash 0 : Int}{\Gamma_1 \vdash 0 : Int}}{\Gamma_1 \vdash \{return\ 0;\} : Int}}{\Gamma_1 \vdash if\ (u > 0)\ \{int\ z; z = f(u) + 3; return\ z + z;\}\ else\ \{return\ 0;\} : Int}}{\Gamma_1 \vdash z = y; if\ (u > 0)\ \{int\ z; z = f(u) + 3; return\ z + z;\}\ else\ \{return\ 0;\} : Int}
 \end{array}$$

Unfortunately, our program is not type correct. We can see it trying to complete the derivation ($Tree_1$):

$$\begin{array}{c}
 \text{Tree}_1 : \\
 \frac{\frac{\frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int} \quad \frac{\frac{(u, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash u : Int}, f : Int \rightarrow Int \quad \frac{\vdash 3 : Int}{\Gamma_1 \oplus (z, Int) \vdash 3 : Int}}{\Gamma_1 \oplus (z, Int) \vdash f(u) + 3 : Int}}{\Gamma_1 \oplus (z, Int) \vdash z = f(u) + 3 : void} \quad \frac{\frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int} \quad \frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int}}{\Gamma_1 \oplus (z, Int) \vdash z + z : Int}}{\Gamma_1 \oplus (z, Int) \vdash \{z = f(u) + 3; return\ z + z;\} : Int}
 \end{array}$$

If we change the call $f(u)$ in $f(u > 0)$, we can type our program:

$$\begin{array}{c}
 \text{Tree}_1 : \\
 \frac{\frac{\frac{(u, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash u : Int} \quad \frac{\vdash 0 : Int}{\Gamma_1 \oplus (z, Int) \vdash 0 : Int} \quad (f, Bool \rightarrow Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash u > 0 : Bool, f : Bool \rightarrow Int} \quad \frac{\vdash 3 : Int}{\Gamma_1 \oplus (z, Int) \vdash 3 : Int}}{\Gamma_1 \oplus (z, Int) \vdash f(u > 0) + 3 : Int} \quad \frac{\frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int} \quad \frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int}}{\Gamma_1 \oplus (z, Int) \vdash z + z : Int}}{\Gamma_1 \oplus (z, Int) \vdash z = f(u > 0) + 3 : void} \\
 \frac{\frac{(z, Int) \in \Gamma_1 \oplus (z, Int)}{\Gamma_1 \oplus (z, Int) \vdash z : Int} \quad \frac{\Gamma_1 \oplus (z, Int) \vdash z = f(u > 0) + 3 : void}{\Gamma_1 \oplus (z, Int) \vdash \{z = f(u > 0) + 3; return z + z; \} : Int}}{\Gamma_1 \oplus (z, Int) \vdash z : Int}
 \end{array}$$

Which rule did we add to the ones presented in the course in order to derive those trees?